PHYSICAL REVIEW C VOLUME 35, NUMBER 2 FEBRUARY 1987

Effect of πd channel coupling on the medium energy nucleon-nucleon interaction

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(Received 10 September 1986)

The effect of the coupling to the πd channel on the intermediate energy NN scattering is studied using the recent NN$\rightarrow$NN and NN$\rightarrow\pi$d partial wave analyses as well as the coupled NN$\rightarrow\pi$NN theory. Contrary to some recent claims, it is found that this coupling is important in the $^1D_2$ and $^3F_3$ partial waves in which the intermediate NΔ configuration dominates.

I. INTRODUCTION

Because of a substantial increase in good quality NN data available from meson factories and from the SATURNE accelerator there has been an active effort, in the past several years, to extend the theory of the nucleon-nucleon interaction into the medium energy region: 300 MeV$\leq T_N^{lab}\leq$1 GeV. Here the dominant source of the inelasticity is the single pion production through the NΔ state, so the basic ingredient in any model is the implementation of the transition NN$\leftrightarrow$NΔ. This is done either in the two-body coupled channel equation,[1-4] its extension to implement relativity using the Bethe-Salpeter equation,[5] or in the semirelativistic or relativistic three-body formulation.[6-12] Independent of whichever of these approaches is adopted, and also independent of whether the short range NN interaction due to heavy meson exchanges is included or not, the coupling to the NΔ state enables one to qualitatively understand the gross features of the $^1D_2$ and $^3F_3$ partial waves which are most strongly dominated by the intermediate NΔ configurations. We want to remind the reader that these two partial waves are also the candidates for indicating the possible dibaryon resonances.[13]

By tuning the heavy meson parameters as well as the cutoff at the πNΔ vertex, it is possible to account for the phase shift and inelasticity in these partial waves rather well. Yet, quantitatively speaking, none of the existing models offers a satisfactory result. Restricting ourselves to these channels, the problem is the lack of sufficient inelasticity; specifically, up to $\sim$500 MeV (and in some cases even up to 1 GeV) in $^1D_2$, and up to 1 GeV in $^3F_3$.

In most of the theoretical models, the intermediate states are restricted to those with an interacting πN pair and the spectator nucleon for which the former is approximated by some number of nucleon isobars. Here, as just mentioned above, the Δ resonance is the principal isobar that must be included. However, the other type of intermediate state which consists of an interacting NN pair plus the spectator pion had not drawn much attention for some time. In an attempt to see whether the inclusion of the latter could fill the gap between existing theories and the data (to be more precise, "data" should be replaced by "the result of the NN phase shift analyses"),[14-17] Lee and Matsuyama[18] (LM), in their unitary πNN model, included the *coupling to the pion-deuteron channel* (CPDC). They found that the effect was strongest in the $^1D_2$, but even this turned out to be too small to lead to any possible improvement in the NN models. Their conclusion was thus that the ordinary two-body coupled channel models are only appropriate for the medium energy NN interactions given the present discrepancy between theory and data.

In a recent paper[19] a point of view quite opposite to that of LM is presented: By exploiting the recent NN$\rightarrow\pi$d amplitude analyses of Bugg,[20] van Faassen and Tjon (vFT) have concluded that the deficiency in the inelasticity as obtained from the standard two-body coupled channel models may be cured by introducing the CPDC at least in the $^1D_2$ channel. So the correlated NN + pion states, in general, could be an important ingredient in the proper description of the medium energy NN interaction.

At this point, it is important to reach a definite resolution of this controversial issue. We have had for some time a point of view similar to what vFT have reached by a more detailed study. So, we think it worthwhile to give an account of our own findings.

II. FACTS FROM DATA AND AMPLITUDE ANALYSES

Here we shall study in general terms the possible importance of the NN$\rightarrow\pi$d channel on the medium energy NN processes. First of all, as a qualitative measure for this we compare the total cross sections, σ(pp$\rightarrow\pi^+$d) and σ(pp$\rightarrow$single π^+ production). According to the convenient parametrization of a vast amount of the NN single pion production cross section data by VerWest and Arndt,[21] the pp$\rightarrow\pi^+$d constitutes $\gtrsim$50% of the pp pion production cross section up to $T_p^{lab}\sim$500 MeV. Above this energy the πNN three-body final state becomes dominant over the π^+d, although the latter gives its maximum cross section of $\sim$3 mb around 600 MeV. Thus it is natural to expect that the CPDC should show up in the NN scattering up to 1 GeV.

Now, the next step is to see which NN partial waves may possibly be influenced by the CPDC. For this purpose we look into the partial wave decomposition of the experimental cross section:

$$\sigma(\mathrm{pp}\to\pi^+\mathrm{d})=\sum_{l,l',s,J}(2J+1)^J\sigma_{ls,l'}\ , \qquad (1)$$

where J is the total angular momentum, l,s are the orbital and spin angular momenta of the initial NN state, while l' is the orbital angular momentum of the final πd system. Up until rather recently this decomposition was impossible to do in a model independent manner, but an accumulation of numerous data in pp$\to\pi^+$d spin observables in recent years enabled the initiation of the NN$\to\pi$d amplitude analysis. There are two such independent analyses currently available.[20,22] After some refinements incorporated in the method of analysis, those two independent works now produce rather similar results, the only noticeable difference being the phases of the partial wave amplitudes, which are difficult to fix for nonelastic processes like this. We have elected to use the result of Ref. 22 because the continuity in energy is used as part of the constraint, a very useful tool in view of the fact that the data are not sufficiently rich in quantity. This analysis gives the partial wave decomposition of the cross section as shown in Fig. 1. Clearly, the partial wave initiated from NN $^1D_2$ turns out, by far, to be the dominant piece, while the second most important piece is what starts out with the initial NN $^3F_3$ channel. The latter becomes noticeable for $T_p^{\mathrm{lab}}\geqq 550$ MeV. Other minor partial waves added to these two channels constitute the observed cross section,

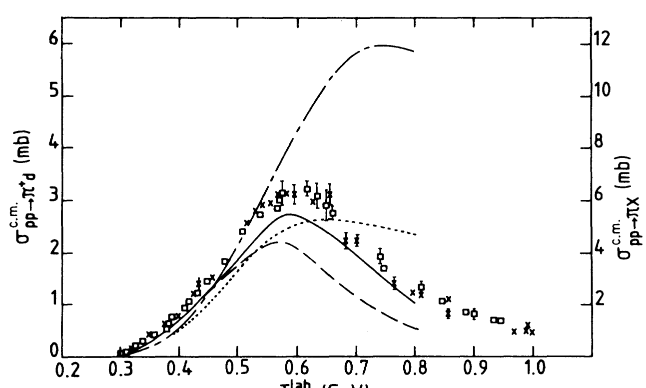

FIG. 1. Partial wave decomposition of single pion production cross sections in pp collisions as a function of proton incident kinetic energy. The dashed and solid curves correspond to the contributions from $^1D_2$ and $^1D_2+{}^3F_3$ partial waves for pp$\to\pi^+$d (left hand scale), using the result of Ref. 22. The dotted and dashed-dotted curves show the contributions from the same partial wave sets, respectively, for the pp$\to\pi$X reaction cross sections (right hand scale), according to Ref. 15. The data (Ref. 27) come from pp$\to\pi^+$d ($\times$, Ref. 28) and π^+d$\to$pp ($\square$, Ref. 29) measurements.

of which the important ones are the $^3P_1$ and $^3P_2$ related contributions at lower and higher energies, respectively. In Fig. 1 we have also included the curves for the partial wave decomposed pp$\to\pi$X (X=NN,d) cross section for $^1D_2$ and $^3F_3$ calculated from the amplitude analysis of Ref. 15. Noting that in Fig. 1 the cross section scale for this quantity is half of that for pp$\to\pi^+$d, we see immediately that about 50% of the pion production cross section in $^1D_2$ for $T_N^{\mathrm{lab}}\leqq 600$ MeV is carried by the final πd channel. On the other hand, the pion production from the $^3F_3$ wave is shown to be completely dominated by the πNN three-body final states.

This $^1D_3$-$^3F_3$ dominance was, of course, anticipated from the coupling to the NΔ state, which prevails in the region of energy under consideration. In fact, any reasonable model for the NN$\to\pi$d reaction including the intermediate Δ excitation would give this trend (not necessarily quantitatively, though!) which we have confirmed here without any recourse to a specific model. The next step is then to see in more quantitative terms how important the CPDC may be in the NN scattering amplitude.

III. EXAMINATION OF THE WORKS OF LEE-MATSUYAMA AND van FAASSEN–TJON

As mentioned in the Introduction, LM (Ref. 18) found that even in the $^1D_2$ NN wave the CPDC turned out to be very small. They reached this conclusion by comparing the phase parameters defined by Arndt *et al.*[15] (denoted here as δ_A and ρ) by turning the CPDC on and off. For uncoupled NN partial waves it is physically more reasonable to use the conventional phase shift (δ) and inelasticity (ρ) parameters as defined through the partial wave S matrix,

$$S(Jls)=\eta\exp(2i\delta)\ . \qquad (2)$$

In particular, we want to emphasize that in the parametrization of Ref. 15 both δ_A and ρ depend on δ and η, so, for example, ρ is no longer the exact measure for the inelasticity. We therefore transform the LM values into this convention along with the parameters of Ref. 15. Figure 2 shows the results in the $^1D_2$ partial wave, the only one discussed in their publication (as the CPDC effect is far smaller in other partial waves in their calculation). Trivially, we confirm the negligible influence of the CPDC within their calculation in terms of these preferred variables.

A close look at the figure reveals that up to about $T_p^{\mathrm{lab}}\sim 700$ MeV there is less inelasticity in this partial wave once the CPDC is introduced. Although the difference between the result with and without the CPDC is very small, this seems somewhat troublesome at first glance: One would naively expect an increase in inelasticity with the CPDC, as this introduces an additional final state, viz., πd, which contributes incoherently to the NN inelastic cross section. In order to see if there is any possible inconsistency present in the LM equation, we have examined its unitarity structure, which has been found to be correct (see the Appendix).

In fact, in the course of examining the LM equation we have become aware that the naive expectation mentioned above does not always hold. The reason is fairly trivial,

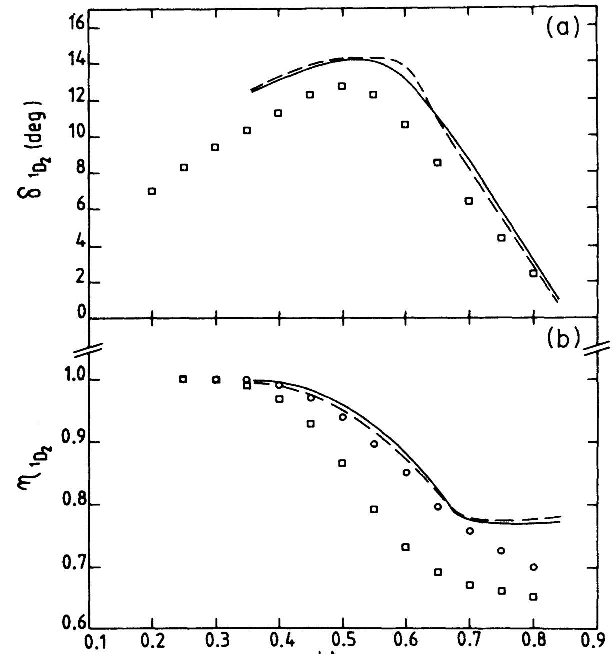

FIG. 2. NN $^1D_2$ (a) phase shift and (b) inelasticity as a function of proton incident energy. The curves have been obtained from Ref. 18 after adequate transformations (see Sec. III) and show the results with (—) and without (— — —) coupling to the πd channel. The square points are from the VPI energy dependent phase shift analysis (Ref. 15) and the circles [in (b)] correspond to Eq. (8) exploiting Refs. 15 and 22.

but its explicit account may be of use for our later purposes. So let us now write the unitarity relation for the NN initiated processes below the two pion production threshold in terms of the partial wave S matrices (we restrict our discussion to the NN uncoupled channels),

$$\eta(Jls)^2 = |S^{\rm el}(Jls)|^2 = 1 - \sum_g |S^{\pi NN}(g,Jls)|^2 - \sum_f |S^{\pi d}(f,Jls)|^2 , \quad (3)$$

where $S^{\rm el}(Jls)$, $S^{\pi NN}(g,Jls)$, and $S^{\pi d}(f,Jls)$ are the partial wave S matrices for NN$\rightarrow$NN (elastic), NN$\rightarrow\pi$NN, and NN$\rightarrow\pi$d reactions, respectively. The summation is to be taken over various possible final state quantum numbers g and f. In the calculation where the CPDC is not included, this relation should be replaced by

$$\eta_0(Jls)^2 = |S_0^{\rm el}(g,Jls)|^2 = 1 - \sum |S_0^{\pi NN}(g,Jls)|^2 . \quad (4)$$

Notice that in Eq. (4) the NN$\rightarrow\pi$d contribution disappears and η as well as the S-matrix elements acquire the subscript "0." This subscript is meant to signify that there is no pion + correlated NN($^3S_1$-$^3D_1$) contribution in either the intermediate or final states. Then we get

$$\eta(Jls)^2 = \eta_0(Jls)^2 + \left\{ \sum [\,|S_0^{\pi NN}(g,Jls)|^2 - |S^{\pi NN}(g,Jls)|^2] - \sum |S^{\pi d}(f,Jls)|^2 \right\} . \quad (5)$$

Therefore, if the second term (inside the curly bracket) on the right hand side of (5) becomes positive, the inelasticity will decrease with the introduction of the CPDC. In other words, the LM result may come out if the NN($^3S_1$-$^3D_1$) correlation in the presence of spectator pion reduces the modulus of the NN$\rightarrow\pi$NN partial wave S matrix to the extent that it overrides the effect of the NN$\rightarrow\pi$d channel.

We now discuss the procedure of vFT.[19] It consists of assessing the effect of the CPDC through

$$\eta^{\rm vFT}(Jls)^2 = \eta_0(Jls)^2 - \sum |S^{\pi d}(f,Jls)|^2 , \quad (6)$$

where $\eta^{\rm vFT}(Jls)$ is identified as the physical inelasticity to be compared with the data.[15] Here, η_0 is taken from their own Bethe-Salpeter calculation, while $S^{\pi d}(f,Jls)$ comes from the pp$\rightarrow\pi^+$d partial wave analysis of Bugg.[20] Since the NN $^1D_2$ initiated partial cross section is large up to $\sim$600 MeV, as discussed in the preceding section, the outcome is the sizable increase in the inelasticity getting closer to the data,[15] as in Fig. 3(a), whereas for the $^3F_3$ channel a corresponding increase is found to be far smaller, see Fig. 3(b). This has made vFT conclude that the

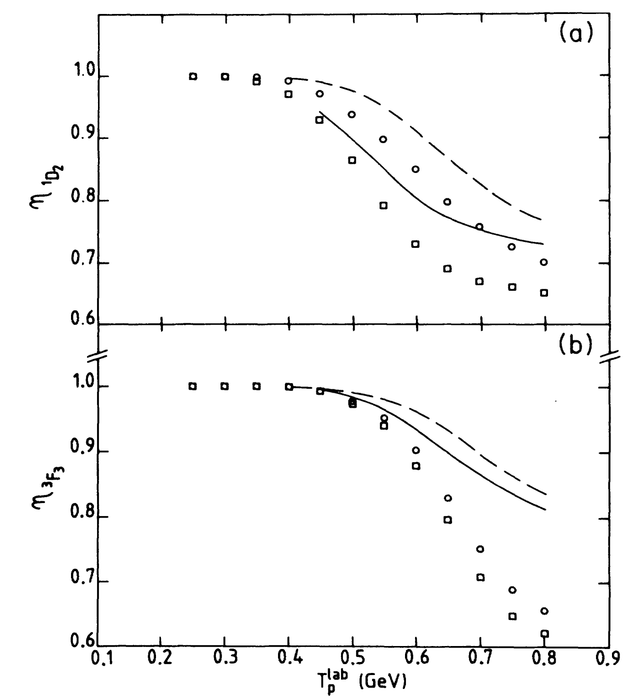

FIG. 3. Inelasticity parameters for (a) $^1D_2$ and (b) $^3F_3$ partial waves from Ref. 19. The curves show $\eta^{\rm vFT}$ (—) and η_0 (— — —), according to Eq. (6). The points are the same as in Fig. 2(b).

coupling of $\pi+(\mathrm{NN})$ states, particularly to CPDC, may play an important role in the medium energy NN processes. Note that since they have not included the CPDC in their dynamical NN equation, no information concerning the change in the phase shifts (δ) can be obtained with this procedure. By comparing Eqs. (3) and (5), it is clear that this conclusion is reached by implicitly assuming

$$\sum |S_0^{\pi\mathrm{NN}}(g,Jls)|^2=\sum |S^{\pi\mathrm{NN}}(g,Jls)|^2 , \qquad (7)$$

which means that the CPDC only introduces the extra NN→πd channel but does not influence the NN→πNN process through the intermediate- and final-state NN($^3S_1$-$^3D_1$) correlations. This assumption will be examined in the next section.

Now we are in a position to start investigating as to which of the two observations discussed above seems more reasonable. In order to still stay away from using specific theoretical models for this purpose, we shall introduce the following quantity for $^1D_2$:

$$\begin{aligned}\eta'(Jls)^2&=\eta(Jls)^2+\sum |S^{\pi\mathrm{d}}(f,Jls)|^2\\ &=\eta_0(Jls)^2+\sum [\,|S_0^{\pi\mathrm{NN}}(g,Jls)|^2\\ &\qquad -|S^{\pi\mathrm{NN}}(g,Jls)|^2] ,\end{aligned} \qquad (8)$$

which is calculated using $\eta(Jls)$ from Arndt *et al.*[15] and $S^{\pi\mathrm{d}}(f,Jls)$ from Ref. 22. This quantity is plotted as circles in Figs 2(b) and 3.

From Fig. 2(b) we find that η' for $^1D_2$ is very close to η and η_0 calculated by LM up to, say, 700 MeV, which from Eq. (8) tells us that (i) their NN→πNN partial wave S matrices, and thus the corresponding cross section, changes very little upon introducing the CPDC, and (ii) the NN→πd S matrices and therefore $\sigma(\mathrm{NN}\to\pi\mathrm{d})$, once calculated explicitly within their model, must be extremely small, totally underestimating the data. As for Fig. 3, this figure shows that within the vFT calculation either η_0 has come out too large or Eq. (7), as implicitly assumed, is not really valid.

IV. A MODEL CALCULATION WITHIN THE COUPLED πNN-NN EQUATIONS

In this section we shall present our result that has come out of our coupled πNN-NN equations in an attempt to simultaneously describe the processes πd→πd, πd↔NN, and NN→NN. Details of the model have been discussed extensively in a separate article,[23] to which the reader is referred. In particular, we have shown that it was necessary to include the heavy meson exchange in the two body NN sector in order to reproduce the behavior of the $^1D_2$ and $^3F_3$ NN phase shifts at energies above ~500 MeV. For the present work we have adopted for the heavy meson parameters a version (TAB4 in Ref. 23) of the Bonn potential[24] in which we have changed values of some meson coupling parameters in order to be consistent with our nonstatic one pion exchange (OPE) transition interactions: NN→NN, NN→NΔ, etc. For the input two-body interactions in the πNN three-body states, we have included πN P_{33} and P_{11} (pole and nonpole) contributions, and NN $^3S_1$-$^3D_1$ partial waves which take care of the CPDC.

To see the general features of our present model calculation, we first show the partial wave contribution to the integrated NN→πd cross section from the NN $^1D_2$ and $^3F_3$ initiated channels. This is depicted in Fig. 4, which may be compared with Fig. 1, where the same quantity obtained from the partial wave amplitude analysis[22] is shown. The result is quite satisfactory, except that the energy where the cross section is peaked turns out to be lower than what the data show, which is found to persist even after including the remaining partial waves. This is due to the fact that our $^1D_2$ related cross section is somewhat overestimated, while the contribution from the $^3F_3$ initiated state is underestimated. We note that the latter trend is the mere reflection of the lack of strength in the NN spin triplet related channel, which is quite universal among the existing theoretical models. See, for example, the discussion in Ref. 23.

Next, we look into the NN partial wave parameters δ and η, with and without the effect of the CPDC. As to what one may intuitively expect and what the authors of Refs. 18 and 19 found, the CPDC effect is appreciable only in $^1D_2$ and, to a lesser extent, in $^3F_3$ partial waves. So our discussion below will be devoted entirely to these waves.

A. $^1D_2$ partial wave

The results are summarized in Fig. 5. Our η agrees fairly well with the result of the phase shift analysis by Arndt *et al.*[15] up to ~700 MeV. Above this energy the model starts underestimating the inelasticity, a common tendency shared by several models; see, for example, Refs. 1–3, 6, and 11. Likewise, the calculated η' turns out very close to what has been obtained empirically from the NN→NN and NN→πd partial wave analyses[15] as done in the preceding section. A slight underestimate of this quantity in our model is simply the reflection that we have somewhat overestimated $|S^{\pi\mathrm{d}}(g,Jls)|$ for $J=1$, $l=2$, and $s=0$, and thus the corresponding NN→πd partial cross section as stated above. Then by comparing our η, η', and η_0, it is clear that, unlike what was found in Ref. 18 and was not assumed in Ref. 19, the introduction

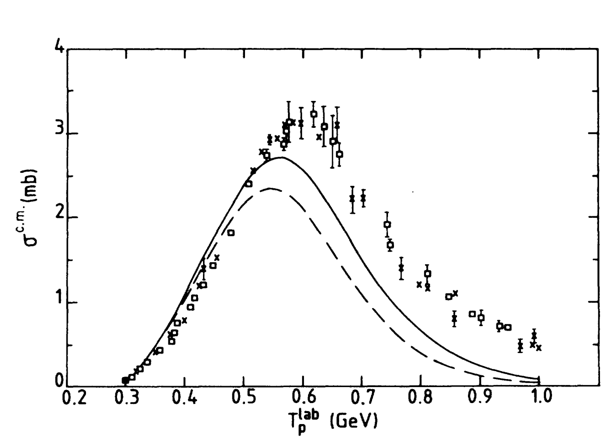

FIG. 4. The same as Fig. 1 for pp→π^+d. The curves show our results for the $^1D_2$ (— — —) and $^1D_2+{}^3F_3$ (—) contributions.

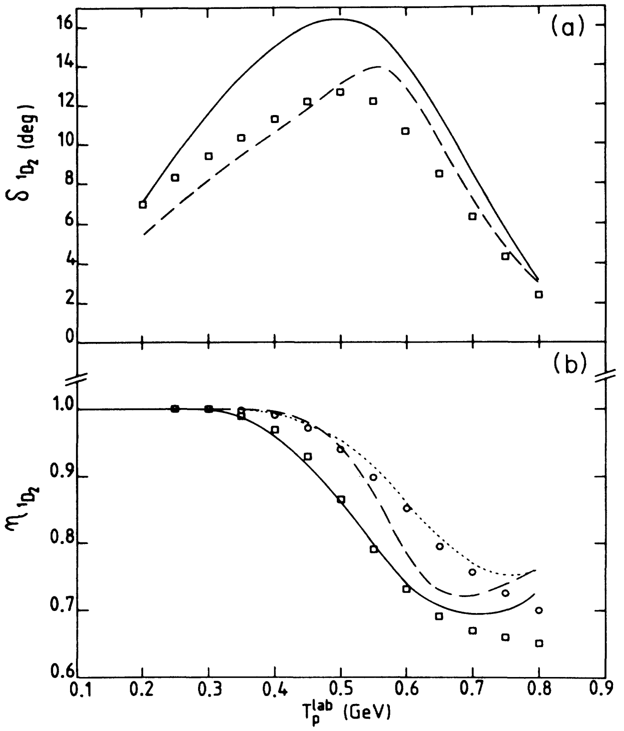

FIG. 5. The same as Fig. 2. The curves are our results with (—) and without (— — —) the CPDC, and the modified inelasticity parameter η' (· · ·).

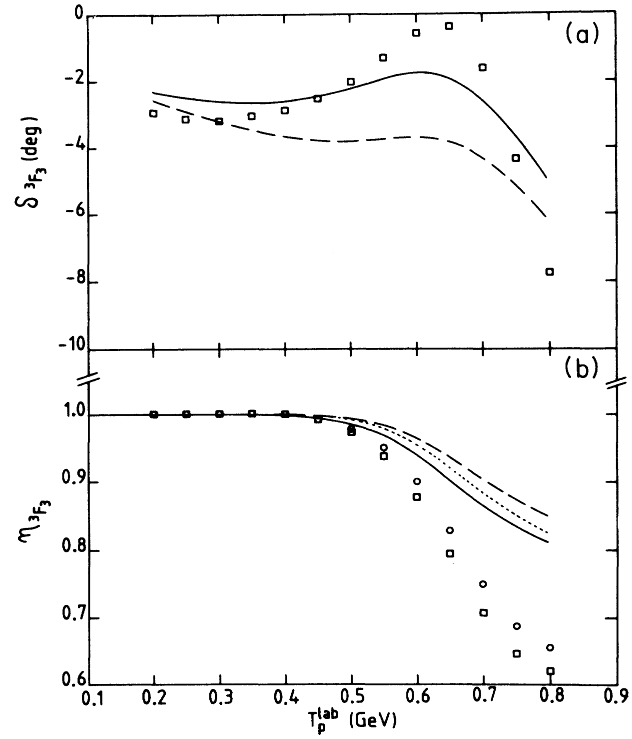

FIG. 6. The same as Fig. 5 for the ${}^3F_3$ partial wave.

of the CPDC is found to reduce the modulus of the NN$\rightarrow\pi$NN S-matrix element between 500 and 700 MeV and thus the corresponding partial cross section appreciably, but not to the extent that it compensates for the NN$\rightarrow\pi$d effect in the inelasticity η. Consequently, one cannot neglect the CPDC.

Regarding the (real) phase shift, Fig. 5(a), our model result is not in very good agreement with the phase shift analysis.[15] This is, to a large extent, due to the fact that our heavy meson parameters have not yet been fine tuned to optimally reproduce the phases, but for this partial wave this is not a very difficult task. Rather, we must emphasize that the qualitative difference between the result with and without the CPDC stays unchanged under the variation of the meson parameters within the acceptable range inferred from existing one-boson-exchange potential (OBEP) models; see, for instance, Ref. 24, and references cited therein. Therefore, we should expect that the CPDC induces two major changes here. The first is the increase in the phase shift value of up to about 5° at relatively low energies, and the second is the shift in the peak energy down to the correct position ($\sim$500 MeV). We note that in Refs. 18 and 19 the published figures indicate a peaking of $\delta({}^1D_2)$ at energy higher than in the phase shift analysis.

B. ${}^3F_3$ partial wave

Figure 6 explains our result. The difficulty is that we cannot get enough inelasticity even with the CPDC. Part of this deficiency comes from our underestimate of $|S^{\pi d}(fJls)|$ in this partial wave, as evidenced from Figs. 1 and 4. But, clearly, the main cause exists already in the calculation of η_0. It must be emphasized that all major model calculations to date have failed to reproduce sufficient inelasticity in this wave, see, e.g., Refs. 3, 5—7, and 11 and Fig. 3(b) in the present article. In our opinion this is one of the main problems which the theory of medium energy NN interaction must resolve, but we will not discuss it here. On the other hand, it is interesting to find that unlike in the ${}^1D_2$ wave the NN(${}^3S_1$-${}^3D_1$) correlation is found to increase the modulus of the NN$\rightarrow\pi$NN S-matrix element here. This means that an improved theory should produce the difference $(\eta_0-\eta)_{\text{theor}}$ of this channel to be greater than $\eta'-\eta$, where the latter is obtained from the NN$\rightarrow$NN and NN$\rightarrow\pi$d amplitude analyses, and can be read off in Fig. 6(b). So one should expect a clear evidence of the CPDC in η also in this partial wave.

The effect is visible as well in the phase shift; see Fig. 6(a): With the CPDC one gains almost 2° at the energy where theory definitely needs this increase to be consistent with the bump present in the data. Note that without the CPDC it appears to be extremely difficult to reproduce this bump correctly.[2–8,11] Again, as in the case of the ${}^1D_2$ channel, our model is not in very good agreement with data due to the use of yet nonoptimal heavy meson parameters.

V. CONCLUSION

In the present work we have focused our attention on the possible influence of the CPDC on the medium energy

NN interaction. Our conclusion is that, at least in the $^1D_2$ and $^3F_3$ partial waves, it is important up to ~ 700 MeV. So the conventional two-body coupled channel approach must, in one way or another, take this into account. There are a few comments to be made before closing our present study. The first is concerned with the fact as to why Lee and Matsuyama[18] have obtained the negligible influence from the CPDC. Upon examining their equation we have found no error concerning its formal aspect: it embodies the appropriate coupling structure with the $NN\rightarrow\pi d$ process so there seems to be no way to miss the required cross section in this channel. In order to clarify this situation, it seems imperative to calculate the $NN\rightarrow\pi d$ cross section within their formalism [see Eq. (A24) in the Appendix].

The van Faassen—Tjon calculation at its present stage is useful for a semiquantitative indication of the CPDC, the limitation being due to the lack of dynamical considerations. Therefore, this method does not allow one to investigate the sensitivity of the phase shifts to the CPDC.

The last point concerns the influence of other intermediate states left out in our present study. We have recently made an investigation into this subject which will be reported elsewhere in a more extensive publication including discussions on the $NN\leftrightarrow\pi d$ spin observables, etc. A preliminary result may be summarized as follows: the overall effect of those remaining intermediate states is smaller than that from the CPDC. However, unlike the CPDC, they also affect partial waves other than $^1D_2$ and $^3F_3$. Concerning its influence on these two latter waves, a reduction in δ and η has been observed due to the $(\pi N)+N$ type of intermediate state with $(\pi N)=S_{11}$, S_{31}, P_{13}, and P_{31} waves,[23] while the other type of contribution, $(NN)+\pi$, with $(NN)={}^1S_0$, $^3P_0$, $^3P_1$, $^3P_2$, etc., tends to work attractively, increasing both δ and η.

To conclude, we would like to stress that although there still remain some unsolved discrepancies between data and theories exploiting the conventional meson-baryon dynamics, careful investigation is required within this type of theoretical framework before concluding that some ingredients external to the conventional picture, like six quark dibaryons, are indispensable. Our present paper is one of such works moving toward that direction.

ACKNOWLEDGMENTS

One of us (T.M.) is grateful to the Institut de Physique Nucléaire, Lyon, and the Service de Physique Nucléaire à Haute Energie, Saclay, for a very pleasant stay during the course of this work. Another of us (B.S.) wishes to thank the Physics Department at the Virginia Polytechnic Institute and State University for the hospitality extended to him. We would like to thank Professor W. Watari for providing us with his most recent result of the $pp\rightarrow\pi^+d$ amplitude analysis. This work was supported in part by the U.S. Department of Energy under Contract No. DE-FG05-84-ER40143.

APPENDIX

Here we shall examine the unitarity structure of the Lee-Matsuyama equation for the NN scattering as presented in Ref. 18. This must be useful since no explicit account of this subject has been given in their original article,[25] which deals with the formulation of the πNN equations in their own fashion: a combination of a two-body coupled channel method and a three-body treatment.

We follow the notation and equations in Ref. 18. What we want to find is the discontinuity relation for the total $NN\rightarrow NN$ t matrix,

$$\mathrm{disc}T(E)\equiv T(E^+)-T(E^-)\ , \tag{A1}$$

across the two- and three-body unitary cut, where

$$E^{\pm}\equiv E\pm i\epsilon\ . \tag{A2}$$

To simplify the presentation we denote

$$A^{\pm}\equiv A(E^{\pm})\ ,$$

where $A\equiv T$, U, U_c, Σ_Δ, etc. Note that unless necessary we drop the plus sign $(A\equiv A^+)$. Now, $T(E)$ obeys the Lippmann-Schwinger equation,

$$T(E)=U(E)+U(E)G_{NN}(E)T(E)\ , \tag{A3}$$

with

$$G_{NN}(E)\equiv\frac{P_{NN}}{E^+-H_0}$$

and since $U(E)$ itself develops a discontinuity structure, we obtain, after simple algebra,

$$\mathrm{disc}T(E)=T^-\delta^2T^++\Omega_{NN}^\dagger\mathrm{disc}U(E)\Omega_{NN}\ , \tag{A4}$$

where

$$\delta^2\equiv-2\pi iP_{NN}\delta(E-H_0) \tag{A5}$$

generates the two-body phase space, and

$$\Omega_{NN}\equiv1+G_{NN}T \tag{A6}$$

is the NN wave operator. In the absence of the second term, Eq. (A4) is the expression for the NN elastic (two-body) unitarity. So the effect of the single pion production channels is taken care of by the second term. Our main task here is then to find the expression for $\mathrm{disc}\,U(E)$.

Since $V_{NN,NN}$ and $V_{NN,N\Delta}$ $(V_{N\Delta,NN})$ possess no right-hand cut within the Lee-Matsuyama formulation, the following relation can be obtained from Eq. (2) of Ref. 18,

$$\mathrm{disc}U(E)=\sum_{i=1,2}V_{NN,N\Delta}G_{N\Delta}^-h_i^+\delta^3h_iG_{N\Delta}^+V_{N\Delta,NN}+\mathrm{disc}U_c(E)\ . \tag{A7}$$

Here, $G_{N\Delta}(E)$ is the free NΔ propagator:

$$G_{N\Delta}(E)=\frac{P_{N\Delta}}{E^+-H_0-\Sigma_\Delta(E^+)}\ , \tag{A8}$$

and

$$\delta^3\equiv-2\pi iP_{\pi NN}\delta(E-H_0) \tag{A9}$$

provides the three-body (πNN) phase space factor. Note that in obtaining the first term in Eq. (A7) we have utilized the resolvant relation,

$$G_{N\Delta}(E)=\frac{1}{E^{+}-H_0}P_{N\Delta}+\frac{1}{E^{+}-H_0}P_{N\Delta}\Sigma_{\Delta}G_{N\Delta}\ , \quad (A10)$$

together with the Δ self-energy expression [Eq. (4) of Ref. 18].

Now we need to find disc $U_c(E)$ in Eq. (A7). Since it contains $T_c(E)$, which then is obtained from $V_c(E)$ by solving the Lippmann-Schwinger equation [Eq. (5) of Ref. 18], we first need disc $V_c(E)$. This quantity contains, within the approximation disregarding the nonresonant πN interactions, the NN t matrix embedded in the πNN three-body space. In this case one finds

$$\mathrm{disc}t=t^{-}\delta^3t^{+}+g_{\mathrm{d}}\delta^{\pi\mathrm{d}}g_{\mathrm{d}}\ , \quad (A11)$$

where g_{d} is the NNd vertex, and

$$\delta^{\pi\mathrm{d}}\equiv-2\pi i\delta(E-H_{\pi\mathrm{d}})\ , \quad (A12)$$

with $H_{\pi\mathrm{d}}$ the free πd Hamiltonian. Then from Eq. (6) of Ref. 18 we can easily find

$$\mathrm{disc}V_c(E)=\sum_{i\neq j}h_i^{+}\delta^3h_j+\sum_{i,j}h_i^{+}(\delta^3tG_3+G_3^{-}t^{-}\delta^3tG_3+G_3^{-}g_{\mathrm{d}}\delta^{\pi\mathrm{d}}g_{\mathrm{d}}G_3)\ . \quad (A13)$$

Here we have

$$G_3(E)\equiv\frac{P_{\pi\mathrm{NN}}}{E^{+}-H_0}\ . \quad (A14)$$

This result is incorporated into

$$\mathrm{disc}T_c=T_c^{-}\mathrm{disc}G_{N\Delta}T_c+\Omega_{N\Delta}^{\dagger}\mathrm{disc}V_c\Omega_{N\Delta}\ , \quad (A15)$$

with

$$\Omega_{N\Delta}\equiv1+G_{N\Delta}T_c\ , \quad (A16)$$

which can be obtained from Eq. (5) of Ref. 18. Then it is straightforward to obtain the equation

$$\mathrm{disc}T_c(E)=\Omega_{N\Delta}^{\dagger}X\Omega_{N\Delta}-\sum_i h_i^{+}\delta^3h_i\ , \quad (A17)$$

where

$$X=\sum_{i,j}h_i^{+}(1+G_3^{-}t^{-})\delta^3(1+t^{+}G_3^{+})h_j+\sum_{i,j}h_i^{+}G_3^{-}g_{\mathrm{d}}\delta^{\pi\mathrm{d}}g_{\mathrm{d}}G_3^{+}h_j\ . \quad (A18)$$

From Eq. (3) of Ref. 18, we derive the following expression:

$$\mathrm{disc}U_c(E)=V_{\mathrm{NN,N}\Delta}G_{N\Delta}^{-}\left[\sum_i h_i^{+}\delta^3h_iG_{N\Delta}^{+}T_c^{+}+\mathrm{disc}T_c+T_c^{-}G_{N\Delta}^{-}\sum_i h_i^{+}\delta^3h_i\right]\times G_{N\Delta}^{+}V_{\Delta\mathrm{N,NN}}\ . \quad (A19)$$

Combining Eqs. (A4) and (A17) and inserting into (A19), we find

$$\mathrm{disc}U(E)=V_{\mathrm{NN,N}\Delta}G_{N\Delta}^{-}\Omega_{\mathrm{NN}}^{\dagger}X\Omega_{N\Delta}V_{N\Delta,\mathrm{NN}} \quad (A20)$$

and

$$\mathrm{disc}T(E)=T^{-}\delta^2T^{+}+\Omega_{\mathrm{NN}}^{\dagger}V_{\mathrm{NN,N}\Delta}\times G_{N\Delta}^{-}XG_{N\Delta}V_{N\Delta,\mathrm{NN}}\Omega_{\mathrm{NN}}\ , \quad (A21)$$

which gives the unitarity relation including NN, πd, and πNN channels:

$$\mathrm{disc}T(E)=T^{-}\delta^2T^{+}+T_{\pi\mathrm{NN,NN}}^{-}\delta^3T_{\pi\mathrm{NN,NN}}+T_{\pi\mathrm{d,NN}}^{-}\delta^{\pi\mathrm{d}}T_{\pi\mathrm{d,NN}}\ , \quad (A22)$$

upon identifying the NN$\rightarrow\pi$NN and NN$\rightarrow\pi$d t matrices as

$$T_{\pi\mathrm{NN,NN}}\equiv\sum_{j=1,2}(1+tG_3)h_j\Omega_{N\Delta}G_{N\Delta}V_{N\Delta,\mathrm{NN}}\Omega_{\mathrm{NN}} \quad (A23)$$

and

$$T_{\pi\mathrm{d,NN}}\equiv\sum_{j=1,2}g_{\mathrm{d}}G_3h_j\Omega_{N\Delta}G_{N\Delta}V_{N\Delta,\mathrm{NN}}\Omega_{\mathrm{NN}}\ . \quad (A24)$$

This identification is found to be consistent with the corresponding expression in Ref. 26 when t in Eq. (A23) is replaced by the complete three-body amplitude for NN$\rightarrow\pi$NN. It may be useful to point out that the final state πd distortion is implicitly included in Eq. (A24), because $T(E)$ and thus Ω_{NN} contains the coupling to the πd channel [through $V_c(E)$, Eq. (6) of Ref. 18].

We therefore conclude that, within its own construction, the Lee-Matsuyama[18] equation is compatible with two- and three-body unitarity. Besides, one notes that Eq. (A22), when expressed in terms of the partial wave S-matrix elements, gives Eq. (3) of the main text.

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