Trinucleon system in a two-body model: Coulomb effect on bound and scattering states

Lauro Tomio,* A. Delfino,† and Sadhan K. Adhikari
Departamento de Física, Universidade Federal de Pernambuco, 50.000 Recife, Pernambuco, Brazil
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Recently it has been suggested that as in the neutron deuteron problem the theoretically calculated spin-doublet S wave proton deuteron scattering length $^2a_{pd}$ is correlated with ^{3}He binding energy. This strongly suggests that $^2a_{pd} \simeq 0$. In this paper we study various properties of the low energy trinucleon system including this correlation among scattering length and binding energy for the spin-doublet neutron deuteron and the proton deuteron systems in an *ad hoc* two-body model. The two-body potential models we employ have a long range tail of r^{-2} type and have been parametrized to produce the correct binding energies of ^{3}H and ^{3}He and the correct s wave spin-doublet neutron deuteron scattering lengths $^2a_{nd}$. The present study predicts the low energy nucleon deuteron phase shifts and predicts the proton deuteron scattering length to be $^2a_{pd} \simeq (0.15 \pm 0.1)$ fm.

I. INTRODUCTION

A strong correlation among the theoretically calculated s-wave spin doublet neutron-deuteron (n-d) scattering length $^2a_{nd}$ and the triton binding energy $B(^3\text{H})$ was first observed by Phillips[1] about 20 years ago. (In this paper we shall only be concerned with the s wave spin doublet trinucleon system, and unless there is a chance of confusion we shall not explicitly write the expression "s wave spin doublet.") More explicitly, whenever neutron-deuteron calculations are performed with different nucleon-nucleon potentials which produce identical two nucleon properties, the resulting $^2a_{nd}$ and $B(^3\text{H})$ are strongly correlated. Phillips[1] used this correlation to fix approximately the experimental value of $^2a_{nd}$. (The spin quartet trinucleon system is, on the other hand, repulsive in nature and is insensitive to the details of nuclear interaction employed. All calculations produce identical results for this system.) Girard and Fuda,[2] later on, studied the asymptotic normalization parameter (ANP) of the triton, and found that this function is strongly correlated with the doublet scattering length. A good value of the ANP of the triton resulted whenever the correct doublet n-d scattering length and the triton binding energy were assumed. Later on, Brayshaw,[3] using his boundary condition model, showed that even the low-energy n-d breakup results are correlated with the doublet scattering length. In other words, all the low energy spin-doublet three-nucleon observables are strongly correlated with the n-d doublet scattering length $^2a_{nd}$.

The three existing experimental analyses[4] of the s wave spin doublet proton-deuteron (p-d) scattering yield for the usual Coulomb modified scattering length $^2a_{pd}$ values ranging from 1 to 4 fm. We, however, neglect throughout this paper the polarization potential arising due to the fact that the charge distribution of the deuteron is neither pointlike (at the deuteron center of mass) nor spherically symmetric, partly because such a treatment is controversial and complicated, and partly because it is not our principal objective. (See discussion in Sec. IV.) The experimental[4] $^2a_{nd}$ ($=0.65$ fm) is less than the $^2a_{pd}$ of Ref. 4. The theoretical estimates[5] of $^2a_{pd}$ are also confusing. Almost all the theoretical estimates of $^2a_{pd}$ yield $^2a_{pd} > {}^2a_{nd}$ and lie close to the experimental estimates. The recent analysis of the Los Alamos group[6–9] suggests a very small value of $^2a_{pd}$ and, in particular, $^2a_{pd} < {}^2a_{nd}$, in contradiction with other theoretical and experimental estimates of $^2a_{pd}$. Their analysis[7] suggests that while calculating $^2a_{pd}$ one should use a model which produces the correct value of the binding energy of ^{3}He, $B(^3\text{He})$. Their calculation[7] also suggests a correlation among the theoretically calculated $^2a_{pd}$ and $B(^3\text{He})$.

It has been shown recently that the correlation among the theoretically calculated $^2a_{nd}$ and $B(^3\text{H})$ can essentially be explained[10] in a two body model using the dispersion theoretic (N/D) approach or in a three-body model using zero range separable nucleon-nucleon interaction.[11] Both these approaches are very simple and do not need detailed information about the two nucleon interaction. Hence it seems that most of the low-energy trinucleon properties can be explained in a two-body model using two-body dynamics.

In this paper we would like to suggest an effective nucleon-deuteron interaction which explains most of these properties. Physically, because of the low binding energy of the deuteron, the one nucleon exchange part of the nucleon-deuteron effective interaction should have a long range tail. We include this long range part in the present effective interaction. For reasons we explain in detail in Sec. II, we conjecture that the long range part should have the r^{-2} form, where r is the separation between the deuteron and the nucleon.

The trinucleon properties can be explained by an effective interaction which is dominated by a truncated r^{-2} interaction at large distances. If the short range part of this effective interaction is varied to reproduce the correct trinucleon properties, the low-energy trinucleon correlations are easily produced by varying only the short range part of the effective interaction while maintaining its long range behavior unchanged. In particular, we explicitly demonstrate that both the $^2a_{pd}$-$B(^3\text{He})$ and $^2a_{nd}$-$B(^3\text{H})$ correlations can essentially be explained by using the same

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nucleon-deuteron effective interaction.

In the present paper we suggest two *ad hoc* nucleon-deuteron effective interactions, each with three parameters. These three constants are fixed by demanding that the effective interaction produces the experimental values of $B(^3\mathrm{H})$, $B(^3\mathrm{He})$, and ${}^2a_{\mathrm{np}}$. Each of these effective interactions yields ${}^2a_{\mathrm{pd}} \simeq 0$ fm, in agreement with the finding of the Los Alamos group, and also produces the correct ${}^2a_{\mathrm{pd}}$-$B(^3\mathrm{He})$ and ${}^2a_{\mathrm{nd}}$-$B(^3\mathrm{H})$ correlations. From our analysis we predict ${}^2a_{\mathrm{pd}}$ to be 0.15 ± 0.1 fm.

The plan of the paper is as follows. In Sec. II we present the effective nucleon-deuteron interaction which we use for studying the low energy trinucleon correlations and the p-d problem. In Sec. III we parametrize the effective interaction by demanding that it produces the correct ${}^2a_{\mathrm{nd}}$, $B(^3\mathrm{H})$, and $B(^3\mathrm{He})$ and study the low energy trinucleon correlations and predict the correct value of ${}^2a_{\mathrm{pd}}$. Finally, in Sec. IV we present a brief summary of our findings.

II. EFFECTIVE NUCLEON-DEUTERON INTERACTION

In this section we shall mainly be concerned with the construction of the present nucleon deuteron effective interaction for the s wave spin doublet system below the breakup threshold. The present effective interaction is, to the best of our knowledge, the first to explain satisfactorily the low energy properties of the nucleon deuteron system.

The usual short range neutron deuteron effective interaction fitted to reproduce correctly the triton binding energy and the spin doublet n-d scattering length ${}^2a_{\mathrm{nd}}$ fails dramatically on several counts.[12] First, it badly fails to reproduce the low energy scattering phase shifts and produces an effective range of wrong sign. Second, it fails to produce the excited virtual state of $^3\mathrm{H}$. Finally, and related to the above, it produces an effective range function $k \cot\delta$ with a pole in energy above the scattering threshold. These difficulties can, however, be resolved satisfactorily by a three-body dynamical calculation. As no two-body model proposed so far explained the low energy properties of the n-d system satisfactorily, one tends to believe that the trinucleon system is sufficiently complex to be amenable to a two-body effective interaction treatment.

We would like to point out that it is possible to introduce a nucleon deuteron effective interaction if one is not strictly limited to the usual class of short ranged interactions, such as the exponential, Yukawa, Gaussian, or Saxon-Woods type interactions. The main problem is associated with the small binding energy of deuteron on the nuclear scale and correspondingly with its large size (extension of its wave function in coordinate space.) Physically, because of the large size of deuteron, the one nucleon exchange part of the nucleon deuteron effective interaction should have a long range tail. The two nucleons in the trinucleon system experience an effective interaction well outside the range of the nucleon nucleon interaction by exchanging the third nucleon. We would like to include this long range part in the present effective interaction. We conjecture that this long range part should have the r^{-2} form based on the work by Fonseca, Redish, and Shanley,[13] who show in a Born-Oppenheimer model that the effective interaction between two heavy particles interacting with each other by exchanging a light particle has a long range tail of the form r^{-2} as the light heavy binding energy reduces to zero. This long range behavior seems to be essential[13] in order to give a satisfactory explanation of the Efimov effect[14] in the limit as the light heavy binding energy reduces to zero. It is interesting to recall that though the discussion of Efimov[12] is limited to an r^{-2} interaction in the hyperspace, the analysis of Ref. 13 suggest an r^{-2} tail in the effective nucleon deuteron interaction in the physical coordinate space.

This long range nucleon deuteron interaction is expected[12,13] to be cut off at distances smaller than approximately the range of nuclear forces r_0 on the one side and for distances greater than Λ on the other side, where Λ is a function of deuteron binding and tends to infinity in the limit when deuteron binding tends to zero. For $r < r_0$ the effective interaction is assumed to be roughly constant as in the Saxon-Woods potential at short distances. The long range part of the effective interaction is independent of the detail of nucleon-nucleon interaction and only reflects the fact that the deuteron is loosely bound. The short range component of the effective interaction will, however, depend[12] on some average properties of nucleon-nucleon interaction. Once these average properties of the nucleon-nucleon interaction are not modified, the trinucleon system is expected to yield the same low energy results. Modifying these average properties holding the two nucleon observables fixed corresponds to modifying the interior part of the nucleon deuteron effective interaction. In calculations involving three-body dynamics this modification corresponds to, for example, modifying the off-shell effect of the nucleon nucleon interaction or introducing a tensor force or three body force, keeping the two nucleon properties unchanged.

Though the recent discussion of Efimov[12] about the low energy trinucleon properties treats the trinucleon in the hyperspace, most of his considerations are applicable to the present nucleon deuteron effective interaction. The distinctive feature of the long range nucleon-deuteron effective interaction is its universality,[12] by which we mean its independence of the details of the nucleon-nucleon interaction. However, to specify the three nucleon wave function, one needs to know the details of nucleon nucleon forces at short distances. According to the universality of the trinucleon problem, the relevant part of the nucleon nucleon interaction can essentially be characterized by one parameter in addition to the two nucleon properties. According to Efimov,[12] fixing this parameter essentially means fixing either $B(^3\mathrm{H})$ or ${}^2a_{\mathrm{nd}}$. This universality clearly manifests itself in three-nucleon calculations. The calculations—with nucleon deuteron effective interaction obeying this universality—which lead to a fixed value of $B(^3\mathrm{H})$ or ${}^2a_{\mathrm{nd}}$, are expected to lead to similar low energy trinucleon properties.

We find that the scattering length binding energy correlation in the s wave spin doublet trinucleon system with and without the Coulomb interaction can essentially be explained by the universal nucleon deuteron effective interaction dominated by a truncated r^{-2} interaction at

large distances, while we vary the short range part of this interaction and maintain the long range part unmodified. Specifically, this means that the origin of the $^2a_{pd}$-$B(^3\text{He})$ correlation is the same as that of the $^2a_{nd}$-$B(^3\text{H})$ correlation, though the Coulomb interaction modifies the detail of the correlation. The Coulomb interaction makes the prediction of the $^2a_{pd}$-$B(^3\text{He})$ correlation as well as the experimental $^2a_{pd}$ quite nontrivial without performing a two-body calculation using a universal effective interaction or a full three-body calculation. Such a calculation is expected to yield the correct experimental $^2a_{pd}$ only if it produces the correct experimental values of $^2a_{nd}$, $B(^3\text{H})$, and $B(^3\text{He})$.

Motivated by the above discussion, we chose to study the trinucleon problem using the following s wave spin doublet effective nucleon deuteron interactions:

$$\text{Choice A: } V(r)=\begin{cases} -V_0\equiv-\widetilde{V}_0 e^{-\mu R}/R^2\ , & r\le R \\ -\widetilde{V}_0 e^{-\mu r}/r^2\ , & r\ge R \end{cases} \tag{1}$$

$$\text{Choice B: } V(r)=-V_0R^2\sin^2(r/R)e^{-\mu r}/r^2\ . \tag{2}$$

In each case the potential has three parameters, V_0 (or $\widetilde{V}_0$), μ, and R. The effective interactions of Eqs. (1) and (2) have a smooth cutoff, in contrast to our discussion of the universality, where we have a sharp cutoff at Λ. The potential with a sharp cutoff has a finite range and no left hand cut. The potentials of Eqs. (1) and (2) have left hand cuts from $k^2=-\mu^2/4$ to $k^2=-\infty$, where k is the wave number and is related to the three nucleon center of mass energy E by

$$E=-B(^2\text{H})+(3k^2/4)\hbar^2/2m\ , \tag{3}$$

where $\hbar^2/2m=41.47$ MeV fm^2, and the deuteron binding $B(^2\text{H})$ is taken to be 2.226 MeV. The realistic three body model also has a left hand cut which extends, however, from $E=-4B(^2\text{H})/3$ to $E=-\infty$. We chose to work with the effective interactions of Eqs. (1) and (2), as they simulate better the cut structure of the realistic trinucleon problem.

It is well known that the triton has an excited virtual state[15,16] whose position is strongly correlated with $^2a_{nd}$. The excited virtual state appears as a pole of the three nucleon t matrix on the real negative energy axis in the second sheet of the complex energy plane associated with the lowest scattering threshold; the position of this pole is limited between the lowest scattering threshold and the left hand cut.

One way of fixing the parameters of the effective interactions A and B is to demand that they produce the correct $^2a_{nd}$, $B(^3\text{H})$, and $B(^3\text{H}^*)$, where $B(^3\text{H}^*)$ is the binding energy of the virtual state. Using the pole dominance argument it has been conjectured in Refs. 16 and 17 that a correct descripton of the virtual state is essential for yielding the correct $^2a_{nd}$ in a model, as it produces a nearby pole in the t matrix. The discussion of the Los Alamos group[7] supports this conjecture. The present authors, however, do not believe in this conjecture because the extreme, small residue[15] of the t matrix at the virtual state pole will make the effect of the virtual state on $^2a_{np}$ a higher order correction on the effect of the triton pole on $^2a_{np}$. Also, the virtual state $^3\text{H}^*$ is not physically observable. This is why we chose not to build in the correct position of the virtual state.

Instead, the parameters V_0, R, and μ of the effective interactions are evaluated by requiring that the effective interactions produce the experimental values of $^2a_{nd}$, $B(^3\text{H})$, and $B(^3\text{He})$, where in the last case the p-d Coulomb interaction is added to the effective interaction. This model is expected to yield the "correct" value of $^2a_{pd}$. As mentioned in Sec. I and discussed in Sec. V, we neglect the p-d polarization potential altogether.

III. PARAMETRIZATION OF THE EFFECTIVE INTERACTIONS

In this section we parametrize[18] the effective interactions by requiring that they produce the correct $^2a_{np}$, $B(^3\text{H})$, and $B(^3\text{He})$. Both interactions—labeled choices A and B—are used for the parametrization. The s wave Schrödinger equations are solved in the coordinate space in both cases with and without Coulomb interaction. Interaction A is very simple, but does not have an analytic form in the momentum space. Interaction B has an analytic form in the momentum space and is attractive for a momentum space treatment. At large r interaction B oscillates because of the $\sin^2(r/R)$ factor, but in practice these oscillations do not lead to serious problems as they are quickly attenuated because of the exponential damping factor. The parameters of the two interactions are given in Table I. In Fig. 1 we plot the potentials with the parametrizations of Table I. Choice A appears to be more suitable as an effective interaction, as it is reasonably constant within the deuteron, and in the exterior it decays as r^{-2}, whereas the potential of choice B starts decreasing directly from $r=0$. We study interaction B not because we think it to be an ideal effective interaction for the n-d system, but because it helps in establishing certain points we are trying to make in this paper, about the universality of the effective interaction

We calculated the n-d and p-d effective range functions at energies below the breakup threshold. The available experimental p-d points are also shown. The effective range

TABLE I. The parameters of interactions A and B of Eqs. (1) and (2) and the calculated observables. The numbers in parentheses are the experimental values.

	Interaction A	Interaction B
V_0 (MeV)	30.325	39.839
R_0 (fm)	1.6275	1.7571
μ (fm^{-1})	0.2000	0.1506
$B(^3\text{He})$ (MeV)	7.718	7.717
	(7.717)	(7.717)
$B(^3\text{H})$ (MeV)	8.481	8.480
	(8.482)	(8.482)
$^2a_{nd}$ (fm)	0.651	0.650
	(0.65)	(0.65)
$^2a_{pd}$ (fm)	0.2±0.05	−0.25±0.05
	(?)	(?)

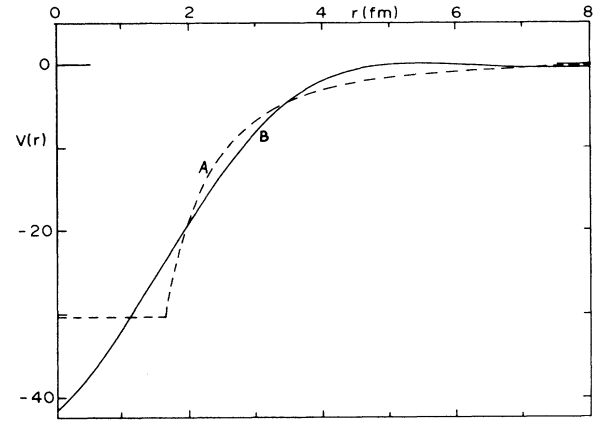

FIG. 1. Interactions A and B in MeV for parameters given in Table I.

function K versus energy leads to highly nonlinear plots because of the occurrence of a subthreshold pole in such functions. The inverses of the effective range functions[19] plotted in Fig. 2 are expected to yield more linear plots. The p-d inverse effective range function was fitted to the following expansion,[19]

$$-K^{-1}=a_c+bk^2+ck^4 , \tag{4}$$

where $k^2=8E_{\rm lab}/9$, and $E_{\rm lab}$ is the incident nucleon energy in the laboratory system expressed in fm^{-2}. For choice A this yields $a_c={}^2a_{\rm pd}\cong(0.2\pm0.05)$ fm, and for choice B this yields $a_c={}^2a_{\rm pd}\cong(-0.25\pm0.05)$ fm.

Several comments are in order in relation to the above calculation. The first one is about the correct value of ${}^2a_{\rm pd}$. Both interactions—choices A and B—produce the

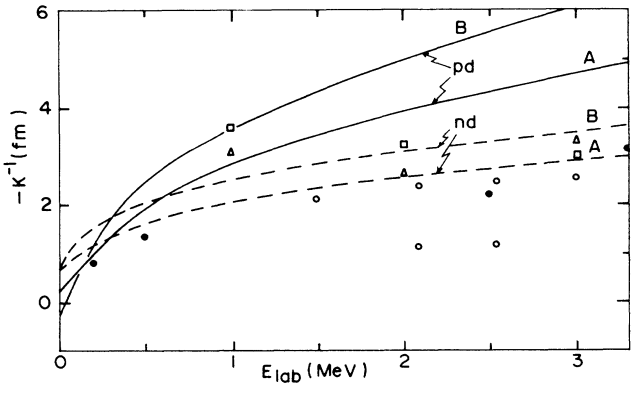

FIG. 2. The inverse effective range function $-K^{-1}$ of Eq. (4) for various incident nucleon energy $E_{\rm lab}$ for n-d and p-d systems for interactions A and B. For n-d system $K^{-1}=k^{-1}\tan\delta$ and for p-d system K^{-1} is defined in Ref. 18. The experimental points are taken from Ref. 4: Huttel *et al.* (p-d), squares; Arvieux (p-d), triangles; van Oers and Brockman (p-d), open circles; van Oers and Brockman (n-d), solid circles.

correct experimental ${}^2a_{\rm nd}$, $B({}^3{\rm H})$, and $B({}^3{\rm He})$, and have the universal r^{-2} behavior at large distances. So which of the above two values of ${}^2a_{\rm pd}$ should be the correct one? Interaction B has an unusual behavior at very small r and at large r. At small r it is not roughly constant, as is expected of an effective interaction, and at large distances it has an unusual oscillating cutoff because of the presence of the $\sin^2(r/R)$ factor. It is because of these two points that we take choice A—and not B—to be the physical effective interaction. Allowing for a safe margin of error arising due to the use of the *ad hoc* interaction A to represent the p-d system, and also due to the uncertainty discussed in Ref. 18—which has a tendency to increase the ^{3}He binding energy and consequently decrease ${}^2a_{\rm pd}$—we predict ${}^2a_{\rm pd}$ to be (0.15 ± 0.1) fm, in agreement with Refs. 6–8.

The next comment is about the extrapolation of experimental p-d phase shifts to zero energy to find ${}^2a_{\rm pd}$. From the p-d effective range curves of Fig. 2 it is clear that a linear extrapolation of the experimental effective range points to zero energy, as done in the analyses of Ref. 4, is expected to yield an incorrect and large value of ${}^2a_{\rm pd}$ because of the nonlinearity of the plot. So, even if the experimental phase shifts are correct, the extrapolation to zero energy to extract ${}^2a_{\rm pd}$ is a very delicate task and this is possibly one of the main sources of error in the analyses of Ref. 4, which yield ${}^2a_{\rm pd}\gtrsim 2$ fm. This last result is at variance with our finding and with that of Refs. 6–8.

The choice A interaction yields an effective range function K with a pole just below the threshold at $k^2\gtrsim -0.001$ fm^{-2}. For the choice B function the pole is at $\widetilde{k}^2\gtrsim 0$ fm^{-2}. As the choice A interaction is considered to be closer to the physical nucleon deuteron effective interaction, the p-d effective range function K is expected to have a pole just below the scattering threshold. This result is in variance with the analyses of Ref. 4.

Next, in order to produce the Phillips plots with interaction A, we modified the short range part of the effective interaction and generated a class of interactions by

$$V(r)=\begin{cases} -V_0'\equiv-\widetilde{V}_0\exp(-\mu R')/R'^2 , & r\le R' \\ -\widetilde{V}_0\exp(-\mu r)/r^2 , & r\ge R' \end{cases} \tag{5}$$

where μ and $\widetilde{V}_0$ have the same values as in Table I. The long range parts of Eqs. (1) and (5) are identical. For distances $r<R'$ two interactions given by Eqs. (1) and (5) are different. By varying V_0' we solved the n-d and p-d problems with the class of interactions (5) and generated the ${}^2a_{\rm nd}$-$B({}^3{\rm H})$ and ${}^2a_{\rm pd}$-$B({}^3{\rm He})$ correlations, which are plotted in Fig. 3. In the case of interaction B we produced the Phillips plots by varying V_0 in Eq. (2). The class of functions so generated is given by

$$V(r)=-V_0'R^2\sin^2(r/R)e^{-\mu r}/r^2 , \tag{6}$$

with μ and R given in Table I. By varying V_0' we solved the n-d and p-d problems with the class of interactions (6) and the resulting ${}^2a_{\rm nd}$-$B({}^3{\rm H})$ and ${}^2a_{\rm pd}$-$B({}^3{\rm He})$ correlations, which are also plotted in Fig. 3. The results of the recent realistic three nucleon calculations[6–8] are also plotted in

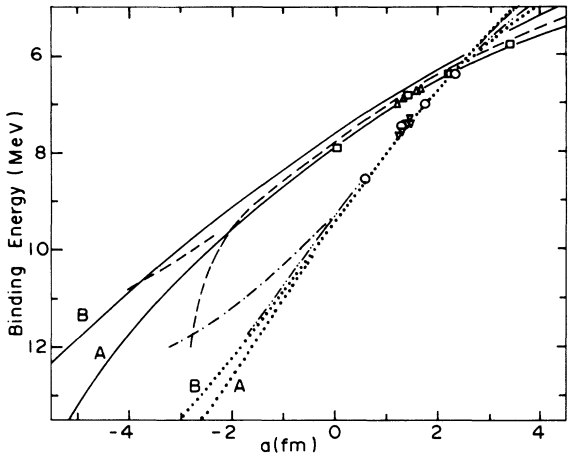

FIG. 3. Binding energy—scattering length plot for trinucleon ground state. The points are taken from the recent calculations of Refs. 7 and 8: circles (n-d), Ref. 7; squares (p-d), Ref. 7; triangles (p-d), Ref. 8; inverted triangles (n-d), Ref. 8; dashed line (p-d), plot of Refs. 7 and 8; dashed-dotted and dashed-dotted-dotted lines (n-d), plot of Refs. 7 and 8; solid lines, the present p-d plots for interactions A and B; dotted lines, the present n-d plots for interactions A and B. The present plot agrees with that of Ref. 9.

Fig. 3. These realistic calculations are in good agreement with our calculations for both n-d and p-d systems.

The difference between the various calculations of the correlation can be explained as follows. The effective interaction A is varied through Eq. (5) in a universal way, in that the long range part of the effective interaction is held fixed while the short range part is varied. This means that the two nucleon properties are held fixed while generating the correlation. So the Phillips plot for this interaction is supposed to be the usual universal plot. But when we vary interaction B by varying V_0' via Eq. (6), we are varying both the long range and short range parts of the interaction. This means that the two nucleon properties are not strictly held fixed during the process. So the Phillips plot in this case is supposed to be a model dependent one and not the usual universal plot. This explains the deviation between the plots of interactions A and B. We see that the deviation is not too large, which means that the deviation from the universality in the case of interaction B is not very severe.

Next we would like to comment that the Los Alamos group generated the a-B correlation by artifically multiplying the $^1S_0$ two nucleon interaction by a multiplier, hence changing the two nucleon properties and, consequently, both the long range and short range parts of the effective interaction. For example, they generated the "experimental" p-d point by multiplying the Reid $^1S_0$ effective interaction by a factor of 1.11, which transforms the two nucleon singlet virtual state into a real bound state. Hence their a-B plots are supposed to be different from our plots for interaction A, specially when the factor multiplying the $^1S_0$ interaction deviates the most from unity. In these regions the Los Alamos correlation is expected to yield model dependent plots and to be different from the universal plots of effective interaction A. This explains the small discrepancy between the various plots of Fig. 3.

If one believes in the universality of the $^2a_{pd}$-$B(^3\text{He})$ correlation as these authors do, one should pick up the correct value of $^2a_{pd}$ consistent with the experimental $B(^3\text{He})$. For the universal curve of interaction A the experimental $B(^3\text{He})$ corresponds to $^2a_{pd}=0.2\pm0.05$ fm. In this region the Los Alamos $^2a_{pd}-B(^3\text{He})$ correlation coincides with the universal curve of interaction A and their value $^2a_{pd}=0.06$ fm is consistent with our result of $^2a_{pd}=0.2\pm0.05$ fm, if we remember that the Los Alamos calculation overbinds the ^{3}He by about 0.17 MeV. We, however, present a more conservative estimate of $^2a_{pd}$ ($=0.15\pm0.1$ fm), as discussed after Eq. (4).

Finally, we end this section with some comments about the excited states of ^{3}H and ^{3}He and their relation to the scattering length ^2a and to the nonlinearity of the plots of the effective range function as shown in Fig. 2. Some recent comments in the literature regarding these aspects are very confusing. The first comment is about the excited state of ^{3}He—^{3}He*. The excited state of ^{3}H is a virtual state. In the case of ^{3}He, because of the presence of long range Coulomb interaction, the left hand cut essentially starts at the scattering threshold and there is no possibility of a virtual state to appear in this case. This contradicts the discussion of Refs. 7 and 9, whose authors conjecture about the possible effect of the virtual state of ^{3}He on $^2a_{pd}$ and the convergence of the effective range expansion. Zankel and Mathelitsch[9] even *calculate* the position of the virtual state of ^{3}He in their three-body model! Actually, there are two possibilities for the fate of the virtual triton state pole when the Coulomb interaction is switched on as in ^{3}He. It may either go to a remote unphysical sheet or may move into the complex energy plane[20] as in the case of ^{2}He. Only after a more precise study can one conclude its actual fate.

The other points of confusion in the literature are conjectures about the possible reason for the nonconvergence of the effective range expansion. This is really due to the nonlinearity of the effective range plots because of a pole in the effective range function below the scattering threshold for both the n-d and p-d cases. In this work we confirm the presence of a subthreshold pole in the p-d effective range function from Fig. 2. But the position of the virtual state pole (in the t matrix) and not the subthreshold pole in the effective range function has been made responsible[7] for the nonconvergence of the effective range expansion in the nucleon deuteron problem. We, however, do not share this point of view.

IV. SUMMARY

We have explained satisfactorily in two-body models the low energy properties of the trinucleon system. This is, to the best of our knowledge, the first dynamical (though *ad hoc*) two body model to explain such properties. The models use effective interactions dominated by an r^{-2} long range tail which is essential for the success of these models. In particular, we predict the correct experi-

mental value of $^2a_{pd}$, and the low-energy $^2a_{nd}$-$B(^3H)$, $^2a_{pd}$-$B(^3He)$ correlations and the low energy phase shifts. The extreme nonlinearity of the p-d effective range function explains why a linear extrapolation of this function to zero energy is expected to yield an incorrect value of $^2a_{pd}$. We confirm for the first time the presence of a subthreshold pole in the p-d effective range function from the present study. We also give a new estimate of $^2a_{pd}$ from our study: $^2a_{pd} \simeq (0.15 \pm 0.1)$ fm, consistent with the Los Alamos calculation.

It is well known that the two nucleon interaction has some ambiguity at short distances—commonly known as the off-shell effect—and the three nucleon interaction is completely unknown at short distances. One of the objectives of performing complicated three nucleon calculations with different "realistic" two- and three-nucleon interactions is to be able to choose between various interactions, so that one can learn about the off-shell behavior of two nucleon interaction and about the short range part of three-nucleon interaction. It is now generally accepted—and now more evident after the conclusion of the present work—that at low energies the sensitivity of the three-nucleon calculations to the two- and three-nucleon interactions can essentially be parametrized by one observable: $B(^3H)$ or ^2a. Hence it will be very difficult, if not impossible, to fix the off-shell behavior of two-nucleon interaction and the short-range behavior of three-nucleon interaction by performing complicated three-nucleon calculations as different admixtures of three-nucleon, and on-shell equivalent two-nucleon interactions will produce identical low-energy trinucleon results once these interactions produce the same trinucleon binding energy.

In our calculation we have neglected effects of polarization potential arising due to the fact that the charge distribution of the deuteron is not pointlike at the deuteron center of mass, but is spherically asymmetric with electric multipole moments. This leads to terms of the type $O(r^{-m})$, $m \geq 4$, in the effective p-d interaction, in addition to the usual Coulomb interaction treated in this paper. A correct treatment of the polarization potential is still problematic and controversial, as discussed by Berthold and Zankel,[9] and no three body calculation has estimated its effect precisely. Since the main objective of the present work is to test whether our two-body model can reproduce the results of the three body calculation including only a Coulomb force and not to predict results with a correct treatment of the polarization potential, we have neglected the polarization potential and compared our results with the available three body calculations in coordinate[6–8] and momentum spaces.[9] Once an accurate estimate of the effect of the polarization potential is available, it will be interesting to verify whether our simple two-body model can reproduce this effect, and this will be a work of future interest.

The present study of the effective interaction will find an interesting application in the construction of a phenomenological nucleon deuteron optical potential. For example, extension to energies above the breakup threshold is easily achieved by introducing an imaginary part to the optical potential in the usual manner, e.g., by

$$V'(r) = (1 + Ci)V(r) ,$$

where C is a constant and $V(r)$ is interaction A. The constant C should be fixed as usual by requiring that $V'(r)$ produces the correct complex phase shifts above the breakup threshold. For the quartet state almost any short ranged interaction should be applicable. Once we have a deuteron nucleon optical potential it can be used to construct a deuteron nucleus optical potential by the usual folding prescription. So the present investigation suggests a new way of constructing deuteron nucleon and deuteron nucleus optical potentials and should be of use to both theoreticians and experimentalists.

Note added in proof. We have received a copy of unpublished work by Bencze *et al.*,[21] which shows how one can correctly define a physically meaningful proton-deuteron doublet scattering length $^2a_{pd}$ when a polarization p-d potential is present in addition to the Coulomb and nuclear interactions treated in this paper. When superposed on a Yukawa-type p-d nuclear interaction, the polarization potential introduced a change of only[21] 0.3% in the value of $^2a_{pd}$ calculated with the complete neglect of the polarization potential, as has been done in this paper. They find that because of the very low polarizability of deuteron the effect of the polarization potential on $^2a_{pd}$ can virtually be neglected. In view of this finding we do not expect the results and conclusions of this paper to change significantly after the inclusion of the polarization potential.

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*Permanent address: Instituto de Física Teórica, Rua Pamplona 145, Caixa Postal 5956, 01.405 São Paulo, São Paulo, Brazil.

†Permanent address: Departamento de Física, Universidade Federal Fluminense, 24.210 Niteroi, Rio de Janeiro, Brazil.

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