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# Six-quark clusters and the energy dependence of the $(\pi^+,\pi^-)$ reaction

Gerald A. Miller
*Institute for Nuclear Theory, Department of Physics, University of Washington, Seattle, Washington 98195*
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Experimental studies of the energy dependence of the $(\pi^+,\pi^-)$ reaction may reveal the presence of six-quark bags in nuclei.

The pion-nucleus double-charge-exchange (DCX) reaction has fascinated physicists for many years. The reaction must proceed on at least two nucleons, so it has been natural to expect that the two-nucleon correlation function might enter.[1] In recent years, many nuclear theorists have attempted to use quark models to describe the short-ranged part of these correlations.[2] This prompted me to compute[3] the effects of a mechanism in which the DCX reaction proceeds via pion absorption and emission on a six-quark bag, Fig. 1. The computed amplitude was large, and this led me to speculate that if the (previously unmeasured) "50 MeV cross section for $^{14}C(\pi^+,\pi^-)^{14}O$(DA) is about 12 $\mu$b/sr near 0° and forward peaked, six-quark components of nuclear wave functions are most likely responsible." For this reaction the final nuclear state is in the same isospin multiplet as the target, hence it is a double analog (DA).

Shortly thereafter data were taken.[4] The DCX result for the $^{14}C$ target was that $d\sigma/d\Omega(0°)\approx 4\ \mu$b/sr. This was too small to conclude that six-quark bags are relevant. The problem was not that the theoretical prediction disagrees with the experiment. As shown in the LAMPF DCX workshop,[5] the predicted cross section is reduced by including initial and final state interactions. Furthermore, the prediction (including distortion) of the $^{18}O$ DCX angular distribution was in agreement with the data.[6] The true difficulty is that one cannot rule out conventional sequential (two-step) mechanisms in which the intermediate nucleus is in a nonanalog state.[7] Indeed the nucleonic mechanisms produce a reasonable description of the data.

However, it should be noted that the conventional mechanisms do include substantial contributions from nucleon-nucleon separations of less than 1 fm. Since this is a region in which nucleons are expected to overlap, quark degrees of freedom might still be the correct ones to use.

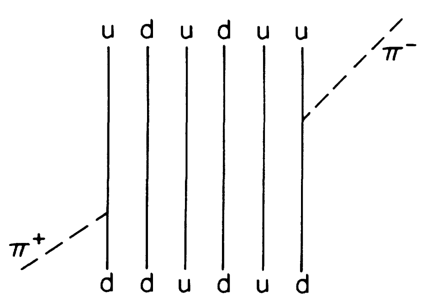


FIG. 1. Six-quark mechanism for double charge exchange. The diagram in which the emission occurs before the absorption is also included.

Thus the problem is that although the six-quark components are expected to give contributions of relevant sizes, it is very difficult to show that the six-quark cluster explanation is unique (or to rule it out). The purpose of this Rapid Communication is to suggest that studies of the energy dependence of the forward $(\pi^+,\pi^-)$ (DA) cross section might provide evidence for the reality of nuclear six-quark clusters. Such experiments may take place shortly.[8]

It is easy to see why this might be so. Consider the quark reaction mechanism in which two down quarks are turned into two up quarks, as in Fig. 1.

One may expect a resonant enhancement of the DCX cross section if the energy of the $\pi^+$ is close to the energy of the six-quark bag intermediate state. Although the energies and widths of the six-quark states are poorly known, it is reasonable to expect some "bumps" to appear for pion kinetic energies above the (3,3) resonance. (The difficulty of distinguishing this energy dependence from that caused by other mechanisms is discussed below.)

To examine this idea more closely, I estimate the term of Fig. 1 (as well as the corresponding one with crossed pions). To proceed one needs wave functions for the initial (i), intermediate ($m$), and final ($f$) six-quark states, a $\pi$-quark interaction Hamiltonian, and the probability to find a six-quark cluster within the two-nucleon valence wave function. We take $^{14}C$ as the target, and neglect core polarization here.

For *each* nuclear state considered here the six-quark cluster is assumed to have a completely symmetric spatial wave function ([6] symmetry) with a single-quark eigenfunction given by the lowest-energy orbital of the MIT bag model. Components consisting of two non-color-singlet baryons coupled to a color singlet dominate (80% of the probability) such clusters. These are the so-called hidden-color components of nuclear wave functions,[9] and are orthogonal to the usual nucleon-nucleon product wave functions. The spin ($S=0$) and isospin ($T=1$) of the initial and final six-quark clusters are the same as for the two valence neutrons or protons.

The requirement[10] that the axial-vector current be partially conserved determines matrix elements of the pion-quark interaction. For example, the term $H_{mi}$ for the absorption of a $\pi^+$ is given by

$$H_{mi}=\frac{if}{m_\pi}\frac{3u(kR_6)}{5(2E)^{1/2}}\langle m\,|\sum_{a=1}^{6}\boldsymbol{\sigma}_a\cdot\mathbf{k}\tau_+(a)\,|\,i\rangle\ , \quad (1)$$

for the six-quark wave functions used here. Here $f$ is the experimentally determined $\pi N$ coupling constant, $\boldsymbol{\sigma}_a(\tau_a)$ are Pauli (iso)spinors, and $u(x)$ is a form factor accounting for the finite size of the six-quark bag radius.[11] $E$ and $\mathbf{k}$ are the pion total energy and momentum.

To estimate the probability, $P_{6q}$, that a six-quark cluster exists, I assume that when two nucleons come within a distance $r_0$ of each other, such a cluster is formed. The ordinary NN wave function is unmodified for $r > r_0$, but set equal to zero for smaller separations. Conservation of the probability current[12] demands that the probability thereby removed from the nucleonic wave function be replaced by an equal amount in other components, which are *chosen* to be six-quark clusters. Then $P_{6q}$ can be obtained from the two-nucleon valence wave function $\psi_{\rm NN}$ as

$$P_{6q}(R)=\int d^3r\,|\psi_{\rm NN}(\mathbf{R},\mathbf{r})|^2\theta(r_0-r)\ , \quad (2)$$

where $R$ is the distance between the cluster and the nuclear centers. The Fourier transform of Eq. (2), $P_{6q}(q)$, where $q$ is the momentum transfer, is used to compute DCX. The function $\Psi_{\rm NN}$ is taken as a product of two $p$-shell harmonic-oscillator wave functions with $b=1.66$ fm.[13] The value of $r_0$ is set to 1 fm here, and then[1] $P_{6q}(q=0)=0.06$.

With these inputs one obtains, in plane-wave approximation (PWA), the expression for the DCX amplitude, $M$,

$$M=P_{6q}(q)\sum_m 2E_m\frac{H_{fm}H_{mi}}{E^2-E_m^2}\ . \quad (3)$$

The calculation is completed by specifying the energies of the intermediate states. Since the pion-absorption operator, e.g., of Eq. (1), is an axial vector, there are only two intermediate states with the spatial wave functions specified above. One has $S=1$ and $T=0$, and the other $S=1$ and $T=2$. The energies are 290 and 580 MeV, respectively, and are taken from the calculation of Mulders and Thomas.[14]

One may now write $M$ in a more explicit fashion:

$$M=\frac{4}{m_\pi^2}\frac{\mathbf{k}\cdot\mathbf{k}'}{2E}u(kR_6)u(k'R_6)P_{6q}(Q)\times\left[\frac{2E_1}{E^2-E_1^2}+\frac{1.28E_2}{E^2-E_2^2}\right]\ , \quad (4)$$

where $E_{1,(2)}$ is the complex energy of the lower (higher) energy six-quark state. The normalization is such that in plane wave approximation

$$\frac{d\sigma}{d\Omega}=\frac{1}{4\pi^2}|M|^2E^2\ . \quad (5)$$

Of course the energies $E_1$ and $E_2$ are of utmost relevance here. If $E$ is close to $E_m$ one expects $M$ to be large. Let us examine the specific values.[14] First note that these are very similar to those of Jaffe.[15] Nevertheless, there is considerable uncertainty in the values. To compute DCX cross sections, I need to know the widths of these states. Since the energies are high enough to allow pionic decay modes ($6q\rightarrow {\rm NN}+\pi$) I expect the widths to be at least as large as that of the delta. In computations I take $\Gamma=140$ MeV for the lower energy state and 280 MeV for the one at higher energy.

Although the energy of one of the states is low (290 MeV) it also corresponds to the energy of the (3,3) resonance. Thus if $E$ is near 290 MeV one cannot expect to easily distinguish the six-quark contributions from other mechanisms. However, the combination of the amplitude with that of other mechanisms could be evident as an interference effect that influences the angular distribution. We plan to report on this elsewhere.[16]

Higher energies for which the $\pi$-nucleon cross section is smaller may be more promising,[8] and such energies will be focused on here.

The formula for $M$ uses plane waves for the $\pi^\pm$ wave functions. The influence of the attenuation of flux caused by distortion is estimated by multiplying $|M|^2$ by $\exp(-l/\rho\sigma)$, where $l$ is the average path length (3.66 fm), $\rho=0.166$ fm$^3$, and $\sigma$ is the average of the $\pi^\pm-p$ total cross sections. The results shown in Fig. 2 are obtained using $M$ only. Other reaction mechanisms are expected to have substantial contributions. I expect that calculations including conventional amplitudes along with $M$ would show a resonant enhancement on top of a smooth background. (It remains to be proven that the energy dependence of all conventional mechanisms is truly smooth. This is so in the calculation of Ernst[17] which includes, e.g., the energy dependence caused by the Roper resonance in the double scattering graph.) Ultimately it will be necessary to include all of the known amplitudes together. The explicit results of Eq. (4) are presented here to facilitate such computations. For now, I discuss only the six-quark mechanism.

Some comments about the results are in order. First, note that the sizes of the cross sections (~9 $\mu$b/sr at the

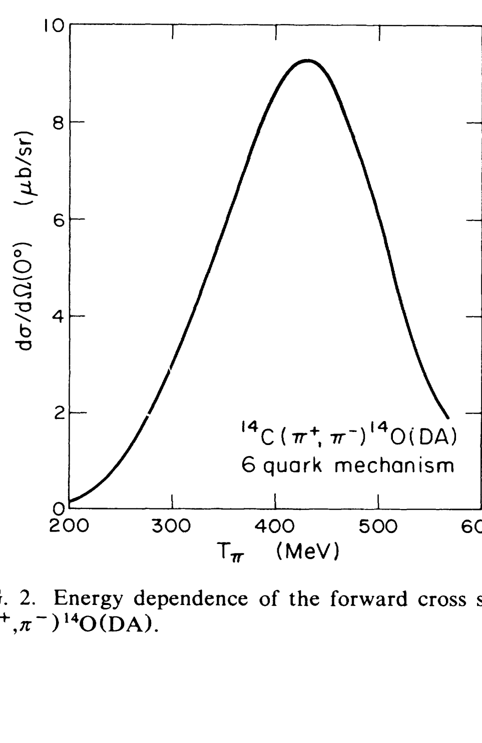


FIG. 2. Energy dependence of the forward cross section for $^{14}$C$(\pi^+,\pi^-)^{14}$O(DA).

peak) are quite large and easily measurable.[8] Second, the peak is at about 425 MeV. The precise location of the peak depends on $R_6$ [through $u(kR_6)$] as well as $E_2$. Furthermore, including the attenuation factor leads to a far greater reduction in the computed cross section at kinetic energies below 300 MeV than above. This effect makes the appearance of the peak more prominent. Of course, the position of the peak is not easy to predict. The energy $E_2$ is uncertain by about 100 MeV due to uncertainties in various bag model terms and parameters. Estimating the magnitude is also difficult because of the dependence on the parameters $\rho$, $r_0$, $R_6$, and $E_m$. The estimate shown here might be changeable by a factor of 3 or so, even if reasonable values of the parameters are used. The simple treatment of distortion also causes uncertainty.

Despite all the uncertainties, if the width of the $L=0$, $S=0$, $T=2$ six-quark bag state is anywhere in the vicinity of what I have chosen, it should be possible to observe a rapid energy dependence. The existence of such a bump could have strong consequences. Excitation of the six-quark bag intermediate states considered here requires six-quark bags (mainly hidden color states) to exist in the nucleus. I hope that the relevant double-charge-exchange experiments may take place soon.

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