Theory of kaonic hydrogen

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We have investigated different forms of the Klein-Gordon equation using phenomenological potentials and attempted to fit the measured strong interaction shifts and widths of kaonic hydrogen and all the heavier kaonic atoms, as well as the scattering lengths obtained from the analysis of low energy $\bar{K}N$ scattering data. We find that we can fit all the data except kaonic hydrogen with various forms of the equation, but in order to fit kaonic hydrogen at the same time it is crucial to have a vector strong interaction term and include the interference between this and the Coulomb force. Our results explain the discrepancy in the sign of the strong interaction shift, which has been a puzzle for many years.

I. INTRODUCTION

The studies of kaonic atoms have followed essentially two major paths. The first starts with the basic $\bar{K}N$ scattering amplitudes, and attempts to use them in extracting kaonic atom's shifts and widths.[1] The second also begins with the scattering amplitudes, but they are replaced by two-body effective potentials which are then used to calculate the kaonic atom's shifts and widths.[2] These approaches have the advantage of using the scattering data as constraints, as opposed to the purely phenomenological method used, for example, in Batty's review of kaonic atoms.[3]

In any case, it seems that all the above procedures fail in the prediction of the kaonic hydrogen width and shift. In Table I the measured widths and shifts of kaonic hydrogen are displayed, together with some of the predictions from the above methods.[4–6] The strong disagreement in the sign of the shift suggests that there may be some inherent defect in the above methods. A possible explanation of this was given by Deloff and Law,[7] but it seems that this suggestion is untenable, owing to the strong distortion needed just below threshold which results in a large effect above threshold, destroying the agreement with the scattering data.[8]

TABLE I. Measured shifts and widths in kaonic hydrogen, and some theoretical predictions.

ϵ (eV)	Γ (eV)	Reference
40 ± 60	0^{+230}_{-0}	Davies *et al.*[a]
370 ± 80	560 ± 260	Izycki *et al.*[b]
193 ± 60	$80^{\pm220}_{-80}$	Bird *et al.*[c]
-293	505	Deloff and Law[d]
-370	502	Alberg, Henley, and Wilets[d]

[a] Reference 4.
[b] Reference 5.
[c] Reference 6.
[d] Reference 6; recalculated by Law and Barrett using AHW4B potential.

In this paper, we reexamine the suggestion of Deloff and Law and find that we can get a consistent fit to all the data on the kaonic atoms, kaonic hydrogen, and the scattering lengths. This comes about because we have included the interference term between the Coulomb and strong interaction potentials, and this turns out to be essential.

II. CHOICE OF WAVE EQUATION

In previous calculations a linear strong interaction term has been added to the Klein-Gordon equation for a meson moving in an electromagnetic field. Two different forms have been used:

$$[\nabla^2+(E-V_c)^2-\mu^2]\psi=2\mu V_K\psi\ , \tag{1a}$$

$$[\nabla^2+(E-V_c)^2]\psi=2EV_K\psi\ . \tag{1b}$$

In form (1a) the strong interaction V_K is treated as a scalar, and terms of order V_K^2 neglected. In form (1b) V_K is treated as the fourth component of a vector, and the Coulomb-nuclear interference term $V_K V_c$ as well as V_K^2 are neglected. For single-channel calculations with local potentials these two forms give essentially the same results when the phenomenological potentials are adjusted to fit the data. In multichannel calculations, however, the two forms give very different results owing to the effect of the different threshold energies in the different channels.

The implication of this is that care must be taken in using published optical potentials which have been fitted to scattering data, and the identical form of the equation should be used for both scattering and atomic energy level calculations.

In our work we have used both scalar and vector potentials and have kept the squared and Coulomb-nuclear interference terms, so that the full equation is

TABLE II. Fitted parameters and overall χ^2 to kaonic hydrogen, scattering lengths, and all kaonic atom's widths and shifts.

Potential:	0	1	2	3
V_p (MeV)	$384.7+i486.6$	$217.6+i503.68$	$250.56+i454.83$	$1372.6+i236.37$
β_p (fm)	0.45	0.611 39	0.708 94	0.466 36
λ_1	0	$-0.462\,71+i4.7388$	$-1.1687+i5.0663$	$-16.22+i5.5836$
β_1 (fm)		0.111 57	0.104 25	0.079 11
λ_2	0	$0.489\,01-i0.115\,89$	$0.563\,28-i0.077\,15$	$2.7408-i1.2337$
β_2 (fm)		0.632 57	0.777 24	0.325 21
V_n (MeV)	$138.3+i149.0$	$1325.5+i693.46$	$138.3+i149.0$	$1325.5+i693.46$
β_n (fm)	0.45	0.2	0.45	0.2
η	0	0	0	0.5
Vector/Scalar	(vector)	(vector)	(vector)	(mixed)
Kaonic H				
ϵ (eV)	-293.8	193	192.4	193.2
Γ (eV)	505.2	81.0	81.4	112.3
χ^2 (all data)		330	276	273
χ^2 (upper levels)		234	182	162

$$\{\nabla^2+[E-V_c-(1-\eta)V_K]^2-[\mu+\eta V_K]^2\}\psi=0\ . \quad (2)$$

V_c is the Coulomb potential, and

$$\mu=M_K M_{nuc}/(M_K+M_{nuc})$$

is the reduced mass. For kaonic hydrogen and $\overline{K}p$ scattering, $V_K=V_{Kp}$. For other nuclei,

$$V_K=\int (ZV_{Kp}+NV_{Kn})\rho_{nuc}dv\ , \quad (3)$$

where Z and N are proton and neutron numbers and

$$\int \rho_{nuc}dv=1\ . \quad (4)$$

Since we are concerned with fairly light nuclei, we assume that the proton and neutron densities are the same. η is a parameter which controls the relative amounts of scalar and vector potential. We have thus chosen these to have the same shape to minimize the number of parameters.

III. CHOICE OF POTENTIALS

Although the $\overline{K}p$ system is strongly coupled to other channels (especially $\Sigma\pi$), we treat it as a single channel

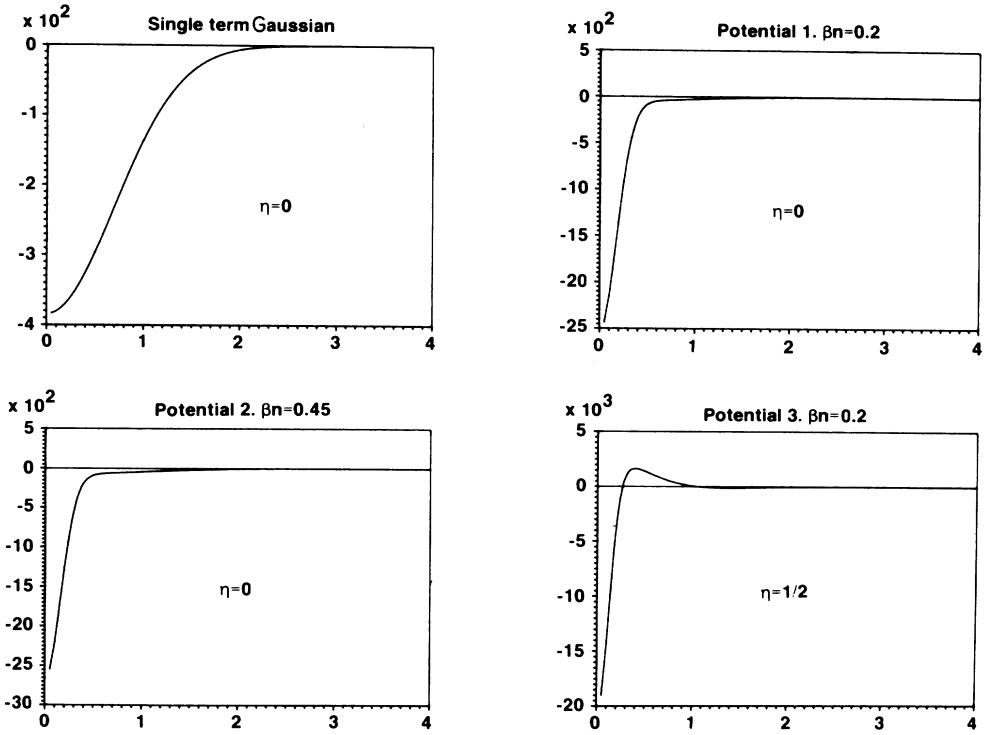

FIG. 1. Real parts of single term Gaussian and three term Gaussian potentials (MeV/fm).

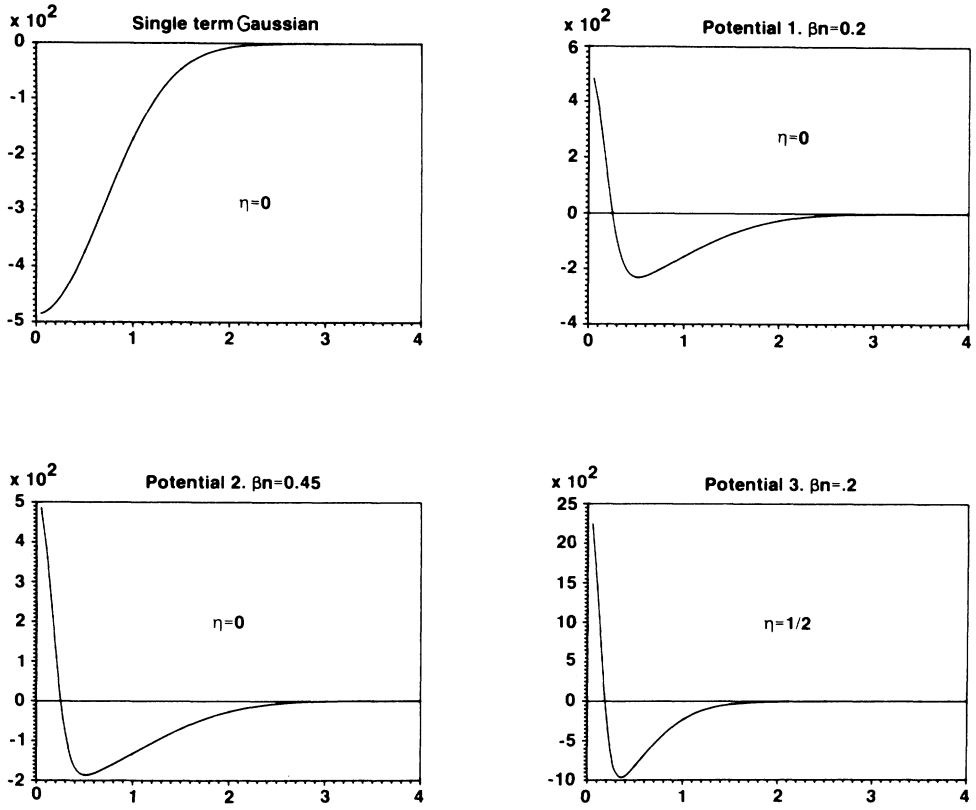

FIG. 2. Imaginary parts of single term Gaussian and three term Gaussian potentials (MeV/fm).

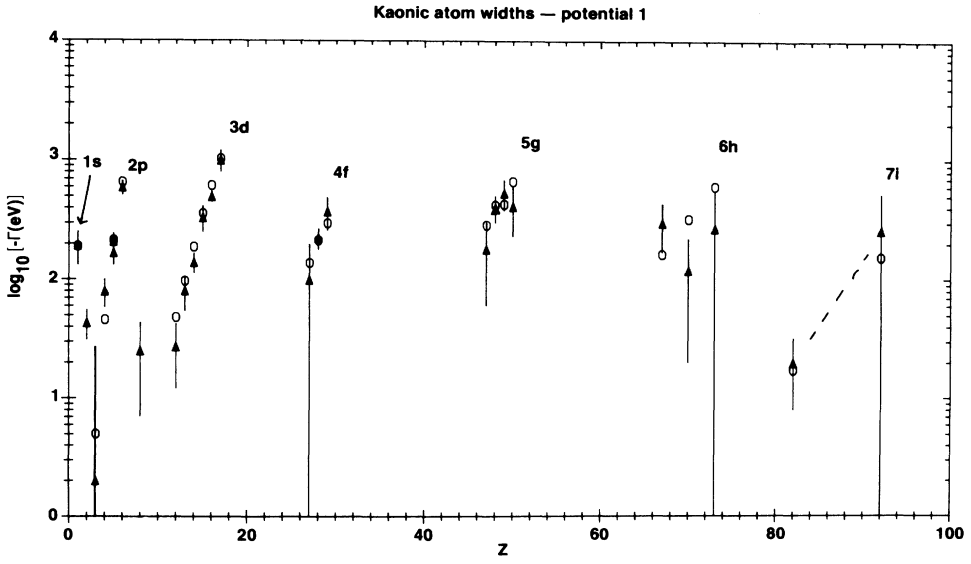

FIG. 3. Calculated versus experimental kaonic atom shifts ($-\epsilon$) for potential 1. The kaonic hydrogen shift is plotted with an opposite sign to get it in the diagram.

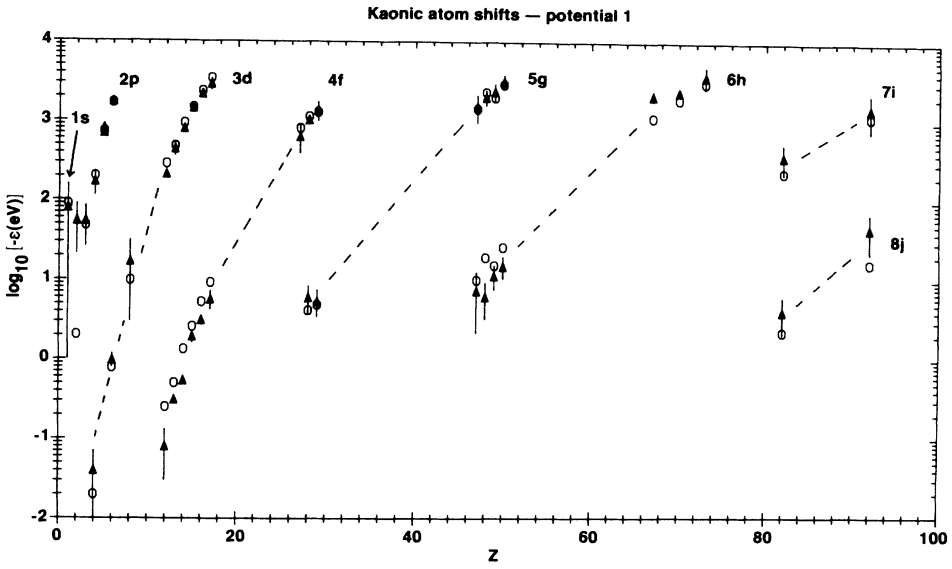

FIG. 4. Calculated versus experimental kaonic atom widths (Γ) for potential 1.

system and simulate the coupling by a local complex potential which we take to be a sum of up to three Gaussians.

$$V_{\mathrm{Kp}} = -V_{\mathrm{p}}[\exp(-0.4586r^2/\beta_{\mathrm{p}}^2) - \lambda_1 \exp(-0.4586r^2/\beta_1^2) - \lambda_2 \exp(-0.4586r^2/\beta_2^2)] \ . \quad (5)$$

The $\overline{\mathrm{K}}$n potential is taken to be a single Gaussian

$$V_{\mathrm{Kn}} = -V_{\mathrm{n}} \exp(-0.4586r^2/\beta_{\mathrm{n}}^2) \ . \quad (6)$$

We thus have four ranges and four complex potential strengths. The three terms used for V_{Kp} were found to be necessary in order to fit kaonic hydrogen and $\mathrm{K}^-\mathrm{p}$ scattering lengths simultaneously. For the nuclear density a Fermi shape was used,

$$\rho = \rho_0\{1 + \exp[(r-c)/a]\}^{-1} \ , \quad (7)$$

and the parameters a and c were taken from Collard *et al.*[9] For the lightest nuclei a Fermi shape may be a rather crude approximation to the actual density, but the strong interaction effects are almost independent of shape in these cases.

IV. CALCULATIONS

The potentials which we used had 12 (real) parameters and these were varied to fit the $\overline{\mathrm{K}}$N scattering lengths of Dalitz *et al.*[10]

$$A_0 = -1.57 + i0.70 \text{ fm}, \quad A_1 = 0.11 + i0.57 \text{ fm} \ ,$$

the kaonic hydrogen shift and width of Bird *et al.*[6]

$$\epsilon = 193 \pm 60 \text{ eV}, \quad \Gamma = 80^{+230}_{-80} \text{ eV} \ ,$$

and the complete set of data for shifts and widths for heavier atoms compiled by Batty.[3] The resulting parameters are given in Table II. Figures 1 and 2 show the sizes and shapes of the real and imaginary parts of the potentials. Figures 3 and 4 give the experimental and calculated values of ϵ and Γ plotted versus atomic number Z, for potential 1. Potentials 2 and 3 give very similar plots and are not shown.

V. DISCUSSION

If the kaonic hydrogen data are omitted, the remaining data can be fitted with a single Gaussian for the shape of V_{Kp} and a pure scalar strong interaction ($\eta = 1$) as found by Deloff and Law.[2] They can also be fitted by a pure vector interaction ($\eta = 0$) or an equal combination $\eta = \frac{1}{2}$.

If we include kaonic hydrogen we cannot fit the data with a pure scalar potential. Moreover we need a much more complicated $\overline{\mathrm{K}}$p potential (with nine real parameters). The shapes of the potentials are shown in Figs. 1 and 2. Apart from the single-term Gaussian, the real parts all have a short range (~ 0.5 fm), and the imaginary part of the potentials have a longer range and a small positive region near the origin. (The latter is far too small to cause unphysical production of particles.) The values of χ^2 (for 68 pieces of data) are given in Table I. The contributions to these values from the widths of the 16 upper levels are also given. Some of these widths have very small errors, e.g., ^{28}Si ($n = 4, l = 3$) has $\Gamma = 0.54 \pm 0.07$ eV and contributes about one third of the value of χ^2. A recalculation using a new cascade code by Turner and Law[11] shows that the new extracted values for the upper widths are more in line with the theoretical models. For example, in the case of kaonic Si, the $4f$ width is extracted as 1.12 eV as compared to the value 0.54 eV from Batty. This number alone accounts for a χ^2 of about 103. With the new extracted value, the effective χ^2's in Table II are all reduced by about 100.

One should, of course, recalculate all the upper level

widths and perform a refitting of the parameters. However, the major trend of the result would not be changed, namely that kaonic hydrogen can be brought into line with the rest of the data if the Coulomb-nuclear interference term is present.

Currently, a calculation based on the cloudy bag quark model[12] is underway to elucidate this interference term on a more fundamental basis.

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