Formal framework for the electroproduction of polarized nucleons from nuclei

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(Received 29 October 1985)

A theoretical framework for the description of the $(\vec{e},e'\vec{p})$ reaction is presented. A set of physically motivated, linearly independent four-vectors is employed to construct a complete set of second rank Lorentz tensors in terms of which the nuclear electromagnetic tensor is expressed. The implications of charge and parity conservation, time reversal, restricted spin dependence, and the final state boundary conditions are developed and applied. The azimuthal angular dependence (in the laboratory frame) of the nuclear tensor is made explicit, leading to the definition of a set of 18 independent response functions. The cross section and the polarization vector of the ejected proton are expressed in terms of the 13 new, spin-dependent response functions. The physical significance of these response functions with regard to spin observables is manifest. The separation of the response functions, both theoretically and experimentally, is discussed. Extensions and restrictions to the general $(\vec{e},e'X)$ reaction, to reactions which violate current and/or parity conservation, and to the case of oriented (polarized) targets are evident. Questions of linear dependence, completeness, and alternative representations of the nuclear tensor for both $(\vec{e},e'\vec{p})$ and the general $(\vec{e},e'X)$ reaction are resolved. Preliminary numerical results indicate the sizes and relative importances of the new response functions and demonstrate their experimental accessibility.

I. INTRODUCTION

High duty factor, continuous-wave electron facilities provide an enhanced capability for performing scattering experiments of the coincidence type. Given appropriate theoretical developments, such experiments may be expected to play an important role in a systematic program of fundamental investigations at these facilities. Although analyses of proton electroproduction (e,e'p) have characteristically focused on the extraction of nuclear bound state information,[1,2] it is also important to explore the extent to which nucleon electroproduction, both with polarized $(\vec{e},e'N)$ and unpolarized (e,e'N) electrons, probes the strong interaction dynamics of the many-body nuclear system. Thus theoretical and experimental investigations of the $(\vec{e},e'N)$ reaction must be performed with both static properties and dynamical degrees of freedom in mind.

In a previous work[3] we examined, in the distorted wave impulse approximation, the effects and the importance of final state interactions (FSI's) for the $(\vec{e},p'p)$ reaction. These effects were found to be large and substantial differences were noted between the microscopic predictions of the nonrelativistic[4] and relativistic[5] (Dirac) approaches to the description of the ejected proton. Future considerations of nucleon electroproduction must include investigations of more fundamental dynamical treatments posed in terms of explicit meson-nucleon and/or quark-dynamic degrees of freedom, as well as of controlling uncertainties which arise from approximation schemes which violate current conservation.

However, there is no reason to neglect the possibility that the ejectile's spin degree of freedom may provide an important source of insight into its fundamental interaction dynamics. At present, although a general form for the (e,e'N) reaction[6,7] has been extended to include electron polarization,[8–11] and although there has been renewed interest in electron scattering from polarized targets,[12] a formal framework for the $(\vec{e},e'\vec{N})$ reaction does not exist. The objective of this paper is to provide such a framework. This is done by the extension of techniques often employed in electron scattering[8,9,13,14] to deal with the problems peculiar to the ejection of polarized nucleons from the many-body system. For convenience, we present our treatment of the electroproduction of polarized nucleons with the $(\vec{e},e'\vec{p})$ reaction in mind. It is, of course, equally applicable to the ejection of neutrons.

As we will see, there are 18 independent response functions for $(\vec{e},e'\vec{N})$, as opposed to five for $(\vec{e},e'N)$, so that there is indeed an additional richness associated with the detection of the ejectile's final-state spin vector. The similarity in form of the cross section to the "super-Rosenbluth" formula of Donnelly[10,11] is an obvious consequence of having a sufficient number of independent four-vectors to span the space. Although we give some preliminary indication of the sizes and importances of the 13 new response functions for $(\vec{e},e'\vec{p})$, we will be mainly concerned with the formal description of the reaction. A detailed study of the new response functions and their physical content will be presented in a subsequent paper.

In Sec. II the general formalism for the electron scattering process is briefly reviewed;[13–15] this serves both to define our notation and to provide the basic groundwork required by the later analysis. Because the ejected nucleon experiences strong interactions with the recoiling residual nucleus, FSI's have important implications not only for detailed calculational predictions, but also for the formal framework itself. Plane wave limits are not adequate.[3] Careful consideration of symmetries and their resultant constraints is therefore necessary. Thus, Sec. III consists of a brief treatment of the constraints on the nuclear electromagnetic tensor. The constraint due to parity conservation is obtained and the relationship between time reversal and the final state scattering boundary conditions is described. The fact that time reversal does not yield an immediately useful constraint upon the nuclear tensor for $(\vec{e},e'\vec{N})$, due to FSI's and the presence of the boundary conditions, is explicated. In Sec. IV a systematic construction of the general form of the nuclear electromagnetic tensor is given. Extensions to the general $(\vec{e},e'X)$, as well as other reactions, are indicated. In Sec. V we employ the results of Sec. IV to separate the (laboratory frame) azimuthal angular dependences and identify a set of 18 response functions. Their relationship to the ejectile's polarization vector defines a natural set of spin variables in terms of the projections of the polarization vector and in terms of the response functions. The experimental separation and identification of the various response functions is discussed. In Sec. VI the theoretical prediction of the complete set of response functions is described. Preliminary indications of the sizes and importances of the response functions are presented. The special features of the plane wave limit are discussed relative to time reversal and the FSI's. Section VI concludes with a brief, qualitative discussion of the physics issues and implications associated with the new response functions. Section VII consists of a summary of our results and a prospectus for further work.

II. ELECTROPRODUCTION OF NUCLEONS

The proton electroproduction process, with the usual assumption of a one-photon—exchange mechanism, is schematically illustrated in Fig. 1. The initial (final) electron spin and four momentum are represented by s_e and k (s'_e and k'), respectively. The initial and final momenta of the target are P and P', while the exchanged photon

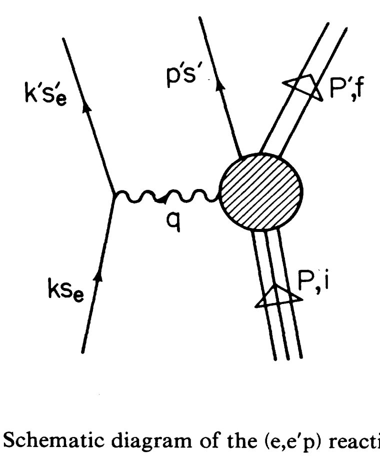

FIG. 1. Schematic diagram of the (e,e'p) reaction.

momentum is $q=k-k'$ and the momentum and spin of the ejectile are p' and s', respectively. In the following, the electron charge is e, m_e (m) is the electron (nucleon) mass, $\epsilon_k^2=\mathbf{k}^2+m_e^2$, $E_p^2=p^2+m^2$, and our conventions for Dirac spinors and gamma matrices follow the standard conventions.[16]

In the laboratory frame (target initially at rest), the differential cross section for the electroproduction of a polarized nucleon of four-momentum p' and (rest-frame) spin $\mathbf{s}'_R$ by electrons of (initial) helicity h is given by

$$d\sigma\,|_{h,\hat{s}'_R}=\frac{2\pi m_e}{|\mathbf{k}|}\frac{e^4}{q^4}\left[\frac{\eta_{\mu\nu}W^{\mu\nu}(\hat{s}'_R)}{m_e^2}\right]\left[\frac{m_e}{\epsilon_{k'}}\frac{d^3k'}{(2\pi)^3}\right]\times\left[\frac{m}{E_{p'}}\frac{d^3p'}{(2\pi)^3}\right], \tag{2.1}$$

where the electron tensor $\eta_{\mu\nu}$ is defined as usual in terms of the electron current matrix elements:

$$\eta_{\mu\nu}=m_e^2\sum_{s'_e} j_\mu^\dagger j_\nu\ , \tag{2.2}$$

where

$$j_\nu=\bar{u}(k',s'_e)\gamma_\nu u(k,s_e)\ . \tag{2.3}$$

For later use we note the explicit form of the electron tensor in the extreme relativistic limit (ERL), which is applicable at intermediate energies,[17–19]

$$\eta^{\mu\nu}=\tfrac{1}{2}(k'^\mu k^\nu+k'^\nu k^\mu-k\cdot k'g^{\mu\nu}+ih\epsilon^{\mu\nu\rho\sigma}k'_\rho k_\sigma)\ , \tag{2.4}$$

where h is the initial electron helicity and $\epsilon^{\mu\nu\rho\sigma}$ is[18] the completely antisymmetric Levi-Civitá tensor. It is convenient to express Eq. (2.4) as

$$\eta^{\mu\nu}=\left[K^\mu K^\nu+\frac{q^2}{4}\left[g^{\mu\nu}-\frac{q^\mu q^\nu}{q^2}\right]\right]+\frac{ih}{2}\epsilon^{\mu\nu\rho\sigma}K_\rho q_\sigma\ , \tag{2.5}$$

where $K=(k'+k)/2$, so that $q\cdot K=0$ in the ERL and $\eta^{\mu\nu}$ manifestly satisfies the current conservation requirements $q_\mu\eta^{\mu\nu}=0=\eta^{\mu\nu}q_\nu$. Thus in the separation of the electron tensor into parts symmetric and antisymmetric under exchange of its indices $\eta^{\mu\nu}=\eta_S^{\mu\nu}+\eta_A^{\mu\nu}$,

$$\eta_S^{\mu\nu}=\left[K^\mu K^\nu+\frac{q^2}{4}\left[g^{\mu\nu}-\frac{q^\mu q^\nu}{q^2}\right]\right] \tag{2.6}$$

and

$$\eta_A^{\mu\nu}=\frac{ih}{2}\epsilon^{\mu\nu\rho\sigma}K_\rho q_\sigma\ . \tag{2.7}$$

Since in view of Eq. (2.2) η is a Hermitian tensor, its symmetric and antisymmetric parts are real and pure imaginary, respectively.

The nuclear electromagnetic tensor $W^{\mu\nu}$ which appears in Eq. (2.1) is given by

$$W^{\mu\nu}(\hat{s}'_R)=\overline{\sum_i}\int\frac{d^3P'}{(2\pi)^3}\Phi(\mathbf{P}')(2\pi)^3\times\delta^4(k-k'+P-P'-p)\Gamma^{\mu\nu}\ , \tag{2.8}$$

where the notation $\overline{\sum}_i$ denotes the usual average over initial states, $\phi(\mathbf{P}')$ is a density of states factor appropriate to the intrinsic spin of the residual nucleus [here the recoil is nonrelativistic, $\phi(\mathbf{P}')=1$], and $\Gamma^{\mu\nu}$ is defined by

$$\Gamma^{\mu\nu}=\sum_F J^{\mu}(q)^{\dagger}J^{\nu}(q)\ , \tag{2.9}$$

where the sum is over the set of undistinguished intrinsic final states of the residual hadronic system and the $J^{\mu}(q)$ are the matrix elements of the nuclear electromagnetic current operator $\hat{J}^{\mu}(q)$ analogous to Eq. (2.3), viz.,

$$J^{\mu}(q)=\langle F\,|\,\hat{J}^{\mu}(q)\,|\,I\rangle\ . \tag{2.10}$$

If the leptonic current is considered to be completely known and the one-photon—exchange mechanism reliable, then the foregoing considerations isolate the unknown quantities of central interest, namely the nuclear current matrix elements, or the tensor $\Gamma^{\mu\nu}$ of Eq. (2.9), from the rest of the (known) ingredients required for the construction of the cross section.

The full physical matrix element of Eq. (2.10) is dependent on both the initial and final physical hadronic circumstances and the interaction dynamics of the theoretical framework adopted, be it nonrelativistic or relativistic potential theory, meson theory, quark models, etc. The separation of Eq. (2.10) into initial and final states and a current operator, however, is not unique, even when the dynamics is fully specified. It depends upon the choice of initial and final basis states and suppressed degrees of freedom, for example.[3]

For these reasons it is important in constructing a general framework for nucleon electroproduction to avoid specifying the details of the description of the nuclear states and the hadronic current operator. Retaining complete generality, we write Eq. (2.10) in more explicit form as

$$J^{\mu}(q)=\langle p',\mathbf{s}'_R,(-);F,P'\,|\,\hat{J}^{\mu}(q)\,|\,I,P\rangle\ , \tag{2.11}$$

so that

$$\Gamma^{\mu\nu}=\sum_F \langle p',\mathbf{s}'_R,(-);F,P'\,|\,\hat{J}^{\nu}(q)\,|\,I,P\rangle \times\langle I,P\,|\,\hat{J}^{\mu}(q)^{\dagger}\,|\,p',\mathbf{s}'_R,(-);F,P'\rangle\ . \tag{2.12}$$

In Eqs. (2.11) and (2.12) the initial state consists of an intrinsic target state, I, with total four-momentum P. The final state is a scattered wave ejectile-nucleus state which asymptotically consists of the intrinsic state F of the residual nucleus with total four-momentum P' and the ejected nucleon of momentum p' and rest-frame spin $\mathbf{s}'_R$. The incoming scattered wave boundary condition is denoted by $(-)$. The distinction between incoming and outgoing scattered wave boundary conditions is important for time reversal considerations. It is also useful in considering the properties of Eqs. (2.11) and (2.12) in the next section to keep in mind the limiting case of a single-particle Dirac model, in which case the current operator in Eqs. (2.11) and (2.12) might be approximated by the free nucleon current operator (although this usually results in current conservation violations), the final ejectile adjoint state is a Dirac adjoint, and the overlap of the initial and final target states is a single-particle shell model state.

III. CONSTRAINTS, SYMMETRIES, AND BOUNDARY CONDITIONS

Current conservation (gauge invariance) requires that the nuclear current matrix element of Eq. (2.11) satisfy $q_\mu J^{\mu}(q)=0$ so that the tensor $\Gamma^{\mu\nu}$ satisfies $q_\mu\Gamma^{\mu\nu}=0=\Gamma^{\mu\nu}q_\nu$ or

$$q_\mu W^{\mu\nu}(\hat{\mathbf{s}}'_R)=0=W^{\mu\nu}(\hat{\mathbf{s}}'_R)q_\nu\ . \tag{3.1}$$

The dependence of the nuclear tensor upon the ejectile's spin four-vector is restricted to be at most linear; a direct result of its spin $\frac{1}{2}$ nature. This general property is easily appreciated in the case that the ejected nucleon's asymptotic state is described by the free Dirac equation upon noting that Eq. (2.12) can be written

$$\Gamma^{\mu\nu}=\bar{u}(p',\mathbf{s}'_R)\hat{\Gamma}^{\mu\nu}u(p',\mathbf{s}'_R)\ , \tag{3.2}$$

where all the complications reside in $\hat{\Gamma}^{\mu\nu}$. But this can be written

$$\Gamma^{\mu\nu}=\mathrm{Tr}\left[\left[\frac{\not{p}'+m}{2m}\right]\left[\frac{1+\gamma_5\not{s}'}{2}\right]\hat{\Gamma}^{\mu\nu}\right]\ , \tag{3.3}$$

where s' is twice the ejectile's spin four-vector, $s'\cdot p'=0$, which is given in terms of $\hat{\mathbf{s}}'_R$ by

$$s'=\left[\frac{\hat{\mathbf{s}}'_R\cdot\mathbf{p}'}{m},\hat{\mathbf{s}}'_R+\frac{(\hat{\mathbf{s}}'_R\cdot\mathbf{p}')\mathbf{p}'}{m(E_{p'}+m)}\right]\ . \tag{3.4}$$

Since Eq. (3.3) contains only one power of s', $\Gamma^{\mu\nu}$ may be restricted to be, at most, linear in s'.

To examine the implications of parity and time reversal symmetries, we consider electroproduction from the (nondegenerate) ground state of the target nucleus, leading to a final residual-nuclear state of intrinsic angular momentum J. We also restrict ourselves to the case where the residual target's spin projection is not observed, so that the sum over F in Eq. (2.9) is a sum over spin projections. (These restrictions are made for simplicity; generalizations are straightforward. For example, the restriction on the initial state can be removed simply by defining Γ to include an appropriate average over intrinsic initial states.) Then, from Eqs. (2.12) and (3.4), we have

$$\Gamma^{\mu\nu}=\Gamma^{\mu\nu}(q,p',P,s',(-))\ . \tag{3.5}$$

We consider first the parity operator Π. From Eq. (2.12) we have

$$\Gamma^{\mu\nu}=\sum_F\{\langle p',\mathbf{s}'_R,(-);F,P'\,|\,\Pi^{-1}[\Pi\hat{J}^{\nu}(q)\Pi^{-1}]\Pi\,|\,I,P\rangle\langle I,P\,|\,\Pi^{-1}[\Pi\hat{J}^{\mu}(q)\Pi^{-1}]^{\dagger}\Pi\,|\,p',\mathbf{s}'_R,(-);F,P'\rangle\}\ . \tag{3.6}$$

The exact form of the parity (and time reversal) operator depends, of course, on the particular theoretical framework involved. However, the physical picture uniformly imposed in the definition of the parity (and time reversal) operation implies certain results of general validity. The parity operator reverses the space components (only) of the momenta of free plane wave states and leaves the rest-frame spin $\mathbf{s}'_R$ unchanged. Since the electromagnetic current operator should transform as a true vector, it follows that

$$\Pi \hat{J}_\mu(q)\Pi^{-1} = \pi(\mu)\hat{J}_\mu(\tilde{q}) , \tag{3.7}$$

where $q=(q_0,\mathbf{q})$ implies $\tilde{q}=(q_0,-\mathbf{q})$ and where, numerically, $\pi(\mu)=g_{\mu\mu}$. No summation convention is assumed to be associated with $\pi(\mu)$, as in Eq. (3.7), for example. Equation (3.7) simply states that the space components of the parity transformed current operator are reversed in the parity transformed reference frame (where $q\rightarrow\tilde{q}$). It is easy to verify that Eq. (3.7) follows for the free nucleon current operator[17]

$$\hat{J}^\mu(q) = \int d^4k d^4k' \,|\, k'\rangle \left[F_1(q^2)\gamma^\mu + \frac{F_2(q^2)}{2m} i\sigma^{\mu\nu}q_\nu \right] \times \delta^4(k'+q-k)\langle k \,| \ , \tag{3.8}$$

where F_1 and F_2 are the electromagnetic form factors of the nucleon, the abstract Dirac kets $|\,k\rangle$ signify free momentum-space eigenvectors with four-momentum k^μ and satisfy

$$\langle k' \,|\, k\rangle = \delta^4(k'-k) , \tag{3.9}$$

and the free, Dirac (momentum-space) parity operator is[15]

$$\Pi = \gamma_0 P_k , \tag{3.10}$$

where $P_k\,|\,p\rangle = |\,\tilde{p}\rangle$. Combining Eqs. (3.6) and (3.7) yields

$$\Gamma^{\mu\nu} = \pi(\mu)\pi(\nu)\sum_{F_\pi} \langle \tilde{p}', \mathbf{s}'_R, (-); F_\pi, \tilde{P}' \,|\, \hat{J}^\nu(\tilde{q}) \,|\, I, \tilde{P}\rangle \times \langle I, \tilde{P} \,|\, \hat{J}^\mu(\tilde{q})^\dagger \,|\, \tilde{p}', \mathbf{s}'_R, (-); F_\pi, \tilde{P}'\rangle , \tag{3.11}$$

where F_π denotes the parity transform of F and where we have employed the previously asserted properties of the parity operator, as well as the following.

(1) The bilinearity of Eq. (3.6) obviates any phases from the parity operation.

(2) The parity operator commutes with the Hamiltonian.

(3) I is nondegenerate, so, in view of (1) and (2), $I\rightarrow I$ in Eq. (3.6).

(4) The set of states F transforms into itself under the parity operation.

(5) The parity operator is a linear (as opposed to antilinear) operator, so that (2) implies that the parity operator commutes with the Møller operator

$$\Omega_\pm = \lim_{t\rightarrow -\infty} [\exp(\pm iHt)\exp(\mp iH_0 t)] , \tag{3.12}$$

so that the boundary condition remains unchanged. Making use of Eq. (3.4), we conclude from Eqs. (3.5), (3.6), and (3.11) that

$$\Gamma^{\mu\nu}(\tilde{q},\tilde{p}',\tilde{P},-\tilde{s}',(-)) = \pi(\mu)\pi(\nu)\Gamma^{\mu\nu}(q,p',P,s',(-)) . \tag{3.13}$$

Equation (3.13), or the equivalent result for $W^{\mu\nu}(\hat{\mathbf{s}}'_R)$, is the parity constraint upon the form of the nuclear tensor.

Turning to an examination of time reversal symmetries, it is convenient to consider rather than the time reversal operator $\mathscr{T}$ the product $\mathscr{T}\Pi$. The general features of the $\mathscr{T}$ operation are analogous to those of Π, except that $\mathscr{T}$ reverses the rest frame spin. Thus the analog of Eq. (3.7) is

$$\mathscr{T}\hat{J}^\mu(q)\mathscr{T}^{-1} = \pi(\mu)\hat{J}^\mu(\tilde{q}) , \tag{3.14}$$

which can easily be verified for the current operator[20] of Eq. (3.18) and the free Dirac (momentum-space) $\mathscr{T}$ operator $\mathscr{T}=i\gamma_1\gamma_3 P_k K$, where K is the complex conjugation operator. To consider the $\mathscr{T}\Pi$ operation we employ the technique analogous to the transition from Eq. (2.12) to (3.6), except using $\mathscr{T}$ instead of Π and starting with Eq. (3.11). If we then employ Eq. (3.14), we find

$$\Gamma^{\mu\nu} = \sum_F \langle \tilde{p}', \mathbf{s}'_R, (-); F_\pi, \tilde{P}' \,|\, \mathscr{T}^{-1}[\hat{J}^\nu(q)]\mathscr{T} \,|\, I, \tilde{P}\rangle \times \langle I, \tilde{P} \,|\, \mathscr{T}^{-1}[\hat{J}^\mu(q)]^\dagger \mathscr{T} \,|\, \tilde{p}', \mathbf{s}'_R, (-); F_\pi, \tilde{P}'\rangle . \tag{3.15}$$

The only changes in going from the five assertions for Π made after Eq. (3.11) to the analogous statements here are the replacement of Π by $\mathscr{T}$ and the fact that $\mathscr{T}$ is an antilinear operator. The antilinearity of $\mathscr{T}$ implies that $\mathscr{T}\Omega_\pm\mathscr{T}^{-1}=\Omega_\mp$, so that the boundary conditions are changed by $\mathscr{T}$ and, further, the initial and final states are interchanged in each of the matrix elements, or, equivalently, μ and ν are interchanged. Thus, Eq. (3.15) becomes

$$\Gamma^{\mu\nu} = \sum_{F_{\pi\mathscr{T}}} \langle p', -\mathbf{s}'_R, (+); F_{\pi\mathscr{T}}, P' \,|\, \hat{J}^\mu(q) \,|\, I, P\rangle \times \langle I, P \,|\, \hat{J}^\nu(q)^\dagger \,|\, p', -\mathbf{s}'_R, (+); F_{\pi\mathscr{T}}, P'\rangle , \tag{3.16}$$

so that, noting Eq. (3.4), we have, in view of Eqs. (3.5) and (3.6),

$$\Gamma^{\mu\nu}(q,p',P,s',(-)) = \Gamma^{\nu\mu}(q,p',P,-s',(+)) . \tag{3.17}$$

Equation (3.17) states that the nuclear tensor is symmetric under the simultaneous interchange of the tensor indices, changing the sign of the spin vector, and switching the boundary conditions. This would provide an immediate constraint upon the nuclear electroproduction tensor if it were not for the change of the boundary conditions. In the general case $\mathscr{T}$ does not provide a constraint on the form of the electroproduction tensor (it relates it to an electroproduction process which, due to the scattered flux at infinity, involves a complicated asymptotic configuration). The effect of Eq. (3.17) in certain limiting cases and approximations is discussed in Sec. VI.

IV. CONSTRUCTION OF THE NUCLEAR TENSOR

In addition to transforming as a Hermitian, second rank Lorentz tensor, the nuclear tensor must satisfy the three constraints of the preceding section: gauge invariance, no dependence on s' beyond linear, and parity conservation. The general form of the nuclear electroproduction tensor for $(\vec{e},e'\vec{N})$ must be constructed from the available independent variables, which are the four-vectors q, p', P, and s'. Several other vectors can be fashioned from this set by making use of the completely antisymmetric Levi-Civitá tensor $\epsilon^{\mu\nu\rho\sigma}$ and any three of the above vectors, e.g.,

$$\xi^\mu = \epsilon^{\mu\nu\rho\sigma} q_\nu p'_\rho P_\sigma \ . \tag{4.1}$$

Clearly, there are three other such vectors.

The independent Lorentz scalars which can be constructed from these four-vectors can be separated according to their parity properties. The true scalars are q^2, $q\cdot p'$, $q\cdot P$, $p'\cdot P$, and $\xi\cdot s'$ (note $p'^2=m^2$, $s'^2=-1$, and $P^2=M_T^2$), while the pseudoscalars are $q\cdot s'$ and $P\cdot s'$ (note $p'\cdot s'=0$). Scalars obtained by contracting two of the vectors constructed from $\epsilon^{\mu\nu\rho\sigma}$ are not independent of those listed. The contraction of higher order tensors constructed from the available four-vectors is also easily seen to yield no new independent scalars: the set of scalars listed above is complete.

Prior to the construction of the nuclear tensor, it is also convenient to construct a set of orthogonal four-vectors with which to span the four-dimensional space. The gauge invariance (current conservation) condition requires that the nuclear response tensor $W^{\mu\nu}$ lies in a subspace orthogonal to q. It is therefore desirable to choose q as one of the basis vectors and to replace p' and P by the usual "gauge invariant" forms

$$V_i = P - \frac{P\cdot q}{q^2} q \tag{4.2}$$

and

$$V_f = p' - \frac{p'\cdot q}{q^2} q \ , \tag{4.3}$$

which satisfy $q\cdot V_i = q\cdot V_f = 0$. If we now define

$$\bar{V}_f = V_f - \frac{V_f\cdot V_i}{V_i^2} V_i \ , \tag{4.4}$$

then the three vectors q, V_i, and $\bar{V}_f$ form an orthogonal, linearly independent set. A fourth independent vector is required to span the space. It turns out to be desirable for the remaining vector to be independent of s' and thus to choose ξ^μ of Eq. (4.1). Note that ξ^μ may also be written

$$\xi^\mu = \epsilon^{\mu\nu\rho\sigma} q_\nu (\bar{V}_f)_\rho (V_i)_\sigma \ , \tag{4.5}$$

so that $q\cdot\xi = \bar{V}_f\cdot\xi = V_i\cdot\xi = 0$. Thus, ξ^μ completes the set of orthogonal, linearly independent vectors. Note that the construction of such a set of four vectors implies that (in general) three are spacelike, one is timelike, and that the set spans the four-dimensional space.[21]

It follows that the nuclear tensor, or for that matter any second rank Lorentz tensor, can be expanded in terms of these vectors as

$$W^{\mu\nu}(\hat{s}'_R) = \sum_{i,j} F_{ij} Z_i^\mu Z_j^\nu \ , \tag{4.6}$$

where the set Z_i, $i=1-4$, is the set $q, V_i, \bar{V}_f, \xi$ and where the coefficient functions F_{ij} are Lorentz scalar functions of the independent scalars identified above. In the following we shall give up the orthogonality of the decomposition of Eq. (4.6) and employ, for simplicity, V_f rather than $\bar{V}_f$. Applying the gauge invariance conditions to Eq. (4.6) yields $(Z_1 = q)$

$$q_\mu W^{\mu\nu}(\hat{s}'_R) = q^2 \sum_j F_{1j} Z_j^\nu = 0 \tag{4.7a}$$

and

$$W^{\mu\nu}(\hat{s}'_R) q_\nu = \sum_i F_{i1} Z_i^\mu q^2 = 0 \ , \tag{4.7b}$$

so that, in view of the linear independence of the Z_i, $F_{1j} = F_{j1} = 0$ for all j and Eq. (4.6) becomes

$$W^{\mu\nu}(\hat{s}'_R) = \sum_{i,j\neq 1} F_{ij} Z_i^\mu Z_j^\nu \ , \tag{4.8}$$

consistent with gauge invariance.

The decomposition of the nuclear electroproduction tensor given in Eq. (4.8) is of more general validity than just the $(\vec{e},e'\vec{N})$ reaction. The basic assumption necessary for its validity is, of course, current conservation [in the absence of current conservation—for example, in the weak reaction $(\nu,\nu'X)$—one simply reverts to Eq. (4.6)]. The nine independent tensors of Eq. (4.8) are also valid for the restriction to the (unpolarized) $(\vec{e},e'N)$ reaction[22] as well as for the general $(\vec{e},e'X)$ reaction[22] with obvious, but generally nonunique,[10] reinterpretations of the momenta p' and P, whether or not any spin observables are detected. These are advantages of constructing the second rank tensors from a set of spin-independent four-vectors. As will be described shortly, another such advantage lies in the effective restriction of the (at most linear) spin dependence to the scalar coefficient functions. This forces the ejectile's spin vector to occur only in scalar products with the momenta that, in turn, permit a simple separation of the ejectile's polarization vector and the spin-summed cross section.

It should also be noted at this point that, although we have restricted ourselves to nondegenerate initial target states for simplicity, virtually all of the *preceding* analysis is also applicable in, and advantageous for, the case of oriented (polarized) targets.[10,14,15] In this case, of course, the initial target angular momentum vector, which transforms like the ejectile spin vector under parity and time reversal, must be included in the set of four vectors available for the construction of a complete, linearly independent set of scalars. Similarly, the extension of the complete framework to charge and/or parity nonconserving reactions will also be seen to be immediate.

The nuclear electroproduction tensor of Eq. (4.8) can be decomposed into its symmetric and antisymmetric parts $(A^\mu B^\nu)_{S,A} \equiv (A^\mu B^\nu \pm A^\nu B^\mu)$, viz.,

$$W^{\mu\nu}(\hat{s}'_R) = W_S^{\mu\nu}(\hat{s}'_R) + W_A^{\mu\nu}(\hat{s}'_R) \ . \tag{4.9}$$

One finds

$$W_S^{\mu\nu}(\hat{s}'_R)=F_1[G^{\mu\nu}]+F_2[V_i^\mu V_i^\nu]+F_3[V_f^\mu V_f^\nu]+F_4[V_i^\mu V_f^\nu]_S+F_5[V_i^\mu \xi^\nu]_S+F_6[V_f^\mu \xi^\nu]_S \tag{4.10}$$

and

$$W_A^{\mu\nu}(\hat{s}'_R)=F_7[V_i^\mu V_f^\nu]_A+F_8[V_i^\mu \xi^\nu]_A+F_9[V_f^\mu \xi^\nu]_A\ , \tag{4.11}$$

where

$$G^{\mu\nu}=g^{\mu\nu}-\frac{q^\mu q^\nu}{q^2}\ , \tag{4.12}$$

and we have used the properties of the Levi-Civitá tensor to replace $\xi^\mu\xi^\nu$ by $G^{\mu\nu}$. In Eqs. (4.10) and (4.11) the coefficient functions F_i, $i=1$—9, are functions of the available scalars listed following Eq. (4.1). Since $W^{\mu\nu}(\hat{s}'_R)$ is at most linear in s', and since this dependence is confined to the coefficient functions by the expansion of Eqs. (4.6) and (4.8), it is useful to manifest any dependence of the coefficient functions F_i upon the spin-dependent scalars and to isolate coefficient functions, W_i, which are functions only of the momentum-space (*true*) scalars q^2, $q\cdot p'$, $q\cdot P$, and $p'\cdot P$.

In the absence of a further constraint, this leads to 36 independent terms, four terms for each of the nine tensors of Eqs. (4.10) and (4.11): a spin-independent term and terms linear in $q\cdot s'$, $P\cdot s'$, and $\xi\cdot s'$. However, under parity, the spin-independent and $\xi\cdot s'$ terms are true scalars, while $q\cdot s'$ and $P\cdot s'$ are pseudoscalars [see Eqs. (3.13) and (4.1)]. Since ξ is a (parity) pseudovector while the other basis vectors are true vectors, terms in Eqs. (4.10) and (4.11) linear in ξ^ν must be matched with coefficient functions F_i which are pseudoscalar. Therefore, only one of the two categories of spin-dependent scalars is permitted to contribute for each of the explicit tensors. Thus, there are only 18 independent terms. It is easily verified that the result for the $(\vec{e},e'\vec{N})$ electroproduction tensor that is consistent with the parity constraint of Eq. (3.13) is

$$W_S^{\mu\nu}(\hat{s}'_R)=(W_1+\widetilde{W}_1\xi\cdot s')G^{\mu\nu}+(W_2+\widetilde{W}_2\xi\cdot s')V_i^\mu V_i^\nu+(W_3+\widetilde{W}_3\xi\cdot s')V_f^\mu V_f^\nu$$
$$+(W_4+\widetilde{W}_4\xi\cdot s')[V_i^\mu V_f^\nu]_S+(W_5 q\cdot s'+\widetilde{W}_5 P\cdot s')[V_i^\mu \xi^\nu]_S+(W_6 q\cdot s'+\widetilde{W}_6 P\cdot s')[V_f^\mu \xi^\nu]_S \tag{4.13}$$

and

$$W_A^{\mu\nu}(\hat{s}'_R)=(W_7+\widetilde{W}_7\xi\cdot s')[V_i^\mu V_f^\nu]_A+(W_8 q\cdot s'+\widetilde{W}_8 P\cdot s')[\xi^\mu V_i^\nu]_A+(W_9 q\cdot s'+\widetilde{W}_9 P\cdot s')[\xi^\mu V_f^\nu]_A\ , \tag{4.14}$$

where the coefficient functions W_i and $\widetilde{W}_i$ are functions of (only) the momentum-space scalars.

V. RESPONSE FUNCTIONS, CROSS SECTION, AND POLARIZATION

In the preceding section we obtained the general form of the nuclear electroproduction tensor $W^{\mu\nu}$. In this section we consider the contraction of the nuclear and leptonic tensors and the construction of expressions for the physical scattering observables in the laboratory frame. We manifest the azimuthal angular dependence of the nuclear tensor and thus separate a set of (azimuthal) angle-independent response functions and obtain expressions for the cross section and the ejectile's polarization vector in terms of a physically motivated set of response functions.

The coordinate system which we choose for the evaluation of the various quantities of interest in the laboratory frame is depicted in Fig. 2. The basic quantity of interest is the contraction of the nuclear and leptonic tensors, which we decompose as

$$\Sigma=\eta_{\mu\nu}W^{\mu\nu}(\hat{s}'_R)=\eta_{\mu\nu}^S W_S^{\mu\nu}(\hat{s}'_R)+\eta_{\mu\nu}^A W_A^{\mu\nu}(\hat{s}'_R)=\Sigma_S+\Sigma_A\ . \tag{5.1}$$

Considering first Σ_S, the symmetric/symmetric contraction, and recalling Eq. (2.6),

$$\eta_S^{\mu\nu}=\left[K^\mu K^\nu+\frac{q^2}{4}\left[g^{\mu\nu}-\frac{q^\mu q^\nu}{q^2}\right]\right]\ , \tag{5.2}$$

we see that, in the coordinate system of Fig. 2, $\eta_S^{\mu\nu}=0$ if either $\mu=1$ or $\nu=1$, unless $\mu=\nu$. Further, the electron tensor satisfies the charge conservation condition which enables us to eliminate any 3 index in $\eta^{\mu\nu}$ in favor of the zero index, e.g., $|\mathbf{q}|\eta^{3\nu}=q_0\eta^{0\nu}$. In view of these conditions, it is evident that the construction of Σ_S depends (in the laboratory frame) only on the four components of $W_S^{\mu\nu}(\hat{s}'_R)$: $W_S^{00}(\hat{s}'_R)$, $W_S^{11}(\hat{s}'_R)+W_S^{22}(\hat{s}'_R)$, $W_S^{11}(\hat{s}'_R)-W_S^{22}(\hat{s}'_R)$, and $W_S^{02}(\hat{s}'_R)+W_S^{20}(\hat{s}'_R)$. Since the momentum-space scalars are all independent of the azimuthal angle β, the only azimuthal dependence in $W^{\mu\nu}(\hat{s}'_R)$ resides in the spin-dependent scalars (which are already explicit) and in the tensors. The azimuthal angular dependence of the four relevant components of the constituent tensors of $W^{\mu\nu}(\hat{s}'_R)$, in the system of Fig. 2, may be obtained by inspection of Eq. (4.13). This can

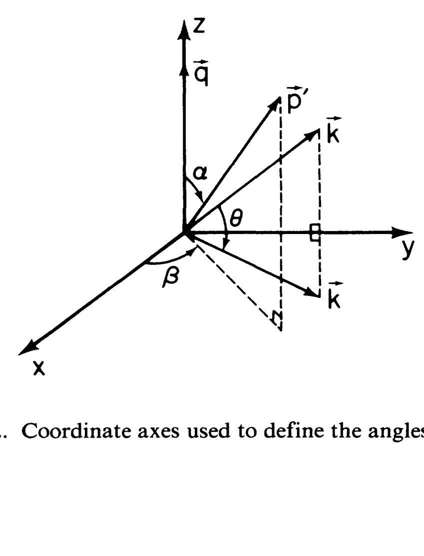

FIG. 2. Coordinate axes used to define the angles α and β.

then be used to define a set of response functions which are independent of the azimuthal angle β; for example,

$$\tfrac{1}{2}(R_L+\tilde{R}_L\xi\cdot s')=\int_{\rm line}dE_{p'}W^{00}(\hat{\mathbf{s}}'_R)\ , \tag{5.3}$$

where we have followed the usual practice of defining response functions in terms of an integral over a linewidth in the missing mass spectrum, and where the factor of $\frac{1}{2}$ is inserted to conform to the usual normalization of spin-independent response functions.

Similarly, we have, from Eq. (2.7),

$$\eta_A^{\mu\nu}=\frac{ih}{2}\epsilon^{\mu\nu\rho\sigma}K_\rho q_\sigma\ , \tag{5.4}$$

so that $\eta_A^{\mu\nu}=0$ for $\mu=\nu$ and, since neither q nor K has a 1 component, either $\mu=1$ or $\nu=1$. Again, gauge invariance allows the elimination of the 3 components in favor of the 0 components. Thus, the antisymmetric nature of $\eta_A^{\mu\nu}$ implies that the only components of $W^{\mu\nu}(\hat{\mathbf{s}}'_R)$ needed to construct Σ_A are $[W^{10}(\hat{\mathbf{s}}'_R)-W^{01}(\hat{\mathbf{s}}'_R)]$ and $[W^{12}(\hat{\mathbf{s}}'_R)-W^{21}(\hat{\mathbf{s}}'_R)]$. The azimuthal dependences are obtained by inspection of Eq. (4.14) and response functions again defined accordingly.

However, it turns out to be advantageous to proceed in a slightly different manner. The response functions, defined as above, are arbitrary functions of the variables q^2, $q\cdot p'$, $q\cdot P$, and $p'\cdot P$ or, in other words, q_0^2, $|\mathbf{q}|^2$, $q_0p_0'-\mathbf{q}\cdot\mathbf{p}'$, M_Tq_0, and M_Tp_0'. Equivalently, they are functions of q_0, $p_0'=(|\mathbf{p}'|^2+m^2)^{1/2}$, $|\mathbf{q}|^2$, and $\mathbf{q}\cdot\mathbf{p}'$. Evaluation of the spin-dependent scalars yield

$$\xi\cdot s'=M_T(\mathbf{q}\times\mathbf{p}')\cdot\hat{\mathbf{s}}'_R\ , \tag{5.5a}$$

$$P\cdot s'=(M_T/m)\mathbf{p}'\cdot\hat{\mathbf{s}}'_R\ , \tag{5.5b}$$

and

$$q\cdot s'=\left[\frac{q_0}{m}-\frac{\mathbf{q}\cdot\mathbf{p}'}{m(E_{p'}+m)}\right]\mathbf{p}'\cdot\hat{\mathbf{s}}'_R-\mathbf{q}\cdot\hat{\mathbf{s}}'_R\ , \tag{5.5c}$$

Thus, we may make the variable replacements

$$\xi\cdot s'\rightarrow\hat{\mathbf{n}}\cdot\hat{\mathbf{s}}'_R \tag{5.6a}$$

and

$$P\cdot s'\rightarrow\hat{\mathbf{l}}\cdot\hat{\mathbf{s}}'_R\ , \tag{5.6b}$$

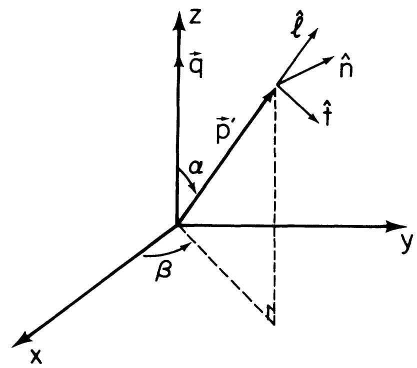

FIG. 3. Unit vectors employed for the ejectile spin projections in relation to the coordinate axes.

where $\hat{\mathbf{n}}$ and $\hat{\mathbf{l}}$ are unit vectors in the direction $\mathbf{q}\times\mathbf{p}'$ and $\mathbf{p}'$, respectively, by simply absorbing the other factors in Eqs. (5.5a) and (5.5b) into redefinitions of the relevant response functions. Similarly, since $q\cdot s'$ and $P\cdot s'$ always occur in combination [same azimuthal dependence, see Eqs. (4.13) and (4.14)], we may make the variable replacement

$$q\cdot s'\rightarrow\hat{\mathbf{t}}\cdot\hat{\mathbf{s}}'_R\ , \tag{5.6c}$$

where $\hat{\mathbf{t}}$ is a unit vector in the direction $\hat{\mathbf{n}}\times\hat{\mathbf{p}}'$, by again redefining the response functions which are associated with $P\cdot s'$ and $q\cdot s'$. Note that $\hat{\mathbf{t}}$ is transverse to the ejectile's momentum vector and that, in the coordinate system of Fig. 2, $\hat{\mathbf{n}}$ is the normal to the photonuclear scattering plane, which is defined by the momenta of the exchanged photon and the ejected nucleon, $\hat{\mathbf{n}}=\hat{\boldsymbol{\beta}}$, and $\hat{\mathbf{t}}=\hat{\boldsymbol{\alpha}}$. The unit vectors $\hat{\mathbf{n}}$, $\hat{\mathbf{l}}$, and $\hat{\mathbf{t}}$ are indicated in Fig. 3.

The explicit relationships between the new set of response functions (defined using three-vector scalar products) and those defined in terms of four-vector scalar products are easily obtained. However, the main point to this is the relationship of the new response functions to the components of the nuclear tensor $W^{\mu\nu}$. The defining relationships are simply

$$\tfrac{1}{2}(R_L+R_L^n\hat{\mathbf{n}}\cdot\hat{\mathbf{s}}'_R)=\int_{\rm line}dE_{p'}W^{00}(\hat{\mathbf{s}}'_R)\ , \tag{5.7}$$

$$\tfrac{1}{2}(R_T+R_T^n\hat{\mathbf{n}}\cdot\hat{\mathbf{s}}'_R)=\int_{\rm line}dE_{p'}[W^{11}(\hat{\mathbf{s}}'_R)+W^{22}(\hat{\mathbf{s}}'_R)]\ , \tag{5.8}$$

$$\tfrac{1}{2}[(R_{TT}+R_{TT}^n\hat{\mathbf{n}}\cdot\hat{\mathbf{s}}'_R)\cos2\beta+(R_{TT}^t\hat{\mathbf{t}}\cdot\hat{\mathbf{s}}'_R+R_{TT}^l\hat{\mathbf{l}}\cdot\hat{\mathbf{s}}'_R)\sin2\beta]=\int_{\rm line}dE_{p'}[W^{22}(\hat{\mathbf{s}}'_R)-W^{11}(\hat{\mathbf{s}}'_R)]\ , \tag{5.9}$$

$$\tfrac{1}{2}[(R_{LT}+R_{LT}^n\hat{\mathbf{n}}\cdot\hat{\mathbf{s}}'_R)\sin\beta+(R_{LT}^t\hat{\mathbf{t}}\cdot\hat{\mathbf{s}}'_R+R_{LT}^l\hat{\mathbf{l}}\cdot\hat{\mathbf{s}}'_R)\cos\beta]=\int_{\rm line}dE_{p'}[W^{02}(\hat{\mathbf{s}}'_R)+W^{20}(\hat{\mathbf{s}}'_R)]\ , \tag{5.10}$$

$$\tfrac{1}{2}[(R'_{LT}+R'^n_{LT}\hat{\mathbf{n}}\cdot\hat{\mathbf{s}}'_R)\cos\beta+(R'^t_{LT}\hat{\mathbf{t}}\cdot\hat{\mathbf{s}}'_R+R'^l_{LT}\hat{\mathbf{l}}\cdot\hat{\mathbf{s}}'_R)\sin\beta]=i\int_{\rm line}dE_{p'}[W^{10}(\hat{\mathbf{s}}'_R)-W^{01}(\hat{\mathbf{s}}'_R)]\ , \tag{5.11}$$

and

$$\tfrac{1}{2}[(R'^t_{TT}\hat{\mathbf{t}}\cdot\hat{\mathbf{s}}'_R+R'^l_{TT}\hat{\mathbf{l}}\cdot\hat{\mathbf{s}}'_R)]=i\int_{\rm line}dE_{p'}[W^{12}(\hat{\mathbf{s}}'_R)-W^{21}(\hat{\mathbf{s}}'_R)]\ . \tag{5.12}$$

The main advantage of the response functions defined by Eqs. (5.7)–(5.12) is that they are more simply related to the scattering observables. This is clear from the resulting expression for the differential cross section, wherein a spin-independent piece is isolated and the dependence upon the rest frame spin vector is decomposed into separate dependences upon the projections of $\hat{\mathbf{s}}'_R$ onto the set of orthogonal unit vectors $\hat{\mathbf{l}}$ (parallel to $\mathbf{p}'$), $\hat{\mathbf{n}}$, and $\hat{\mathbf{t}}$ (both transverse to $\mathbf{p}'$). Combining Eqs. (2.1) and (5.7)—(5.12) with an explicit evaluation of the components of the electron tensor, we find, for the differential cross section for the ejection of nucleons of spin $\mathbf{s}'_R$ by electrons of initial helicity h,

$$\left[\frac{d^3\sigma}{d\epsilon_{k'}d\Omega_{k'}d\Omega_{p'}}\right]_{s',h}=\frac{m\,|\mathbf{p}'|}{2(2\pi)^3}\left[\frac{d\sigma}{d\Omega_{k'}}\right]_{\rm Mott}\times\left\{\left[\frac{q}{|\mathbf{q}|}\right]^4(R_L+R_L^n\hat{\mathbf{n}}\cdot\hat{\mathbf{s}}'_R)+\left[\tan^2(\theta/2)-\frac{q^2}{2|\mathbf{q}|^2}\right](R_T+R_T^n\hat{\mathbf{n}}\cdot\hat{\mathbf{s}}'_R)\right.$$
$$-\frac{q^2}{2|\mathbf{q}|^2}[(R_{TT}+R_{TT}^n\hat{\mathbf{n}}\cdot\hat{\mathbf{s}}'_R)\cos(2\beta)+(R_{TT}^t\hat{\mathbf{t}}\cdot\hat{\mathbf{s}}'_R+R_{TT}^l\hat{\mathbf{l}}\cdot\hat{\mathbf{s}}'_R)\sin(2\beta)]$$
$$+\frac{q^2}{|\mathbf{q}|^2}\left[\tan^2(\theta/2)-\frac{q^2}{|\mathbf{q}|^2}\right]^{1/2}[(R_{LT}+R_{LT}^n\hat{\mathbf{n}}\cdot\hat{\mathbf{s}}'_R)\sin\beta+(R_{LT}^t\hat{\mathbf{t}}\cdot\hat{\mathbf{s}}'_R+R_{LT}^l\hat{\mathbf{l}}\cdot\hat{\mathbf{s}}'_R)\cos\beta]$$
$$+h\left[\frac{q^2}{|\mathbf{q}|^2}\right]\tan(\theta/2)[(R'_{LT}+R_{LT}^{\prime n}\hat{\mathbf{n}}\cdot\hat{\mathbf{s}}'_R)\cos\beta+(R_{LT}^{\prime t}\hat{\mathbf{t}}\cdot\hat{\mathbf{s}}'_R+R_{LT}^{\prime l}\hat{\mathbf{l}}\cdot\hat{\mathbf{s}}'_R)\sin\beta]$$
$$\left.+h\tan(\theta/2)\left[\tan^2(\theta/2)-\frac{q^2}{|\mathbf{q}|^2}\right]^{1/2}(R_{TT}^{\prime t}\hat{\mathbf{t}}\cdot\hat{\mathbf{s}}'_R+R_{TT}^{\prime l}\hat{\mathbf{l}}\cdot\hat{\mathbf{s}}'_R)\right\}, \tag{5.13}$$

with the Mott cross section, in the ERL assumed valid here, given by

$$\left[\frac{d\sigma}{d\Omega_{k'}}\right]_{\rm Mott}=\frac{\alpha^2\cos^2(\theta/2)}{4|\mathbf{k}|^2\sin^4(\theta/2)}, \tag{5.14}$$

where, in Eq. (5.14), α is the fine structure constant.

In order to take advantage of the decomposition evident in Eq. (5.13), it is only necessary to note that the differential cross section for the observation of a spin-$\frac{1}{2}$ particle with (rest-frame) spin vector $\mathbf{s}'_R$, $\sigma(\mathbf{s}'_R)$, has the general form

$$\sigma(\mathbf{s}'_R)=\frac{\sigma(0)}{2}(1+\boldsymbol{\pi}_{\rm out}\cdot\hat{\mathbf{s}}'_R), \tag{5.15}$$

where $\sigma(0)$ is the unpolarized differential cross section and $\boldsymbol{\pi}_{\rm out}$ is the polarization vector. In the present circumstance let us introduce the notation

$$\sigma_h(\mathbf{s}'_R)=\left[\frac{d^3\sigma}{d\epsilon_{k'}d\Omega_{k'}d\Omega_{p'}}\right]_{s',h}, \tag{5.16}$$

as well as

$$\sigma_h(0)=\sigma_h(\mathbf{s}'_R)+\sigma_h(-\mathbf{s}'_R) \tag{5.17}$$

for the unpolarized cross section from electrons with helicity h, and $\boldsymbol{\pi}_h$ for the ejectile polarization vector.

These observables follow immediately from Eq. (5.13), using Eq. (5.15). Equation (5.13) is the expression for $\sigma_h(\mathbf{s}'_R)$, while[3]

$$\sigma_h(0)=\frac{m\,|\mathbf{p}|}{(2\pi)^3}\left[\frac{d\sigma}{d\Omega_{k'}}\right]_{\rm Mott}\left\{\left[\frac{q}{|\mathbf{q}|}\right]^4R_L+\left[\tan^2(\theta/2)-\frac{1}{2}\left[\frac{q}{|\mathbf{q}|}\right]^2\right]R_T-\frac{1}{2}\left[\frac{q}{|\mathbf{q}|}\right]^2R_{TT}\cos2\beta\right.$$
$$\left.+\left[\frac{q}{|\mathbf{q}|}\right]^2\left[\tan^2(\theta/2)-\left[\frac{q}{|\mathbf{q}|}\right]^2\right]^{1/2}R_{LT}\sin\beta+h\left[\frac{q}{|\mathbf{q}|}\right]^2\tan(\theta/2)R'_{LT}\cos\beta\right\}, \tag{5.18}$$

and the ejectile polarization vector for incident electrons of helicity h may be decomposed into three orthogonal components according to

$$\boldsymbol{\pi}_h=\pi_h^n\hat{\mathbf{n}}+\pi_h^l\hat{\mathbf{l}}+\pi_h^t\hat{\mathbf{t}}, \tag{5.19}$$

where the component of the polarization vector normal to the photonuclear scattering plane is given by

$$\sigma_h(0)\pi_h^n=\frac{m\,|\mathbf{p}'|}{(2\pi)^3}\left[\frac{d\sigma}{d\Omega_{k'}}\right]_{\rm Mott}\left\{\left[\frac{q}{|\mathbf{q}|}\right]^4R_L^n+\left[\tan^2(\theta/2)-\frac{1}{2}\left[\frac{q}{|\mathbf{q}|}\right]^2\right]R_T^n\right.$$
$$-\frac{1}{2}\left[\frac{q}{|\mathbf{q}|}\right]^2R_{TT}^n\cos2\beta+\left[\frac{q}{|\mathbf{q}|}\right]^2\left[\tan^2(\theta/2)-\left[\frac{q}{|\mathbf{q}|}\right]^2\right]^{1/2}R_{LT}^n\sin\beta$$
$$\left.+h\left[\frac{q}{|\mathbf{q}|}\right]^2\tan(\theta/2)R_{LT}^{\prime n}\cos\beta\right\}, \tag{5.20}$$

and the components of the polarization vector within the photonuclear scattering plane are given by

$$\sigma_h(0)\pi_h^i=\frac{m\,|\mathbf{p}'|}{(2\pi)^3}\left[\frac{d\sigma}{d\Omega_{k'}}\right]_{\text{Mott}}\left\{\frac{-1}{2}\left[\frac{q}{|\mathbf{q}|}\right]^2 R_{TT}^i\sin2\beta+\left[\frac{q}{|\mathbf{q}|}\right]^2\left[\tan^2(\theta/2)-\left[\frac{q}{|\mathbf{q}|}\right]^2\right]^{1/2} R_{LT}^i\cos\beta\right.$$
$$\left.+h\left[\frac{q}{|\mathbf{q}|}\right]^2\tan(\theta/2)R_{LT}^{\prime i}\sin\beta+h\tan(\theta/2)\left[\tan^2(\theta/2)-\left[\frac{q}{|\mathbf{q}|}\right]^2\right]^{1/2}R_{TT}^{\prime i}\right\}, \quad (5.21)$$

with $i=l,t$. Note that the circumstance of no incident electron polarization is obtained from Eq. (5.13) by simply averaging over the incident electron helicity $h=\pm1$. In effect, this simply removes from Eq. (5.13) the terms which are linear in h. As a consequence, all of the other equations for the scattering observables, Eq. (5.18) for the unpolarized (nucleon) cross section and Eqs. (5.19)–(5.21) for the nucleon polarization vector, are also modified only by the elimination of the terms linear in h and by the notational change $\sigma_h\to\sigma_0$ and $\pi_h^i\to\pi_0^i$.

Equations (5.13) and (5.18)–(5.21), and their analogs in the $h=0$ circumstance, completely specify the relationships between the response functions and the experimental observables expected to be accessible in the near future (observations of the final electron spin do not appear to be in the offing). Thus, this set of equations governs the extraction of the individual response functions from experimental data. Of practical relevance to this process are the sizes and relative importances of the various response functions; this is discussed in the next section. Here we only indicate an "in principle" procedure for the experimental discrimination of the various response functions. We suppose that each of the four observables, Eqs. (5.18) and (5.20) and (5.21), is experimentally accessible, both with and without polarized initial electrons. The response functions which contribute to Eqs. (5.18)–(5.21) through terms linear in h are then immediately separated from the rest. This determines R_{LT}' [from the unpolarized cross section $\sigma_h(0)$] and $R_{LT}^{\prime n}$ (from the component of the polarization normal to the photonuclear scattering plane, π_h^n), while the separation of $R_{LT}^{\prime l}$ from $R_{TT}^{\prime l}$ (in the longitudinal component of the polarization vector, π_h^l) and of $R_{LT}^{\prime t}$ from $R_{TT}^{\prime t}$ (in the remaining component of the polarization vector, π_h^t) is perhaps most easily made on the additional basis of a sign difference in the contribution of $R_{LT}^{\prime l}$ and $R_{LT}^{\prime t}$ for $\beta=\pm\pi/2$. This is especially attractive due to the fact that the other contributors to π_h^l and π_h^t vanish in the electron-scattering plane. Note that the determination of R_{LT}' and $R_{LT}^{\prime n}$ requires measurements out of the electron-scattering plane. The remainder of the response functions may be determined without a polarized electron beam. The unpolarized differential cross section yields R_L, R_T, R_{TT}, and R_{LT}. The discrimination of R_{LT} is on the basis of a sign difference in its contribution for $\beta=\pm\pi/2$, that of R_{TT} is on the basis of β dependence (and so requires measurements out of the electron-scattering plane), and that of R_L and R_T is through a Rosenbluth separation. The response function R_L^n, R_T^n, R_{TT}^n, and R_{LT}^n are obtained in a completely analogous fashion from the nucleon polarization projection π_o^n. Finally, the separation of R_{TT}^l from R_{LT}^l (in π_0^l) and of R_{TT}^t from R_{LT}^t (in π_0^t) can be most easily made by the Rosenbluth technique; however, since all of these contributions vanish in the electron-scattering plane, their determination requires measurements out of the plane. Note that all of the required out-of-plane information can be obtained from experiments performed at a single angle $\beta\neq\pm\pi/2$, say $|\beta|=\pi/4$.

VI. CALCULATION AND CHARACTERISTICS OF THE RESPONSE FUNCTIONS

Methods for the (approximate) calculation of the current matrix elements $J^\mu(q)$ of Eqs. (2.10) and (2.11), in the laboratory frame, are treated in detail in Sec. III of Ref. 3. Briefly, the calculation of the current matrix elements is carried out in a Dirac-spinor representation and entirely in momentum space. Among the advantages of such an approach is the flexibility of obtaining theoretical predictions within the context of both the nonrelativistic and relativistic (Dirac) descriptions of the nuclear bound state and ejectile scattering dynamics. The major approximations employed in Ref. 3 (and in general) are single-particle descriptions of the bound states, an optical model treatment of the ejectile-nucleus final state interaction, and the use of the free-nucleon current operator. The potential severity of these approximations, particularly in regard to violations of current conservation, is discussed in Ref. 3, as is a simple error estimate. The numerical predictions of the sizes and relative importances of the various response functions presented later in this section are based upon these methods.

It is straightforward to extend the computational methods of Ref. 3 to the calculation of the new, spin-dependent response functions. The main complication of this extension lies in relating the present considerations to those of Ref. 3. In this paper the treatment of the ejectile has been given in terms of its spin unit vector $\hat{\mathbf{s}}'_R$; no choice of a quantization axis was necessary. However, in most calculations of electromagnetic current matrix elements it is convenient (especially when a multipole expansion is used) to construct the operators and states for the case in which the axis of quantization is chosen to be along the three-momentum transfer vector $\mathbf{q}$. This choice of quantization axis is used in Ref. 3. In view of Eqs. (5.7)–(5.12), it is necessary to deal with spins pointing along each of the directions $\hat{\mathbf{l}}$, $\hat{\mathbf{n}}$, and $\hat{\mathbf{t}}$ in order to obtain the spin-dependent response functions. It is therefore necessary to reference the description of these spin directions to the $\hat{\mathbf{q}}$ choice of quantization axis and to construct a projection technique for extracting the individual response functions on this basis.

Since the problem is primarily associated with the description of spin, we first manifest the dependence of the current matrix element of Eq. (2.11) on the rest-frame spin vector by means of the notation $J^{\mu}(\hat{\mathbf{q}},s)$. Here, s is the (rest-frame) spin projection along the axis of quantization, which is taken[3] to be the $\hat{\mathbf{q}}$ or $\hat{\mathbf{z}}$ axis (see Fig. 2). It is the $J^{\mu}(\hat{\mathbf{q}},s)$ which are explicitly calculated in Ref. 3. Given the currents $J^{\mu}(\hat{\mathbf{q}},s)$, the determination of the various response functions proceeds as follows. Define a set of tensors $\omega^{\mu\nu}_{s's}(\hat{\mathbf{q}})$ to be given by the combination of Eqs. (2.8) and (2.9), except for the replacement, in Eq. (2.9),

$$J^{\mu}(q)^{\dagger}J^{\nu}(q)\rightarrow J^{\mu}(\hat{\mathbf{q}},s)^{\dagger}J^{\nu}(\hat{\mathbf{q}},s') , \tag{6.1}$$

so that, in view of Eq. (2.12),

$$\omega^{\mu\nu}_{s's}(\hat{\mathbf{q}})=\overline{\sum_{I}}\int\frac{d^3P'}{(2\pi)^3}\phi(\mathbf{P}')(2\pi)^3\delta^4(k-k'+P-P'-p)\times\sum_{F}\langle p',s'\hat{\mathbf{q}},(-);F,P'\,|\,\hat{J}^{\nu}(q)\,|\,I,P\rangle\langle I,P\,|\,\hat{J}^{\mu}(q)^{\dagger}\,|\,p',s\hat{\mathbf{q}},(-);F,P'\rangle . \tag{6.2}$$

Note that the operators $\omega^{\mu\nu}_{s's}(\hat{\mathbf{q}})$ are defined such that

$$W^{\mu\nu}(\hat{\mathbf{s}})=\omega^{\mu\nu}_{ss}(\hat{\mathbf{q}}) . \tag{6.3}$$

From Eq. (6.2) one can define $\omega^{\mu\nu}(\hat{\mathbf{q}})$ such that

$$\omega^{\mu\nu}_{s's}(\hat{\mathbf{q}})=\langle s',\hat{\mathbf{q}}\,|\,\hat{\omega}^{\mu\nu}(\hat{\mathbf{q}})\,|\,s,\hat{\mathbf{q}}\rangle , \tag{6.4}$$

where $|\,s,\hat{\mathbf{q}}\rangle$ denotes a standard Pauli spinor of (rest-frame) spin projection s pointing along $\hat{\mathbf{q}}$.

The operators $\hat{\omega}^{\mu\nu}(\hat{\mathbf{q}})$ are 2×2 matrices in the spin space of the ejectile, with matrix elements given by Eq. (6.4). Clearly, these operators follow immediately from knowledge of $J^{\mu}(\hat{\mathbf{q}},s)$. It is also evident from Eqs. (6.3) and (6.4) that the spin-summed nuclear tensor $W^{\mu\nu}$ is given by

$$W^{\mu\nu}=W^{\mu\nu}(\hat{\mathbf{s}}'_R)+W^{\mu\nu}(-\hat{\mathbf{s}}'_R)=\mathrm{Tr}[\hat{\omega}^{\mu\nu}(\hat{\mathbf{q}})] . \tag{6.5}$$

The nuclear tensor appropriate to the case wherein the ejectile spin is required to be along an arbitrary direction $\hat{\mathbf{a}}$, $W^{\mu\nu}(\hat{\mathbf{a}})$, can also be written [see Eqs. (2.8) and (2.12)] as

$$W^{\mu\nu}(\pm\hat{\mathbf{a}})=\langle \tfrac{1}{2},\pm\hat{\mathbf{a}}\,|\,\hat{W}^{\mu\nu}\,|\,\tfrac{1}{2},\pm\hat{\mathbf{a}}\rangle , \tag{6.6}$$

or

$$W^{\mu\nu}(\hat{\mathbf{a}})=\sum_{s,s'}\langle \tfrac{1}{2},\hat{\mathbf{a}}\,|\,s',\hat{\mathbf{q}}\rangle\langle s',\hat{\mathbf{q}}\,|\,\hat{\omega}^{\mu\nu}(\hat{\mathbf{q}})\,|\,s,\hat{\mathbf{q}}\rangle\times\langle s,\hat{\mathbf{q}}\,|\,\tfrac{1}{2},\hat{\mathbf{a}}\rangle . \tag{6.7}$$

Making use of the spin projection operator

$$P_{\hat{\mathbf{a}}}=\frac{1+\sigma\cdot\hat{\mathbf{a}}}{2} \tag{6.8}$$

yields

$$W^{\mu\nu}(\hat{\mathbf{a}})=\mathrm{Tr}[\hat{\omega}^{\mu\nu}(\hat{\mathbf{q}})P_{\hat{\mathbf{a}}}] , \tag{6.9}$$

so that

$$W^{\mu\nu}=W^{\mu\nu}(\hat{\mathbf{a}})+W^{\mu\nu}(-\hat{\mathbf{a}})=\mathrm{Tr}[\hat{\omega}^{\mu\nu}(\hat{\mathbf{q}})] , \tag{6.10}$$

and upon defining

$$\Delta W^{\mu\nu}(\hat{\mathbf{a}})=W^{\mu\nu}(\hat{\mathbf{a}})-W^{\mu\nu}(-\hat{\mathbf{a}}) , \tag{6.11}$$

together with

$$\Sigma_{\hat{\mathbf{a}}}=\sigma\cdot\hat{\mathbf{a}} , \tag{6.12}$$

that

$$\Delta W^{\mu\nu}(\hat{\mathbf{a}})=\mathrm{Tr}[\Sigma_{\hat{\mathbf{a}}}\hat{\omega}^{\mu\nu}(\hat{\mathbf{q}})] . \tag{6.13}$$

Equations (6.5) and (6.11)—(6.13) allow the effective projection of each of the 18 independent response functions from their defining equations, Eqs. (5.7)—(5.12).

In particular, upon defining the spin-space operators

$$\begin{aligned}
\hat{R}_L&=\int_{\mathrm{line}}dE_{p'}\hat{\omega}^{00}(\hat{\mathbf{q}}) ,\\
\hat{R}_T&=\int_{\mathrm{line}}dE_{p'}[\hat{\omega}^{11}(\hat{\mathbf{q}})+\hat{\omega}^{22}(\hat{\mathbf{q}})] ,\\
\hat{R}_{TT}&=\int_{\mathrm{line}}dE_{p'}[\hat{\omega}^{22}(\hat{\mathbf{q}})-\hat{\omega}^{11}(\hat{\mathbf{q}})] ,\\
\hat{R}_{LT}&=\int_{\mathrm{line}}dE_{p'}[\hat{\omega}^{02}(\hat{\mathbf{q}})+\hat{\omega}^{20}(\hat{\mathbf{q}})] ,\\
\hat{R}'_{LT}&=i\int_{\mathrm{line}}dE_{p'}[\hat{\omega}^{10}(\hat{\mathbf{q}})-\hat{\omega}^{01}(\hat{\mathbf{q}})] ,
\end{aligned} \tag{6.14}$$

and

$$\hat{R}'_{TT}=i\int_{\mathrm{line}}dE_{p'}[\hat{\omega}^{12}(\hat{\mathbf{q}})-\hat{\omega}^{21}(\hat{\mathbf{q}})] ,$$

we find that the complete set of response functions is given by

$$\begin{aligned}
&R_L=\mathrm{Tr}[\hat{R}_L];\quad R^n_L=\mathrm{Tr}[\hat{R}_L\Sigma_{\hat{\mathbf{n}}}] ,\\
&R_T=\mathrm{Tr}[\hat{R}_T];\quad R^n_T=\mathrm{Tr}[\hat{R}_T\Sigma_{\hat{\mathbf{n}}}] ,\\
&\cos(2\beta)R_{TT}=\mathrm{Tr}[\hat{R}_{TT}];\quad \cos(2\beta)R^n_{TT}=\mathrm{Tr}[\hat{R}_{TT}\Sigma_{\hat{\mathbf{n}}}] ,\\
&\sin(2\beta)R^t_{TT}=\mathrm{Tr}[\hat{R}_{TT}\Sigma_{\hat{\mathbf{t}}}];\quad \sin(2\beta)R^l_{TT}=\mathrm{Tr}[\hat{R}_{TT}\Sigma_{\hat{\mathbf{l}}}] ,\\
&\sin(\beta)R_{LT}=\mathrm{Tr}[\hat{R}_{LT}];\quad \sin(\beta)R^n_{LT}=\mathrm{Tr}[\hat{R}_{LT}\Sigma_{\hat{\mathbf{n}}}] ,\\
&\cos(\beta)R^t_{LT}=\mathrm{Tr}[\hat{R}_{LT}\Sigma_{\hat{\mathbf{t}}}];\quad \cos(\beta)R^l_{LT}=\mathrm{Tr}[\hat{R}_{LT}\Sigma_{\hat{\mathbf{l}}}] ,\\
&\cos(\beta)R'_{LT}=\mathrm{Tr}[\hat{R}'_{LT}];\quad \cos(\beta)R'^n_{LT}=\mathrm{Tr}[\hat{R}'_{LT}\Sigma_{\hat{\mathbf{n}}}] ,\\
&\sin(\beta)R'^t_{LT}=\mathrm{Tr}[\hat{\mathbf{R}}'_{LT}\Sigma_{\hat{\mathbf{t}}}];\quad \sin(\beta)R'^l_{LT}=\mathrm{Tr}[\hat{\mathbf{R}}'_{LT}\Sigma_{\hat{\mathbf{l}}}] ,\\
&R'^t_{TT}=\mathrm{Tr}[\hat{R}'_{TT}\Sigma_{\hat{\mathbf{t}}}];\quad R'^l_{TT}=\mathrm{Tr}[\hat{R}'_{TT}\Sigma_{\hat{\mathbf{l}}}] .
\end{aligned} \tag{6.15}$$

This completes the description of the method we have used to obtain the theoretical predictions of the 18 independent response functions.

A detailed study of the new response functions and their physical content will be presented in a subsequent paper. Here we consider only the predicted sizes and relative importances of the various response functions. This is, of course, a crucial consideration for their experimental investigation and for their separation and determination from experimental data.

A preliminary calculation of the response functions as defined by Eqs. (6.14) and (6.15) has been performed using the Dirac dynamical approach described in Ref. 3. The results summarized in Table I are for the ejection of 135 MeV protons from the $1p_{1/2}$ shell of ^{16}O at a momentum transfer of 2.64 fm^{-1}. Column 2 of Table I contains the magnitude of the maximum value of the response function specified in column 1. For reference, the ratio of each magnitude to that of R_L is shown in column 3. Column 4 indicates whether or not the corresponding response function survives in the case in which the $\mathscr{T}\Pi$ operation implies an additional restriction on the form of the nuclear tensor. As discussed in Sec. III, this is the case when the difference in boundary conditions on the left and right hand sides of Eq. (3.17) can be ignored. This obtains, for example, when the FSI's produce no scattered flux at infinity, as in a plane wave limit. When the boundary condition differences can be neglected, Eq. (3.17) requires the nuclear tensor to be symmetric under simultaneous interchange of its tensor indices and spin flip. Thus, any terms linear in spin must vanish in the symmetric part of the tensor and any terms not linear in spin must vanish in the antisymmetric part of the tensor. The symmetric part of the tensor then makes no contribution to the polarization; the only such contribution comes from the antisymmetric part of the tensor (which makes no spin-independent contribution) and this can only occur for nonvanishing electron helicity. This makes evident the crucial role of the FSI's for certain of the response functions and spin observables, so that a summary of these results is included in Table I.

Column 5 indicates whether or not the response function can be measured in the electron-scattering plane and column 6 indicates the reflection symmetry through the xz plane of the term in the cross section containing the specified response function. The most easily isolated response functions are those which can be measured internal to the electron-scattering plane and which are odd under reflection through the xz plane. From Table I it can be seen that the new "polarization" response functions range in relative size from 7% to 164% of R_L. An interesting feature of our results is that while the unpolarized response function R_{TT} is small, being on the order of 6% of R_L, the corresponding response function for longitudinally polarized protons is 7% of R_L, the two response functions for transversely polarized protons are more than half as large as R_L. It is also interesting to note that the largest response functions are $R_{LT}^{\prime n}$ and $R_{LT}^{\prime t}$, both of which require polarized incident electrons as well as the detection of the ejected proton's spin.

A detailed presentation of our results for the various response functions will be provided in a subsequent paper. However, of general interest are the physics implications of the new response functions and the underlying dynamics issues which are intertwined with obtaining an understanding of their properties. Of course, the independence of the new response functions, in and of itself, ensures that they will provide additional access to both the initial hadronic system and the dynamical mechanisms which occur during the electroproduction process. For example, the new response functions are intimately connected to the ejectile spin. Since both elementary hadronic interactions and the current operators exhibit considerable spin dependence, one expects the new response functions to be especially adapted to probing spin structure in the reaction

TABLE I. Summary of the properties of the various response functions and their characteristics. See text for discussion.

	$\max\|R_i\|$ (fm^3)	$\frac{\max\|R_i\|}{\max\|R_L\|}$	$\mathscr{T}\Pi$	In plane	Symmetry
R_L	28.1	1.00	yes	yes	even
R_L^n	4.7	0.17	no	yes	even
R_T	39.0	1.39	yes	yes	even
R_T^n	4.9	0.17	no	yes	even
R_{TT}	1.6	0.06	yes	yes	even
R_{TT}^n	15.8	0.56	no	yes	even
R_{TT}^l	2.1	0.07	no	no	odd
R_{TT}^t	16.0	0.52	no	no	odd
R_{LT}	10.3	0.37	yes	yes	odd
R_{LT}^n	3.3	0.12	no	yes	odd
R_{LT}^l	10.5	0.37	no	no	even
R_{LT}^t	3.7	0.13	no	no	even
$R_{LT}^{\prime}$	15.7	0.56	no	no	even
$R_{LT}^{\prime n}$	44.1	1.57	yes	no	even
$R_{LT}^{\prime l}$	14.3	0.5	yes	yes	odd
$R_{LT}^{\prime t}$	46.0	1.64	yes	yes	odd
$R_{TT}^{\prime l}$	38.6	1.37	yes	yes	even
$R_{TT}^{\prime t}$	3.9	0.14	yes	yes	even

mechanism. Such a spin-selective probe may be expected to be of great value in studying the electroproduction process since the important dynamical degrees of freedom (and thus the character of the response functions) will vary among nonrelativistic, relativistic, mesonic effects and baryon resonances, and subhadronic domains and their interfaces as the four-momentum transfer carried by the virtual photon is varied. Because of the crucial and distinctive role played by spin couplings in these various domains, they and the interfaces between them may be expected to be characterized by strong and distinctive spin-dependent effects.

On the other hand, the complexities associated with treating these diverse dynamical domains, as well as off-shell considerations,[3,23] questions concerning the appropriate elementary current operator,[23] current conservation violations,[3] and the truncations necessary to arrive at an impulse approximation,[3] indicate the primitive state of current models. Because of these difficulties, as well as intricate questions of consistency between the interactions and the currents, present approaches possess a large degree of model dependence. Thus, care must be exercised in drawing general conclusions about the physics attributes of individual response functions. In fact, this complicated interplay of many dynamical aspects makes unlikely the simple, straightforward extraction of specific quantitative results, at least without considerable further preparatory work. It seems more likely that comprehensive analyses will permit inferences to be drawn on the basis of simultaneous comparisons with a set of response functions. This approach takes advantage of the distinct dependences of the various response functions. The complicated interplay of the different dynamical aspects of the problem then enhances the versatility of the set of response functions as a dynamical probe.

However, with the foregoing caveats in mind, one can make some general qualitative, but model-dependent, inferences concerning the various response functions. First of all, the 13 new response functions depend on the ejectile spin vector for their existence and hence should prove useful as spin filters; for example, in regard to verifying relativistic (Dirac) effects expected on the basis of the properties of the usual five response functions,[3] resolving and studying virtual baryon resonance production and its effects, and perhaps in studying subhadronic spin distributions, helicity effects, etc. The new response functions listed in Table I can also be divided into two categories according to their behavior under time reversal in the limit where dependence on the boundary conditions is ignored: those which are then allowed under the combination of parity and time reversal ($\Pi\mathcal{T}$) and those which are not. In the context of the distorted wave impulse approximation, and for restricted four-momentum transfer, the interpretation of the physics of these two classes of response functions is straightforward. The first class of response functions, which are $\Pi\mathcal{T}$ allowed, all arise from the antisymmetric part of the nuclear response tensor $W_A^{\mu\nu}$. As a result, these response functions, $R_{LT}'^{n}$, $R_{LT}'^{l}$, $R_{LT}'^{t}$, $R_{TT}'^{l}$, and $R_{TT}'^{t}$, are all directly dependent on the electron helicity and are all interference response functions. Since these response functions survive in the plane wave limit, where the FSI's vanish, the main source of ejected nucleon polarization is the electromagnetic current operator. In particular, the polarization derives from the spin dependence of the magnetization current, which is purely transverse and in the usual nonrelativistic models has the form $\mathbf{J}_{\mathrm{mag}}=i[G_M(q^2)/2m]\boldsymbol{\sigma}\times\mathbf{q}$. The polarization process in the plane wave limit is essentially the result of a transfer of the intrinsic angular momentum (helicity) of the virtual photon to the target nucleon. The antisymmetric combinations of the currents in $W_A^{\mu\nu}$ preserve this spin dependence, while the symmetric combinations, which give rise to the remaining spin dependent response functions, tend to eliminate this explicit spin dependence due to the fact that they vanish in the plane wave limit. Of course, once the FSI's are included this first class of response functions is also affected by the properties of the FSI's. In particular, any *relative* variation in a class of response functions after the introduction of FSI's must be the result of the spin dependence of the final state interaction. It is interesting to note that two members of the first class, $R_{TT}'^{l}$ and $R_{TT}'^{t}$, are relatively large and have no spin independent analog. From this point of view, these response functions represent an entirely new feature of electroproduction which is inaccessible through the usual complement of response functions.

The second class of response functions, those which are not allowed under $\Pi\mathcal{T}$ when boundary condition effects are ignored, vanish in the limit of no FSI scattering. Thus they depend directly on the outgoing scattered flux and hence on the unitarity properties of the optical potential and the truncation of the many-body problem. The outgoing scattered flux (at infinity) in the optical model directly involves only the on-shell projectile-target elastic T matrix, which is measured in elastic proton scattering. Thus some of these response functions may be insensitive to off-shell effects, and others may show varying degrees of sensitivity. At any rate, the ability to isolate FSI effects from other mechanisms and target structure is enormously advantageous for the study of them all. The response functions R_L^n and R_T^n are closely related to the usual longitudinal and transverse response functions. Currently, the apparent relative quenching of the longitudinal response function in inclusive scattering has generated considerable interest.[24] Studies of their spin-dependent analogs, R_L^n and R_T^n, might shed some light on the underlying mechanism at work; such studies would certainly provide more stringent tests of models which are devised to describe the spin-independent response functions. The remaining response functions of this category, R_{TT}^i and R_{LT}^i ($i=n,l,t$), are interference response functions in the sense that they arise from interference of the different components of the hadronic current. Interference response functions seem to be very sensitive to the off-shell behavior of the FSI's;[3] thus the present ones may be complementary to the other members of the second category in this regard. Moreover, the usual interference response functions seem to be insensitive to relativistic effects. Thus, such sensitivity in the spin-dependent interference response functions would provide a new versatility in separating off-shell from relativistic effects. On the other hand, a lack of sensitivity would also be interesting since it would indicate substantial spin effects which are independent of relativistic issues and thus might be useful in isolating other spin-dependent mechanisms for study, such as virtual resonance production, for example.

The main point here is, of course, that the different dependences of the response functions on the target dynamics, the many-body problem and the associated FSI's, as well as the elementary interactions and currents, provide a substantial opportunity for sorting out the basic physics puzzle. The fact that the crucial ingredients of the elementary interactions and currents will change as the four-momentum transfer varies both complicates matters and is the source of considerable versatility for unraveling the underlying physics of the electroproduction process. It should be stressed that the present characterization of the physics content of the spin-dependent response functions given above is certainly incomplete. Much further work is needed to adequately uncover the physics of these new response functions, even in a qualitative fashion.

VII. SUMMARY

In this paper we have developed a complete theoretical framework for the description of the electroproduction of polarized nucleons from nuclei. By careful consideration of the applicable symmetries, general forms for the $(\vec{e},e'\vec{N})$ cross section and polarization projections have been obtained. Extensions of this systematic construction to the general $(\vec{e},e'X)$ reaction, to the case of polarized targets, and to more general current and/or parity nonconserving processes, have been indicated. The $(\vec{e},e'\vec{N})$ physical observables are expressed in terms of 18 response functions, the 13 new members of which are intimately connected to the spin of the ejectile. These new response functions appear to provide a host of interesting opportunities. Since both the electromagnetic current operator and the nucleon-nucleon interaction are spin dependent, it is likely that the spin-dependent response functions will show special sensitivity to this quantity. The availability of a complete (or at least large) set of experimentally determined response functions with which to determine the underlying hadronic electromagnetic current would provide a much more rigorous test of theoretical models. Moreover, this is expected to be especially valuable due to the variety of dynamical circumstances which can be probed via electroproduction. By varying the four-momentum transfer carried by the virtual photon, one can move among the natural domains of nonrelativistic potential theory, relativistic and boson exchange models (including a variety of resonance excitations and subhadronic models), as well as the interfaces between them. The variety of conceptual physics issues which are intertwined with the properties of the response functions further enhances their versatility as a dynamical probe. These include questions concerning the appropriate elementary off-shell current operator, current conservation violations, consistency between currents and dynamical mechanisms, and the important aspects of truncating the many-body problem to a manageable level; many of the answers to these questions will be intertwined with the particular dynamical circumstance. The differing behaviors of the response functions in limiting circumstances, partly as a consequence of their symmetry properties (see Table I), also seem to indicate versatility in focusing on, and isolating, some of the general subdivisions of the reaction; for example, the host of distinct dynamics-dependent spin features, the importance of FSI's, on- versus off-shell FSI's, dependences on uncertainties associated with the effective current operator, and models of the initial target dynamics.

Preliminary results suggest that it will be possible to extract the new response functions from experimental data, provided polarized electron beams of sufficiently high current become available. (The high current is necessary in order to compensate for the requirement of a nucleon polarimeter.) Indeed, these preliminary results indicate that several of the spin-dependent response functions are surprisingly large (see Table I). The existence of "switches," such as the electron spin, the proton spin, and asymmetry properties under *xz*-plane reflection, can provide for the extraction of new, detailed information without the difficulties associated with angular separations. A more detailed discussion of the new response functions will be presented along with a full account of our numerical results in a subsequent paper.

This work was supported in part by the U.S. Department of Energy.

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