Muon and neutrino production in inclusive proton-^{12}C reactions

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We present an estimate for the cross section for the inclusive reaction $p+{}^{12}C\rightarrow\mu^{+}+\nu+X$, where X is a nuclear state. For an incident proton c.m. energy of 1076 MeV, a cross section of 4.41×10^{-42} cm^2 is obtained. The possibility of observing anomalous threshold states is also discussed.

I. INTRODUCTION

Recently[1] we calculated the cross section near threshold for the reaction $p+{}^{12}C\rightarrow\mu^{+}+\nu+{}^{13}C_{g.s.}$. This reaction was interesting because q^2 for the process is always timelike. Thus its observation would provide a means to study the nuclear form factors in the timelike region. In particular it would open the possibility to look for anomalous threshold contributions[2] to the matrix element of the divergence of the weak axial current, which should become important below threshold and physically realizable above threshold. However, the cross section for this reaction is quite small.[1]

We therefore consider the inclusive reaction $p+{}^{12}C\rightarrow\mu^{+}+\nu+X$, where X is a nuclear state. We are led to look at this reaction by a calculation of Weiss and Walker.[3] They calculated the cross section for proton-^{12}C reactions leading to a few of the excited states of ^{13}C and found them to be substantially larger than the ground state cross section. Thus the possibility of obtaining a more measurable cross section for the inclusive process exists.

Although it would be more desirable to be able to observe a single final state, it is still useful to look at the inclusive process. As noted, q^2 is timelike and the timelike region for q^2 is not easily accessible for most weak processes.[4] Furthermore, it might still be possible to observe the anomalous threshold states as changes in the cross section not corresponding to the excited nuclear states. In addition, inclusive pion production data exist,[5] making it possible to tie the calculation presented here to experimental results.

In Sec. II of this paper we discuss the matrix elements needed in undertaking this calculation. In Sec. III we obtain an estimate for the inclusive cross section. Finally, in Sec. IV we discuss our results.

II. MATRIX ELEMENTS

We may write an expression for the cross section for the inclusive process $p+{}^{12}C\rightarrow\mu^{+}+\nu+X$ as

$$d\sigma=\sum\int\frac{d^3p_xM_xd^3p_\mu m_\mu d^3p_\nu m_\nu\,|M_{ix}|^2}{E_x(2\pi)^3E_\mu(2\pi)^3E_\nu(2\pi)^3}\times\delta^4(p+P\rightarrow P_X+p_\mu+p_\nu)\ , \tag{1}$$

where x is summed over all final states. The matrix element squared, $|M_{ix}|^2$, may be written as

$$|M_{ix}|^2=\langle x\,|J_\lambda(0)|\,{}^{12}C,p\rangle\langle x\,|J_\rho(0)|\,{}^{12}C,p\rangle^\dagger\times L^{\lambda\rho}G^2\cos^2\theta/2\ , \tag{2}$$

where $L^{\lambda\rho}$, the lepton tensor, may be written as

$$L^{\lambda\rho}=(-2/m_\mu m_\nu)(\nu^\lambda\mu^\rho+\nu^\rho\mu^\lambda-\nu\cdot\mu g^{\lambda\rho}+i\epsilon^{\lambda\rho}_{\eta\kappa}\nu^\eta\mu^\kappa)\ . \tag{3}$$

The form of $L^{\lambda\rho}$ is well known. The physics of the problem lies in the determination of the quantity

$$\langle x\,|J^\lambda(0)|\,{}^{12}C,p\rangle\langle x\,|J^\rho(0)|\,{}^{12}C,p\rangle^\dagger$$

summed over x. We previously examined this problem in some detail in Ref. 6, where we have shown that the integral over

$$\langle x\,|J^\lambda(0)|\,p,{}^{12}C\rangle\langle x\,|J^\rho(0)|\,p,{}^{12}C\rangle^\dagger$$

may be replaced by a tensor given by

$$Q^{\lambda\rho}(p,P_i,\langle q\rangle)=\alpha g^{\lambda\rho}+(\gamma/M^2)P_i^\lambda P_i^\rho+(\beta/m^2)p^\lambda p^\rho+(\delta/m^2)\langle q^\lambda\rangle\langle q^\rho\rangle+\zeta p_i^\lambda p^\rho+\cdots+\eta p^\lambda P_i^\rho+\cdots+\xi\epsilon^{\lambda\rho}_{\tau\nu}P_i^\tau p^\nu+\cdots\ , \tag{4}$$

where P_i^ν and M are the four-momentum and mass, respectively, of the initial nucleus; p^ν and m are the four-momentum and mass, respectively, of the proton; and $\langle q^\nu\rangle$ represents the average four-momentum transferred. Here the ellipses indicate mixed terms of a similar type. By an analysis equivalent to that given in Ref. 6, the contributions to $Q^{\kappa\eta}$ from the mixed terms, $\eta p^\kappa p_i^\eta$, $\chi p^\kappa\langle q^\eta\rangle$, etc. are expected to be small. This is because[6] if form factors corresponding to the individual channels are substituted into the expression

$$\langle x\,|J^\lambda(0)|\,p,{}^{12}C\rangle\langle x\,|J^\rho(0)|\,p,{}^{12}C\rangle^\dagger\ ,$$

the mixed terms come about either from the products of different form factors or from the product of two terms in the coefficient of a single form factor. In the former case, because we expect the signs of the form factors to be random, a sum over a large number of intermediate states should involve substantial cancellation and terms so generated will be small. In the latter case the mixed term will not be the dominant contribution from its particular form factor and may be neglected in this very approximate calculation which are presenting. Thus we may approximate $Q^{\kappa\eta}$ as

$$Q^{\kappa\eta}=\alpha g^{\kappa\eta}+(\beta/m^2)p^\kappa p^\eta+(\gamma/M^2)P_i^\kappa P_i^\eta \ , \tag{5}$$

where we have also neglected the term proportional to δ because $\langle q\rangle$ will be relatively small for the process considered here.

The result which we have just obtained is essentially equivalent to that obtained by means of the allowed approximation.[6,7] In the allowed approximation the weak current is written as[7]

$$J_\alpha^{(\pm)}(\mathbf{x},0)=\sum_a(\Gamma_\alpha)_a\tau_a^{(\pm)}\delta(\mathbf{x}-\mathbf{r}_a) \ , \tag{6a}$$

$$(\Gamma_\alpha)_a=ig_V\delta_{\alpha,0}-(1-\delta_{\alpha,0})g_A(\sigma_\alpha)_a \ . \tag{6b}$$

This expression for the current leads to a tensor of the same form as $Q^{\kappa\eta}$ when the sums and integrations are performed[7] and small terms are neglected.

We note that in the allowed approximation, the large contributions to Q_{00} come from the vector part of the weak hadronic current and the large contributions to Q_{ii} come from the axial part of the weak hadronic current. We shall make use of this in what follows. We shall therefore use A as a superscript to denote values of α, β, and γ corresponding to Q_{00} and V as a superscript to denote values of α, β, and γ corresponding to Q_{jj}. We may perform the angular integratio■s Eq. (1). Making use of Eqs. (3) and (5) we obtain

$$\sigma=(mM_xG^2\cos^2(\theta_C)/4\pi^3p\,|E_p+E|^2)\int E_\nu^2E_\mu p_\mu[\alpha^V+\beta^Vp_0^2/m^2+\gamma^V-(3\alpha^A-\beta^A\mathbf{p}^2/m^2)]dE_\mu \ , \tag{7}$$

where M_x is an average mass of the final state nucleus, E is the energy of the initial nucleus, E_p is the proton energy, M is the mass of the initial nucleus, and m and $(p_0,\mathbf{p})$ are the proton mass and four-momentum, respectively.

The problem is to determine the coefficients α^A and β^A, and α^V, β^V, γ^V. In Ref. 7 it is seen that this integral [Eq. (1)] will be proportional to $g_V^2+3g_A^2$. Comparing Eq. (1) with this we assume that

$$\frac{|3\alpha^A-\beta^A\mathbf{p}^2/m^2|}{|\alpha^V+\beta^Vp_0^2/m^2+\gamma^V|}=\frac{3g_A^2}{g_V^2} \ , \tag{8}$$

since these are the contributions from the hadronic weak current matrix elements squared in the method used here and in the allowed approximation, respectively. Because both g_V and g_A are known, if the numerator of the left-hand side (lhs) can be determined, the denominator can also be immediately found.

We obtain the numerator of the lhs of Eq. (8) by considering the corresponding pion reaction $p+{}^{12}C\to X+\pi^+$. As we have mentioned above, data exist for this inclusive reaction.[5] The cross section for this reaction can be obtained by making use of the Gell-Mann-Levy form of the partial conservation of axial-vector current (PCAC) relation:

$$\partial_\rho A^\rho=f_\pi m_\pi^2\phi_\pi \ , \tag{9}$$

which for the process of interest here becomes

$$\langle X|\partial_\rho A^\rho(x)|{}^{12}C,p\rangle=f_\pi m_\pi^2\langle X|\phi_\pi(x)|{}^{12}C,p\rangle \ . \tag{10}$$

We make use of the standard relationship between the pion field and its source current $(\Box+m_\pi^2)\phi_\pi=j_\pi(x)$ to obtain

$$\langle X|j_\pi(0)|{}^{12}C,p\rangle=[(-iq_\rho)(-q^2+m_\pi^2)/f_\pi m_\pi^2]\langle X|A^\rho(0)|{}^{12}C,p\rangle \ . \tag{11}$$

The cross section for the inclusive reaction, $p+{}^{12}C\to X+\pi^+$, is given by

$$\sigma=\frac{mM_x}{2Mp(2\pi)^2}\int\frac{p_\pi|\mathcal{M}|^2\delta(E-E_\pi)d\Omega\,dE}{2|M+E_p|} \ . \tag{12}$$

The matrix element $|\mathcal{M}|^2$ is given by the expression

$$|\mathcal{M}|^2=\frac{(-q^2+m_\pi^2)^2}{f_\pi^2m_\pi^4}(\alpha^Aq^2+\beta^Ap\cdot q\,p\cdot q/m^2+\gamma^AP\cdot q\,P\cdot q/M^2) \ . \tag{13}$$

Because the pion produced here is a real pion, $q^2=m_\pi^2$, so it is clear that only the pole terms will survive in Eq. (13). We shall denote the pole terms by primes. Using the expression Eq. (13), Eq. (12) can now be integrated yielding the expression for the total cross section:

$$\sigma=\frac{mM_xp_\pi^3[\beta'^A(p_0^2/m^2)+\gamma'^A]}{4\pi Mpm_\pi^4f_\pi^2(M+E_p)} \ , \tag{14}$$

where we have ignored small terms guided by the allowed approximation. It is necessary to find a connection between the primed terms in Eq. (14) and the α^A and β^A which we actually need. To do this we make use of the fact that an impulse approximation calculation of the process $p+{}^{12}C\to X+\pi^+$ is proportional to g_P^2, but g_P is very closely related to g_A, so that

$$g_P=g_Am_\pi^2/(q^2-m_\pi^2) \ . \tag{15}$$

From Eq. (8) we have that $|3\alpha^A+\beta^A\mathbf{p}^2/m^2|$ is proportional to $3g_A^2$, and so we are lead to the relationship

$$\frac{\alpha^A-\beta^A\mathbf{p}^2/3m^2}{(q^2-m_\pi^2)^2[\beta'^A(p_0^2/m^2)+\gamma'^A]}\cong\frac{g_A^2}{(q^2-m_\pi^2)^2g_P^2}\cong 1 \ , \tag{16}$$

so that we obtain the relation

$$\alpha^A-\beta^A\mathbf{p}^2/3m^2\cong\beta'^A p_0^2/m^2+\gamma'^A \ . \qquad (17)$$

We now have enough information to evaluate Eq. (17), and from Eq. (8), to evaluate Eq. (1).

III. THE CROSS SECTION

Values for the lhs of Eq. (17) are obtained from the pion production data[5] via Eq. (14). The data available are extrapolated by approximately 10 MeV and converted to the c.m. frame in which E_p is 1076 MeV. Our experience[6] has shown that the inclusive curves are reasonably smooth and so this should not introduce an unacceptable error in this approximate calculation.

The mass M_x must also be obtained. Because we are examining a region up to approximately 20 MeV above the ^{13}C ground state, we take M_x to be 10 MeV above the ^{13}C ground state. Again, because the ^{13}C ground state mass is very large, this choice has only a very small effect upon the result.

With these considerations we find that

$$\alpha^A-\beta^A\mathbf{p}^2/3m^2=5.83\times10^{-14}/m_\pi^2 \ . \qquad (18)$$

This leads to a value by Eq. (8) for the vector part of the weak matrix element of

$$\alpha^V+\beta^V p_0^2/m^2+\gamma^V=4.41\times10^{-14}/m_\pi^2 \ , \qquad (19)$$

where all units are compatible with energies and momenta given in MeV. Equation (7) may now be evaluated and the cross section obtained. The result for a c.m. proton energy of 1076 MeV is given by

$$\sigma=4.41\times10^{-42}\ \text{cm}^2 \qquad (20)$$

in the c.m. frame.

IV. DISCUSSION OF RESULTS

We first note that the values for σ given by Eq. (20) is for a proton energy a little above the threshold for pion production. It would be particularly desirable to run an inclusive experiment below the pion production threshold to avoid the large background. However, we are really only making a theoretical estimate rather than an exact calculation here. From our earlier result[1] for the ground state of ^{13}C and from the results of Weiss and Walker[3] for a number of the excited states, we expect the dropoff in the cross section to be smooth, and so in the near threshold region we would expect cross section in the 10^{-42} cm^2 range.

We also remark the observation of direct production of muons above the pion production threshold is not completely out of the question. This is because the kinematical conditions are somewhat different. If a proton with c.m. kinetic energy of 160 MeV is assumed on a ^{12}C target, the maximum muon energy from decaying pions is about 157.4 MeV, while the maximum muon energy from direct production is 172.2 MeV. If the quantity $d\sigma/dE_\mu$ were observed, there would be a small region in which only muons formed by direct production would be observed. This result is of course true for both inclusive and exclusive processes.

The number which we have obtained, which is in the 10^{-42} cm^2 region, is still small but is closer to experimental accessibility. Although observation of exclusive processes is clearly more desirable, it might still be possible to observe anomalous threshold states in the inclusive cross section. This would be particularly true if it should prove practical to go above the pion threshold, where it is generally assumed that these[8,9] states would be most observable. Of course, for both the exclusive and inclusive processes q^2 continues to be timelike, thus making observation[6] of these states possible, as they would be physically realizable.

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