Analysis of slow neutron capture by ^{9}Be, ^{12}C, and ^{13}C

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We have analyzed the data on the primary radiative transitions following slow neutron capture by ^{9}Be, ^{12}C, and ^{13}C. We have made estimates of direct capture from optical potential models with physically realistic parameters. These parameters were varied to allow for the effects of a local s-wave level on the initial state. We find that the model estimates are in reasonable agreement with the measured cross sections of the 6.810- and 0.854-MeV transitions in ^{10}Be, the 1.262-MeV transition in ^{13}C, and, possibly, the 8.174-MeV transition in ^{14}C, indicating a major role played by a direct capture mechanism. In all these cases, the estimate of direct capture is greatly affected and reduced by large cancellation effects in the integrand of the radial dipole matrix element. The strengths of the 3.443-MeV transitions in ^{10}Be and, to a lesser extent, the 4.945-MeV transition in ^{13}C, cannot be explained by our optical model formulations of direct capture, but these strengths are consistent with extrapolations to these light nuclides of generally accepted formulations of compound nucleus capture.

I. INTRODUCTION

Information that has been accumulated on the radiative transitions that follow slow-neutron capture in a vast range of nuclides has been one of the main sources of knowledge on their detailed nuclear level structure. Most of this knowledge comes from interpretation of the gamma-ray energies, the relative yields, branching ratios of secondary transitions between low-lying states, and angular correlations among successive secondary transitions. In some cases spectroscopic information on the level structure may also be contained in the cross sections of the primary transitions originating from the capturing state, and this fact is demonstrated by the success of "direct" theories of neutron capture for a considerable number of nuclides, especially those of light and near closed-shell character. In this respect, a simple analytical formula for channel capture[1] has been found to be very successful in estimating cross sections for primary radiative transitions to final states with a considerable degree of single-particle character, and this success has even led to the use of this formula as a tool for determining other nuclear quantities of interest, such as spectroscopic factors of final states, total thermal absorption cross sections, neutron scattering lengths, and nuclear potential radii.[2–5] Recently, we have discussed elsewhere in general terms[6] the somewhat uncritical use of the channel-capture formula as such a tool, especially for the determination of nuclear potential radii, and we have made a specific comment[7] on the recent use of this formula to extract evidence for spin-dependent potential radii in the case of slow neutrons interacting with ^{9}Be. In that comment we stated our belief that the magnitudes of the cross sections for two out of three significant radiative transitions in the ^{9}Be(n,γ) reaction were reasonably consistent with direct-capture theory without invoking the need for spin-dependent potential radii. In the current paper we attempt more quantitative estimates of these cross sections using numerical methods for calculating scattering and capture within a realistic optical-model framework, as developed for the analysis of neutron capture data on the sulfur isotopes.[8]

II. SPECIALIZED OPTICAL MODEL TREATMENT

In the simple channel-capture theory,[1] the capture cross section depends on the binding energy of the final state (the primary gamma-ray energy), the nuclear potential radius, and the neutron scattering length, in addition to the spectroscopic factor and spin factors of the final state. In this simple theory, (a) the potential well is treated as extremely sharp-edged, (b) the contribution to the radial electric-dipole matrix element from the wave functions within the potential well is ignored, (c) the compound-nucleus fluctuations of the wave function are similarly ignored, and (d) a crude approximation is used for the value of the final-state radial wave function. In the more detailed version of the

theory,[8] these limitations are overcome by using the scattering and final-state wave functions numerically computed from a realistic optical-model potential. The potential capture so derived is still sensitive, in greater or lesser degree, to factors relative to those listed above, and the optical-model approach must come to terms with reproducing the measured parameters, principally the final-state binding energy and, especially, the neutron scattering length. The methods for doing this, and the formalism, are described fully in Ref. 8 dealing with slow-neutron capture by the sulfur isotopes. It is shown there that much of the complication due to the range of optical-model parameters available can be overcome by use of the quantity $C_{\rm opt}$, defined as the ratio of the potential-capture cross section (calculated from the chosen set of optical model parameters) to the value of the channel-capture cross section (calculated using as input parameters the radius of the optical-model potential, the binding energy of the single-particle p-wave final state for the real part of the optical potential, and the optical-model potential scattering length). Over considerable ranges of variation of the optical-model parameters, $C_{\rm opt}$ is fairly insensitive to changes, although it can differ considerably from the value of unity (which would imply exactitude for the channel-capture formula). However, in certain situations where the potential-capture cross section becomes very small, owing to strong cancellations in the integrand of the electric-dipole radial matrix element, $C_{\rm opt}$ can depend very strongly on the optical-model parameters. Such situations occur when the s-wave initial single-particle state is close to zero binding. The light nuclides close to ^{9}Be are known to be associated with such a single-particle state, so special care must be taken in the quantitative interpretation of their slow-neutron capture cross sections.

The relevant experimental data on the slow-neutron capture transitions of ^{9}Be are listed in Table I. In addition we have the important information on the total and coherent elastic-scattering cross sections of ^{9}Be (σ_s = 6.15 ± 0.01 b, $\sigma_{\rm coh}$ = 6.15 ± 0.01 b), which imply that the neutron scattering lengths are virtually spin-independent with numerical values a_J = 7.0 fm. To estimate the capture cross section we start with the "standard" set of optical-model parameters used in Ref. 8. This is a Woods-Saxon form:

$$\mathcal{U}(r) = \mathcal{V}(r) + i\mathcal{W}(r) + \mathcal{V}_{so}(r) \ , \tag{1}$$

where

$$\mathcal{V}(r) = \mathcal{V}_o/\{1 + \exp[(r - R)/d]\} \ , \tag{2}$$

$$\mathcal{W}(r) = \mathcal{W}_o/\{1 + \exp[(r - R)/d]\} \ , \tag{3}$$

$$\mathcal{V}_{so}(r) = (\mathbf{k}\cdot\sigma) K_s \mathcal{V}_o \exp[(r - R)/d]/[rd\{1 + \exp[(r - R)/d]\}^2] \ . \tag{4}$$

Here r is the radial separation of neutron and nucleus, k is the orbital angular momentum of the system, j is the spin formed by coupling the neutron spin to the orbital angular momentum, and K_s is the spin-orbit coupling coefficient. For slow-neutron capture the spin-orbit coupling term is nonzero only for the real potential that binds the final single-particle state. The two prescriptions quoted in Ref. 8 for the potential radius R [$R = 1.35\,A^{1/3}$ fm and $R = (1.16\,A^{1/3} + 0.6)$ fm] give alternative values of 2.808 fm and 3.013 fm for mass number 9. The calculation of final p-state binding energy E_f (spin-orbit coupling j = 3/2), theoretical scattering length $a_{\rm scatt}$, potential-capture cross section $\sigma_{\rm pot,\gamma}$, and $C_{\rm opt}$ are shown in the left-hand side of Fig. 1 as a function of real potential well depth $\mathcal{V}_0$ for the case R = 2.808 fm and diffuseness parameter d = 0.69 fm. (A similar set of calculations for ^{12}C + n is shown on the right-hand side of Fig. 1).

We find that the potential well depths that fit the binding energies of the final states listed in Table I also give potential scattering lengths that differ grossly from the observed value of the neutron scattering length for ^{9}Be (and also for ^{12}C). We find also that no reasonable variation of the imaginary component of the well depth can give the required value of the potential scattering length; for the radius and diffuseness of the "standard" optical models, a real well depth that reproduces modest binding energies of the p-wave states gives an unbound s state and hence a low rather than a high value of the scattering length.

Hence, we have attempted to find special sets of optical-model parameters that will reproduce both the binding energy of the final state E_f and the neutron scattering length a. For the ground-state transition we have done this by varying both the real well depth $\mathcal{V}_0$ and the diffuseness parameter d, keeping the imaginary well depth constant at $\mathcal{W}_0 = -2.5$ MeV. For the choice of potential radius R = 2.808 fm, it is found that d = 0.863 fm gives close agreement with the required values of E_f and a. The variation of E_f and potential scattering cross section with real well depth about its optimum value is shown in Fig. 2(a) together with the behavior of

TABLE I. Experimental data relevant to the current analysis.

Final nucleus	Level energy (MeV)	J_f^π	(d,p) θ_f^2	Primary E_γ (MeV)	Experiment $\sigma_{\gamma(i\to f)}$ (mb)
		(A). ^{9}Be(n,γ)^{10}Be reaction[a]			
		$\sigma_s = 6.15 \pm 0.01$ b; $\sigma_{coh} = 6.15 \pm 0.01$ b; $\sigma_\gamma = 7.6 \pm 0.8$ mb; $a = 7.0 \pm 0.1$ fm			
^{10}Be	0.0	0^+	2.1[b]	6.810	(4.9 ± 0.5)[d]
	3.368	2^+	0.35[b]	3.443	(0.86 ± 0.09)[d]
	5.958	2^+	0.785[c]	0.854	(2.0 ± 0.2)[d]
		(B). ^{12}C(n, γ)^{13}C reaction[a]			
		$\sigma_s = 4.75 \pm 0.01$ b; $\sigma_\gamma = 3.53 \pm 0.07$ mb; $a = 6.1 \pm 0.1$ fm			
^{13}C	0.0	$1/2^-$	1.1[e]	4.945	(2.38 ± 0.05)[f]
	3.684	$3/2^-$	0.1[e]	1.262	(1.14 ± 0.02)[f]
		(C). ^{13}C(n,γ) ^{14}C reaction[a]			
		$\sigma_s = 4.19 \pm 0.12$ b; $\sigma_{coh} = 4.16 \pm 0.12$ b; $\sigma_\gamma = 1.37 \pm 0.04$ mb; $a_{J=1} = 5.5 \pm 0.1$ fm; $a_{J=0} = 6.6 \pm 0.4$ fm			
^{14}C	0.0	0^+	2.09[g]	8.174	(1.15 ± 0.05)[g]

[a]Values for σ_s (scattering cross section), σ_{coh} (coherent scattering cross section), σ_γ (thermal neutron radiative capture cross section), and a (scattering length) that immediately follow are from S. F. Mughabghab, M. Divadeenam, and N. E. Holden, *Neutron Cross Sections,* Vol. 1, Part A, (Academic, New York, 1981).

[b]S. E. Darden, G. Murillo, and S. Sen, Nucl. Phys. **A266,** 29 (1976).

[c]Calculated value from S. Cohen and D. Kurath, Nucl. Phys. **A101,** 1 (1967).

[d]From σ_γ and branchings, the latter from E. T. Jurney, in *Neutron Capture Gamma Ray Spectroscopy,* edited by R. E. Chrien and W. R. Kane (Plenum, New York, 1978) p. 461.

[e]S. E. Darden, S. Sen, H. R. Hiddleston, J. A. Aymar, and W. A. Yeh, Nucl. Phys. **A208,** 77 (1973).

[f]From σ_γ and branchings, the latter from S. F. Mughabghab, M. A. Lone and B. C. Robertson, Phys. Rev. C **26,** 2698 (1982).

[g]S. K. Datta, G. P. A. Berg, and P. A. Quin, Nucl. Phys. **A312,** 1 (1978).

the potential scattering cross section and C_{opt}. While the latter varies fairly rapidly, it is not particularly unstable and a value of $C_{opt} = 0.40$ is indicated. Similar behavior of the potential scattering and capture cross sections and C_{opt} is found for the case $R = 3.013$ fm, for which $d = 0.84$ fm gives agreement with the required values of E_f and a, whereupon $C_{opt} = 0.45$.

Either of the choices for the potential radius and the associated value of the diffuseness parameter gives closely similar values of the potential-capture cross section, $\sigma_{pot,\gamma} = 5.9$ and 5.6 mb, respectively, close to the experimental value of 4.9 mb for the ground-state transition. The calculated $\sigma_{pot,\gamma}$ values are smaller by a factor of more than 2 compared to the value calculated from the channel-capture formula, which also gives results that vary more strongly with potential radius. The agreement of the optical-model value with experiment validates the hypothesis that the ground-state transition in the ^{9}Be(n,γ) reaction is principally a direct transition and indicates that there is little admixture of the compound-nucleus mechanism (which we suggested as a possibility in Ref. 6).

The remaining two transitions listed in Table I have been treated similarly, with the difference that the final state has been assumed to have spin-orbit coupling character $j = 1/2$ and the energies of the final state have been adjusted by varying the spin-orbit coupling constant as well as the real potential well depth. Some results for the 3.44-MeV transition are shown in Fig. 2(b). The most significant indication from this figure is

the extreme sensitivity of C_{opt} to variations in $\mathcal{V}_0$ (and, hence, presumably to other optical model parameters) because of cancellation effects in the wave function. Using the two sets of potential radii and diffuseness parameters principally employed for the study of the ground-state transition with the real well depth and spin-orbit coupling coefficient adjusted to give precisely the experimental values of binding energy and scattering length, we find a difference of a factor of 4 in the potential-capture cross section and of 2.5 in C_{opt}. We conclude that it is not possible to make a reliable, precise estimate of $\sigma_{pot,\gamma}$ for this transition. However, such model estimates as we have done indicate $\sigma_{pot,\gamma} \leq 0.01$ mb, almost two orders of magnitude lower than the measured cross section of 0.86 mb. This result indicates that the 3.44-MeV transition may well be dominated by a compound-nucleus mechanism. We shall return to this point later.

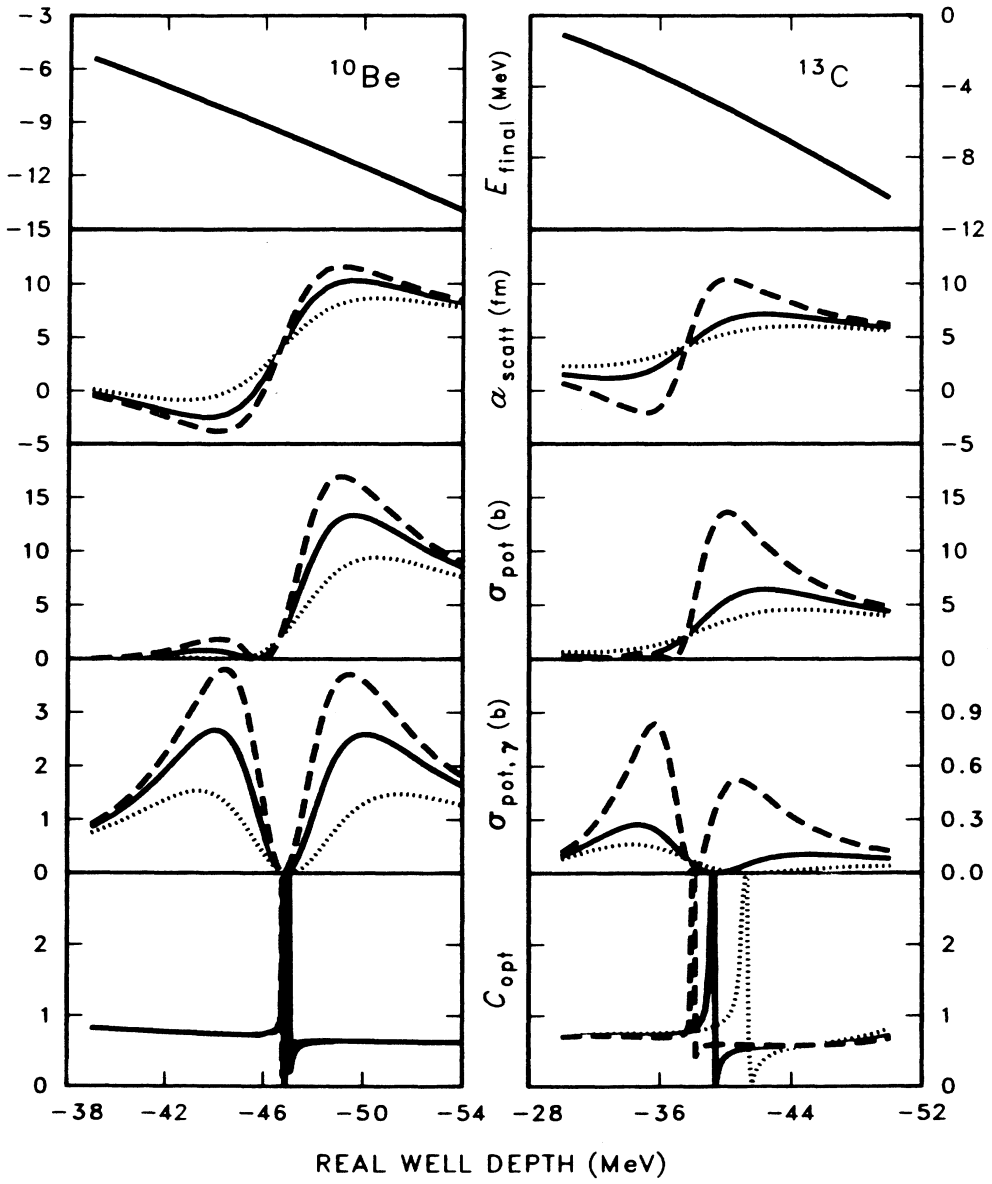

FIG. 1. Results of optical-model calculations with a Woods-Saxon potential well of radius $R = 2.808$ fm for ^{9}Be and $R = 3.174$ fm for ^{12}C and diffuseness parameter $d = 0.69$ fm in both cases. The real well depth $\mathcal{V}_0$ (in MeV) is shown in the abscissae. For the final nucleus ^{10}Be and $j = 3/2$, the dashed curve corresponds to an imaginary well depth $\mathcal{W}_0 = -2.5$ MeV, the solid to $\mathcal{W}_0 = -3.0$ MeV, and the dotted to $\mathcal{W}_0 = -4.0$ MeV. For ^{13}C and $j = 1/2$, the dashed curve corresponds to $\mathcal{W}_0 = -2.5$ MeV, the solid to $\mathcal{W}_0 = -5.0$ MeV, and the dotted to $\mathcal{W}_0 = -7.5$ MeV. In these calculations, θ^2 and $\mathcal{W}_{J_f}$ have been set to unity.

The position is much clearer for the 0.853-MeV transition. The spin-orbit coupling coefficient must be increased by almost a factor of 2 to reproduce the final-state binding energy, but the C_{opt} factors turn out to be relatively insensitive to the optical-model parameters. For $R = 3.013$ fm, $d = 0.84$ fm, we obtain $C_{opt} = 1.04$ and for $R = 2.808$ fm, $d = 0.863$ fm, we obtain $C_{opt} = 1.19$. There is even better agreement (to within 4%) between the two estimates of $\sigma_{pot,\gamma}$ which, in the latter case, is 2.0 mb if the theoretical spectroscopic factor given in Table I is applied. This estimate is in exact agreement with the experimental value of 2.0 mb.

III. GLOBAL OPTICAL MODEL WITH LOCAL LEVEL ADJUSTMENT

It can be argued that forced adjustment of the optical-model parameters to give agreement with both the binding energies of bound states of largely single-

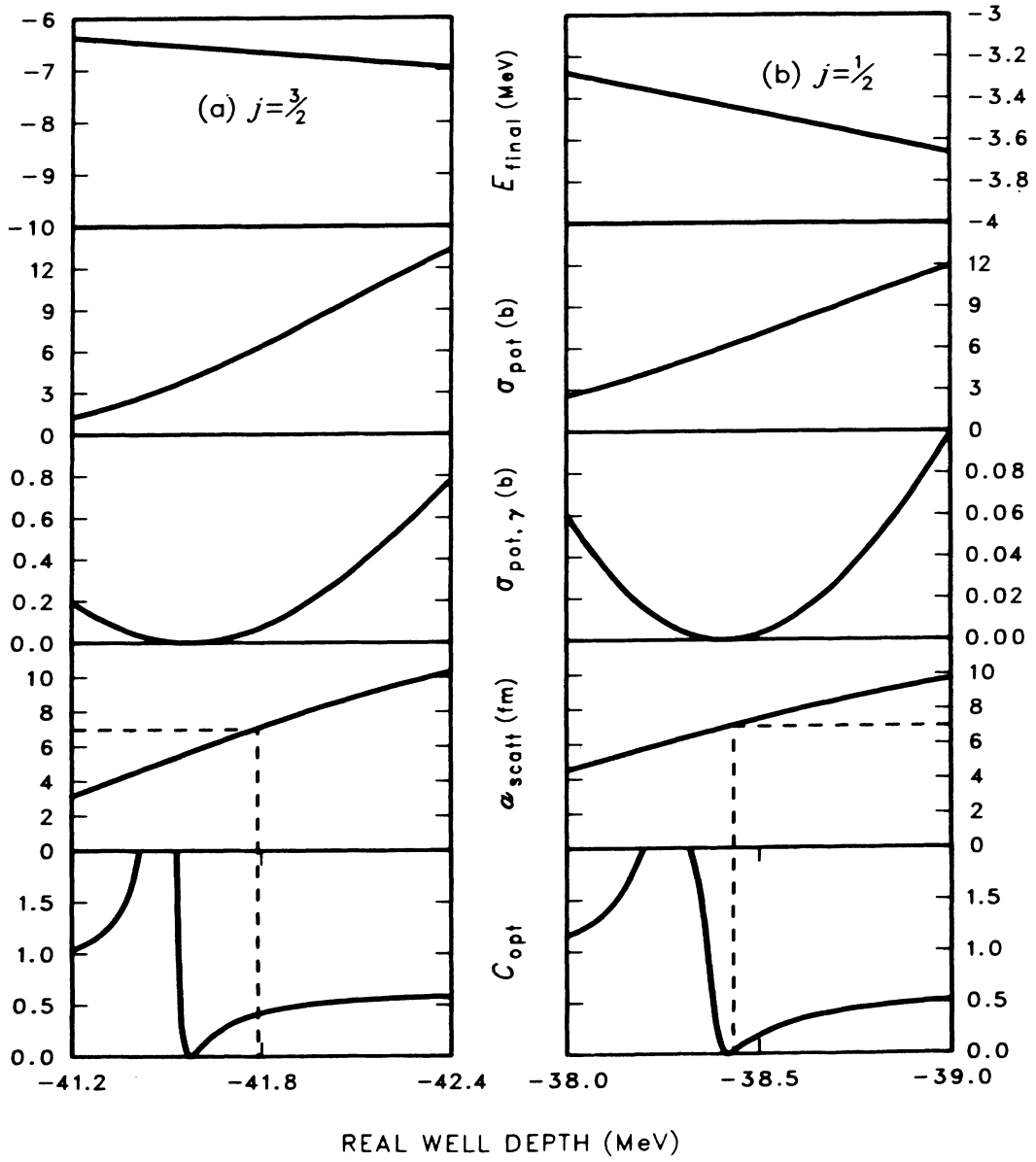

FIG. 2. (a) Results of specialized optical-model calculations for the ground-state transition in ^{10}Be with optical model parameters $R = 2.808$ fm, $d = 0.863$ fm, and $\mathcal{W}_0 = -2.5$ MeV. The known scattering length of 7.0 fm implies $C_{opt} = 0.40$ and $\sigma_{pot,\gamma} = 5.9$ mb. (b) Results of similar calculations for the 3.44-MeV transition in ^{10}Be with $R = 3.013$ fm, $d = 0.84$ fm, and $\mathcal{W}_0 = -2.5$ MeV. In this case C_{opt} is extremely sensitive to the real well depth $\mathcal{V}_0$.

particle character and with thermal neutron scattering lengths (the latter quantities may be strongly affected by local compound-nucleus states) is not the best way to proceed to explain thermal-capture cross sections. In fact, at least for the ground-state transition from the ^{9}Be(n,γ) reaction, the procedure we have adopted can be thought to be reasonable for two reasons. First, the adjustment to the optical potential mainly by increasing the diffuseness parameter by some 20% will not greatly affect the character of the tail of the final-state wave function, from which comes the major contribution to the radial matrix element, provided the binding energy is correct. Second, for such light systems as ^{9}Be, the distinction between the single-particle and compound-nucleus character of the nucleus is becoming blurred. More specifically, the s-wave "local" levels nearest to zero binding are bound states at -0.851 MeV ($J=1$) and -0.547 MeV ($J=2$), and these have a very significant single-particle character.[9] Hence, the optical-model ought to reproduce their main properties.

Nevertheless, it is worth exploring a method in which local level properties can be introduced explicitly within a "near-global" optical-model framework that gives a potential-capture cross-section background. This is similar to an approach adopted by Ho and Lone[10] to estimate the capture cross section for the ground-state transition in the ^{12}C(n,γ) reaction and is in fact a treatment of interference between valence and potential capture as described in Refs. 11 and 12. By slight extension of the formalism given in Ref. 8, (see especially Eqs. 28 and 36), we obtain, for off-resonant situations,

$$\sigma_{\gamma(i\to f)} \approx \sum_J g_J \left| \sqrt{4\pi} R \mathcal{R}_J^{\rm loc} \left(\frac{\bar{\Gamma}_{\lambda\gamma(i\to f)(\rm val)}}{\Gamma_{\lambda n}} \right)^{1/2} + \sigma_{\rm pot,\gamma}^{1/2} \right|^2 . \tag{5}$$

Here, g_J is the spin statistical weight factor for scattering in total angular momentum state J, $\mathcal{R}_J^{\rm loc}$ is the contribution from local levels to the reduced $\mathcal{R}$-function describing the scattering function, $\Gamma_{\lambda\gamma(i\to f)(\rm val)}$ is the valency radiation width for the transition to final state f from a local level λ, and $\Gamma_{\lambda n}$ is the neutron width. The magnitude of the reduced $\mathcal{R}$-function is determined from the measured neutron scattering length,

$$a_J = R\left(1-\mathcal{R}_J\right) , \tag{6}$$

and is the sum of terms from local levels $\mathcal{R}_J^{\rm loc}$ and distant levels $\mathcal{R}^{\infty}$,

$$\mathcal{R}_J = \mathcal{R}_J^{\rm loc} + \mathcal{R}^{\infty} . \tag{7}$$

The distant-level contribution $\mathcal{R}^{\infty}$ is, in turn, related to the potential scattering length

$$a_{\rm pot} = R\left(1-\mathcal{R}^{\infty}\right) , \tag{8}$$

whence $R\,\mathcal{R}_J^{\rm loc} = (a_{\rm pot} - a_J)$ for substitution in Eq. (5).

The estimation of the valency radiation width is also derived from the calculation of capture in an optical-model potential, being related to the imaginary component of the capture amplitude (Eqs. 55 and 56 of Ref. 8). The neutron width to employ in Eq. (5) is derived from the neutron strength function calculated from the optical-model scattering. To obtain the most accurate values of the valency radiation width and potential-capture cross section, it is necessary to make certain adjustments to the chosen global optical potential to reproduce the observed final-state binding energy. In the context of direct capture, the main physical characteristics that seem important to preserve in the global optical potential are radius R, surface diffuseness d, and imaginary well depth $\mathcal{W}_0$. The final-state binding energy is then achieved by adjusting the real well depth $\mathcal{V}_0$, giving, for each final state considered, a set of "near-global" optical parameters from which, in conjunction with the observed neutron scattering length, the quantities required for Eq. (5) can be calculated.

For the main parameters of our global potential we have chosen $R = 2.808$ fm, $d = 0.69$ fm, $K_s = 0.00435$, and $\mathcal{W}_0 = -2.5$ MeV. In the case of the 6.81-MeV transition, we can achieve the binding energy of the ground state of ^{10}Be by adopting $\mathcal{V}_0 = -41.67$ MeV. In this case, $j = 3/2$. The computation gives

$$\sigma_{\rm pot} = 0.604 \text{ b} ,$$
$$\mathcal{R}^{\infty} = 1.781 ,$$
$$\sigma_{\rm pot,\gamma} = 1.995\,\theta^2 \mathcal{W}_{J_f} ,$$
$$C_{\rm opt} = 0.77 ,$$
$$\bar{\Gamma}_{\gamma(i\to f)(\rm val)}/DE_\gamma^3 = 5.94 \times 10^{-8}\,\theta^2 \mathcal{W}_{J_f}(\text{MeV}^{-3}) ,$$

and

$$\bar{\Gamma}_n/D = 0.674 \times 10^{-4} .$$

Here D is the compound-state level spacing, θ^2 the relative single-particle reduced width of the final state, and $\mathcal{W}_{J_f}$ the spin factor of the electric-dipole matrix element of the transition (see Ref. 8 for numerical details).

Using $\mathcal{R}_J = -1.49$ for both values of total angular momentum, Eq. (5) finally results in the value $\sigma_{\gamma(i\to f)} = 8.2$ mb, to be compared with the experimental value of 4.9 mb and the adjusted optical-model estimate of 5.6 mb. The calculation, in this case, turns out to be insensitive to the chosen value of $\mathcal{W}_0$, within the reasonable range $\mathcal{W}_0 = -2.5$ to -4.0 MeV.

In the case of the 3.44-MeV transition, the binding energy is achieved, for an assumed $j = 1/2$ final state with $K_s = 0.00435$ and $\mathcal{V}_0 = -46.57$ MeV. The computation of other quantities then yields

$$\sigma_{\text{pot}} = 1.430 \text{ b} ,$$
$$\mathcal{R}^{\infty} = -0.201 ,$$
$$\sigma_{\text{pot},\gamma} = 0.096\, \theta^2\, \mathcal{W}_{J_f} ,$$
$$C_{\text{opt}} = 0.86 ,$$
$$\bar{\Gamma}_{\gamma(i\to f)(\text{val})}/DE_{\gamma}^3 = 59.2 \times 10^{-8}\, \theta^2 \mathcal{W}_{J_f}(\text{MeV}^{-3}) ,$$

and

$$\bar{\Gamma}_n/D = 3.27 \times 10^{-4} .$$

The capture cross section from Eq. (5) is then found to be 0.11 mb (compared to the experimental value of 0.86 mb and an adjusted optical-model value of $\leqslant 0.01$ mb). If $j = 3/2$ is assumed for the final state, its binding energy is reproduced by $\mathcal{V}_0 = -34.96$ MeV. This real potential value shifts the s-wave scattering properties considerably, but the final computed capture cross section is changed relatively little to 0.12 mb. This cross section is still considerably smaller than the experimental value of 0.86 mb. Increasing the magnitude of the imaginary well depth from $\mathcal{W}_0 = -2.5$ MeV to $\mathcal{W}_0 = -4$ MeV has the effect of decreasing the capture cross section by 10%.

The binding energy of the final state of the 0.854-MeV transition can be reproduced (assuming $j = 1/2$) with $\mathcal{V}_0 = -38.66$ MeV. For $\mathcal{W}_0 = -2.5$ MeV, we calculate

$$\sigma_{\text{pot}} = 0.0010 \text{ b} ,$$
$$\mathcal{R}^{\infty} = 1.102 ,$$
$$\sigma_{\text{pot},\gamma} = 0.080\, \theta^2\, \mathcal{W}_{J_f} ,$$
$$C_{\text{opt}} = 0.72 ,$$
$$\bar{\Gamma}_{\gamma(i\to f)(\text{val})}/DE_{\gamma}^3 = 27.2 \times 10^{-8}\, \theta^2\, \mathcal{W}_{J_f}(\text{MeV}^{-3}) ,$$

and

$$\bar{\Gamma}_n/D = 0.308 \times 10^{-4} .$$

Hence $\sigma_{\gamma} = 1.36$ mb compared to the experimental and adjusted optical-model values of 2.0 mb.

These results for the three transitions cannot be discussed confidently as a body because the potential scattering properties of the system differ strongly in the treatment of each transition. A more systematic approach can be devised by allowing the potential parameters for the p-wave and s-wave states to differ somewhat. The s-wave optical potential and, hence, the potential scattering length remain the same for each transition. The p-wave potential depth is then considered to be adjusted separately to reproduce the final-state binding energy for each transition. This procedure is justifiable if the behavior of C_{opt} is stable in the chosen region of the s-wave optical-model parameters; the different well depth of the p-wave potential has the effect of representing with good accuracy the bound-state wave function in the external region, even though the departure of the bound-state eigenvalue from the "ideal" single-particle energy will be due to residual interactions and configuration mixing.

In the case of a target nucleus like ^{9}Be with nonzero target spin, the s-wave optical-model parameters can be allowed to differ for the two total angular momentum values J that appear in the initial scattering state. The local resonance scattering properties, in the form of $\mathcal{R}_J^{\text{loc}}$, are now to be constructed from known properties of local s-wave levels:

$$\mathcal{R}_J^{\text{loc}} = \sum_{\lambda(\text{local})} \frac{\gamma_{\lambda(n)}^2}{E_{\lambda J} - E} , \tag{9}$$

where the $\gamma_{\lambda(n)}^2$ are the reduced neutron widths of the levels λ. In the case of ^{9}Be only two such levels of differing J are known.[13] These are bound levels, with binding energies $| E_{\lambda,J=1} | = -0.851$ MeV and $| E_{\lambda,J=2} | = -0.547$ MeV. The reduced neutron width of the latter can be deduced from the (d,p) stripping strength.[9] It is $\gamma_{\lambda(n)}^2 = 0.608$ MeV, giving $\mathcal{R}_{J=2}^{\text{loc}} = -1.11$ and, hence, [from Eq. (7)] $\mathcal{R}_{J=2}^{\infty} = -0.38$. However, the width of the former level is very poorly known because it lies so close to the p-wave level at -0.854 MeV. If the width is taken equal to that of the -0.547 MeV level, we find $\mathcal{R}_{J=1}^{\text{loc}} = -0.71$ and, hence, $\mathcal{R}_{J=1}^{\infty} = -0.78$. These $\mathcal{R}$ values give rise to a potential scattering length just in the critical region where C_{opt} is violently unstable for the global optical potentials we have discussed and thus presents difficulty in making a reliable estimate of the potential-capture cross section.

Alternatively, if we assume $\mathcal{R}_{J=1}^{\text{loc}} = \mathcal{R}_{J=2}^{\text{loc}}$ (hence, $\mathcal{R}_{J=1}^{\infty} = \mathcal{R}_{J=2}^{\infty} = -0.38$), we can proceed smoothly. For the global optical-model parameters, $R = 2.808$ fm, $d = 0.69$ fm, our earlier numerical calculations have established that $C_{\text{opt}} = 0.77$ for the 6.81-MeV transition, $C_{\text{opt}} = 0.86$ for the 3.44-MeV transition, and $C_{\text{opt}} = 0.72$ for the 0.85-MeV transition. We now calculate the potential-capture cross section for each case from the formula

$$\sigma_{\text{pot},\gamma} = C_{\text{opt}}\, \sigma_{\text{CH},\gamma} , \tag{10}$$

where $\sigma_{\text{CH},\gamma}$ is deduced from the channel-capture formula given in Ref. 8 with full center-of-mass corrections.

We employ the potential scattering length consistent with the above values of $\mathcal{R}^{\infty}$. The values of the final capture cross sections for each transition are then calculated from Eq. (5). The results are

6.81-MeV transition: $\sigma_\gamma \approx 9.2$ mb ,

3.44-MeV transition: $\sigma_\gamma \approx 0.08$ mb ,

and

0.85-MeV transition: $\sigma_\gamma \approx 1.35$ mb .

These results are quite consistent with the previous approach.

IV. COMPOUND NUCLEUS RADIATIVE MECHANISM

In all the approaches we have investigated to establish a reliable estimate of the direct-capture cross section (including the valence mechanism) for the principal primary electric-dipole transitions resulting from slow-neutron capture by ^{9}Be, a similar pattern has emerged. The direct-capture estimate lies within a factor of 2 of the observed capture cross section for the 6.81-MeV and 0.85-MeV transitions, but for the much weaker 3.44-MeV transition it is at least an order of magnitude smaller. These differences prompt us to speculate on the magnitude of the cross section that would result from a more complex compound-nucleus mechanism for the radiative transition. We can deduce the magnitude of a compound-nucleus transition undiluted by a direct mechanism by subtracting the amplitude of the direct part from that of the observed cross section. The result for the compound-nucleus capture cross section for the 3.44-MeV transition is

$$\sigma_{\gamma(\mathrm{CN})} = 0.34 \text{ or } 1.62 \text{ mb} .$$

From this result an estimate of the compound-nucleus radiation width can be deduced as

$$\Gamma_{\gamma(\mathrm{CN})} = 0.05 \text{ or } 0.25 \text{ eV} .$$

These values are consistent with Cameron's semi-empirical statistical formula[14] for radiation widths (with s-wave level spacing D equal to 1 and 4 MeV, respectively) or to Brink's deduction[15] from the giant-dipole-resonance model (with D equal to 4 and 15 MeV, respectively). The value of D for s-wave levels is unknown, but for p-wave levels with $J = 2$ it appears to be $\approx$2 MeV. This spacing would imply either value for $\sigma_{\gamma(\mathrm{CN})}$ to be reasonable in the sense of being in qualitative agreement with our current knowledge concerning the systematics of compound-nucleus radiation widths.

Extrapolation of the lower value of $\sigma_{\gamma(\mathrm{CN})}$ to the 6.81-MeV transition (assuming $\Gamma_{\gamma(\mathrm{CN})} \propto E_\gamma^3$) would result in a final cross section (which includes both direct and compound-nucleus effects in destructive interference) of $\sigma_\gamma \approx 4$ mb, in close agreement with the observed value. Extrapolation to the 0.85-MeV transition increases the cross section to only 1.5 mb, but this agrees as closely with the observed value of 2.0 mb as we can reasonably expect, considering the uncertainty of our knowledge of the spectroscopic factor of the final state.

Further evidence of the magnitude of a possible compound-nucleus effect in such radiative transitions may be sought from neighboring light nuclides. In the slow-neutron capture of ^{12}C there are two primary transitions of $E1$ multipolarity, with gamma-ray energies of 4.945 MeV and 1.262 MeV with cross sections of 2.38 and 1.14 mb, respectively,[4] while for ^{13}C useful data exist on the transition to the ground state with a gamma-ray energy of 8.174 MeV and cross section of 1.15 mb [Ref. 4]. The experimental data on these are given in Table I.

The 4.945-MeV transition of ^{12}C(n,γ) has been discussed by Ho and Lone,[10] who calculated the thermal-neutron capture cross section to be in near agreement with the experimental value. Furthermore, they found the calculated value to be relatively insensitive to the potential radius. This value is low—very much smaller than the hard-sphere capture cross section estimate—due to large cancellations in the integrand of the radial matrix element. Our estimates, derived from a range of optical-model parameterizations and treatments, are generally even smaller. A specialized optical-model treatment with surface absorption (of Gaussian form) results in a value of 0.17 mb. A global optical-model plus local-level treatment results in values ranging from $\sim$1 μb to 0.4 mb for R ranging from 2.8 fm to 3.1 fm. These calculations would suggest that the compound-nucleus contribution to the cross section of this transition is dominant and is of the order of 1 mb. This is consistent with the Cameron semi-empirical rule for electric-dipole radiative widths and the parameters of the known bound s-wave level at -2.02 MeV.

The cross section for the 1.262-MeV transition is harder to calculate (see Ref. 10), because the final state is of $j = 3/2$ character and the $1p_{3/2}$ single-particle state is usually bound by some 10 MeV. However, it is the tail of the wave function in the channel that is of chief importance, and this effect can be represented realistically by adjusting the depth of the real well to achieve the observed binding energy. With the following set of parameters,

$$R = 3.091 \text{ fm}, \quad d = 0.69 \text{ fm} ,$$
$$\mathcal{V}_0 = -41.4 \text{ MeV (for } \ell = 0) ,$$
$$\mathcal{V}_0 = -25.07 \text{ MeV (for } \ell = 1) ,$$
$$K_s = 0.00435, \text{ and } \mathcal{W}_0 = -8 \text{ MeV} ,$$

and a local level with the properties of the -2.02-MeV s-wave level, it is found that the direct-capture cross sec-

tion is 1.32 mb, close to the experimental value of 1.14 mb. This agreement again is consistent with the Cameron estimate of the compound-nucleus effect, which estimate would fall to about 10 μb and only perturb the total cross section for this transition in a minor way.

Considering its very high gamma-ray energy, the cross section for the 8.174-MeV transition of ^{13}C(n,γ) is extremely small. A specialized optical model for this transition has the parameters:

$$R = 3.174 \text{ fm}, \quad d = 0.69 \text{ fm} ,$$

$$\mathcal{V}_0 = -45.84 \text{ MeV}, \quad K_s = 0.00435 ,$$

and

$$\mathcal{W}_0 = -9 \text{ MeV} .$$

The ground state is treated as $j = 1/2$. These parameters give the observed final-state binding energy and s-wave scattering length, and a calculated potential-capture cross section of 0.40 mb—yet another example of extreme cancellation effects in the radial integral. The discrepancy between this value and the observed value of 1.15 mb could be an indication of a compound-nucleus contribution of the same order of magnitude. The required radiation width ($\Gamma_{\gamma(CN)} \sim 0.5$ eV) is well within the Cameron estimate (~3 eV for s-wave level spacing of 3 MeV).

V. QUASI-REAL POTENTIAL MODELS

A serious difficulty in the concept of the local level approach (as described in Sec. III) for this group of very light nuclei is the apparently special significance of the chosen local level. The concept requires that the single-particle s-wave state be dissolved among a considerable number of more complex states over an energy interval related to the imaginary component of the optical potential; nearby local levels, which are otherwise quite typical of the range of complex states, are included in the formalism to account for the distortion of the nuclear scattering wave function from the average. However, the bound state chosen as the local level in these $A \approx 10$ nuclei is characterized by the following properties: (1) in each compound nucleus it is the only known bound level—for a given J-component in s-wave neutron scattering—to be consistent with zero orbital angular momentum; (2) not only that, but no strong s-wave resonances are known in the neutron cross section; (3) there is an apparently systematic behavior of the bound state with mass number (it exhibits an almost monotonic increase in binding energy with increasing A); and (4) previous (d,p) stripping studies have established that in many cases the spectroscopic factor of the state is a large fraction of the $\ell=0$ single-particle state.

The possibility must be considered therefore that the bound state, rather than being a local level accompanied by more distant fellows of a statistically similar nature, is in fact a virtually pure single-particle s state. If so, the scattering properties of the system and its radiative properties would be describable by a real potential well rather than an optical-potential plus local-level model.

We first of all examine the scattering cross section we expect from a real potential model which has the Woods-Saxon form of Eq. 1. In Fig. 3 we show the behavior of the potential scattering length computed for a real potential with radius given by $R = 3.0$ fm. (The scattering length, for a given eigenvalue, is very insensitive to the potential radius). The well depth was varied to give the changing single-particle eigenvalue shown as the binding energy of the state on the abscissa. The known potential scattering lengths and bound s-state energies for several light nuclei are also plotted as discrete points. It is apparent that in most cases the measured scattering length is considerably lower than is required by the potential well that reproduces the binding energy. The exceptions are for ^{12}C, ^{14}N, and ^{15}N.

In the above calculations we have chosen the diffuseness parameter d (the only free parameter remaining in the Woods-Saxon form) to be 0.69 fm (i.e., within the range of what is considered physically reasonable in nuclear-potential models). If the diffuseness parameter is lowered in value, the potential scattering length is diminished for a given single-particle state binding energy as shown in Fig. 3. We have explored this trend to see if a parameterization can be found that fully reproduces the experimental data. It is apparent that for most nuclides in the group shown in Fig. 3 the diffuseness parameter

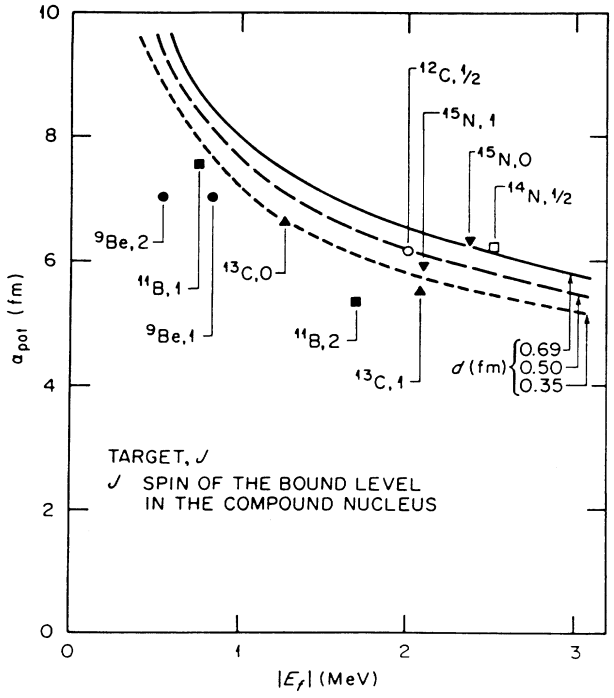

FIG. 3. The s-wave potential scattering length calculated for a potential radius $R = 3.0$ fm, three different values for the diffuseness parameter d, and varying real well depth plotted against the bound s-wave eigenvalue for the same potential. Experimental data (Ref. 13) are also shown.

would be required to be very small and the well depth greater than 60 MeV if the real-potential model were to reproduce the binding energy and scattering length simultaneously.

The deficiency of the potential scattering length can be used to give estimates of the positions and reduced widths of other possible levels. Since the computation of the single-particle eigenvalue also yields the wave function $u(a)$ at a chosen channel radius of $a = 8$ fm (the Woods-Saxon potential form is cut off to zero beyond this point), we can determine the reduced neutron width of the single-particle level as

$$\gamma_{(n)}^2 = \frac{\hbar^2}{2Ma} u^2 \tag{11}$$

and from this deduce the principal contribution to the $\mathcal{R}$-function of the real potential

$$\mathcal{R}^{\mathrm{pr}} \approx \frac{\gamma_{(n)}^2}{(E_o - \Delta)[1 + \kappa a (\mathcal{R} - \mathcal{R}^{\mathrm{pr}})]^2} , \tag{12}$$

where the level shift $\Delta = -\kappa a \gamma_{(n)}^2$ is required to adjust the physical eigenvalue to the $\mathcal{R}$-matrix eigenvalue with the continuum boundary condition of zero. In Eq. 12 the quantity $\kappa = \sqrt{(2M|E_o|/\hbar^2)}$ is the attenuation coefficient of the tail of the eigenstate wave function, and M is the reduced mass of the target-neutron system.

The net contribution to $\mathcal{R}$ from lower and higher levels can be calculated from the eigenvalues of a real potential well with imposed boundary condition $B = 0$ at the chosen channel radius of 8 fm. On the hypothesis that the bound s-state is reduced below the single-particle strength by a given fraction, we can then evaluate the possible position of other $\mathcal{R}$-matrix states in the region above the binding energy. We do this by modifying Eq. 12 to reproduce the observed s-wave scattering cross section, also allowing for the net contribution to $\mathcal{R}$ as described above. It is clear that we cannot seek the extra states above the known bound state and below the neutron binding energy, for this would yield a scattering cross section greater than the value for the real potential-scattering model. In the case of ^{9}Be + n, reductions in the single-particle strength of the bound state by 10 to 40% result in the location of the remaining strength at an energy of 0.2 MeV to 1.0 MeV. At these low energies such a state causes prominent resonance or interference features in the cross section, and these are not found in the experimental data. On the other hand if the bound state contains only about 30% of the single-particle strength, the bulk of the single-particle state could then be located in one or a few unbound $\mathcal{R}$-matrix levels at about $E_n \approx 4.5$ MeV. At this much higher energy the resulting s-wave cross section, which is only a small fraction of the total cross section, shows only a relatively small kink due to the slow change of the phase shift through a multiple of $\pi/2$. A fluctuation of similar magnitude is observed in the measured cross section at $E_n = 4.2$ MeV (Ref. 16).

Proof of the hypothesis that the bound $J = 1$ and $J = 2$ states at -0.85 MeV and -0.55 MeV, respectively, in ^{10}Be contain only about 30% of the single-particle s-wave state, with the remaining strength centered at about 4 to 5 MeV neutron energy, will require a full analysis of polarization and/or angular distribution data on the scattering of neutrons by ^{9}Be. Such an analysis is so far not available in the relevant energy region. Meanwhile, this hypothesis seems to offer the most plausible explanation of the slow neutron scattering properties of ^{9}Be and supports the interpretation of the ^{9}Be(n,γ) data through the method of a global optical potential and local-level valence contribution as outlined in Sec. III.

On the other hand, in the case of ^{12}C (n,γ) the data suggest that a real-potential model with $\mathcal{V}_0 = -50.27$ MeV, $d = 0.5$ fm, $R = 3.25$ fm could describe the scattering. The potential-capture cross section calculated for the ground-state transition in such a model is 0.66 mb. This is still a factor of 3 smaller than the observed value but considerably larger than is given by any of the optical-model calculations presented earlier. In such a picture there would be no scope for any compound-nucleus contribution to the capture.

VI. CONCLUSIONS

The light nuclides in the region of mass number 10 would appear to be good candidates for the manifestation of direct effects in reactions with slow- to medium-energy neutrons. There are few nucleons in the system, so states at low-to-moderate excitation energies should have comparatively simple structure, and indeed it is known that the single-particle $1p$ state dominates the shell-model structure near the ground state while the $2s$ single-particle state dominates the region close to zero binding energy. In these circumstances, it is particularly expected that the slow-neutron capture reaction should show strong direct effects for $E1$ transitions. Yet it is found that the cross sections for such transitions are particularly weak. It has been noted that the cause of this weakness is almost certainly due to major cancellation in the components of the radial electric-dipole matrix element, and it has been our aim in this paper to study (a) whether this effect is quantifiable, (b) how sensitive are the cross sections to parameterization of the nuclear model, and (c) whether other (compound nucleus) effects play a signficant role.

With this aim, we have discussed some methods for making estimates of the thermal neutron direct-capture cross sections for some electric-dipole transitions associated with capture by ^{9}Be, ^{12}C and ^{13}C. In doing so we have tried to avoid the shortcomings that are inherent in the use of the simple channel-capture formula that leads to results deviating from experiments in such a way that often these deviations cannot be reconciled by use of a

single value of the potential radius for a given target nucleus. We have shown that the cross sections of the two strongest transitions of ^{9}Be(n,γ) can both be accounted for as potential-capture cross sections calculated from optical-model parameters adjusted to reproduce both the potential-scattering cross section and the final-state binding energies. However, this approach requires an optical-model diffuseness parameter that is somewhat larger than is normally acceptable. Hence, we have also attempted approaches that use optical-model parameters much closer to a global set. They give both a potential-capture amplitude and an estimate of the valency radiation width, correlated to the single-particle p-wave content of the final state, that can be used to calculate the amplitude of the valency transition from local levels. The relative phase of the two amplitudes is completely determined by the theory, and hence an unambiguous estimate of the "direct" cross section (potential plus valency) results. Such estimates for the two strongest transitions of ^{9}Be(n,γ) deviate a little more from the experimental values but are not in unreasonable agreement.

However, the cross section for the 3.44-MeV transition as estimated by both these methods is much smaller than the observed value. We have speculated that the discrepancy is due to the further admixture of a more complex "compound-nucleus" contribution to the capture mechanism. The sign of the compound-nucleus amplitude is undetermined with respect to the direct-capture amplitude. The likely magnitude of the former appears to be close to what we would expect from the statistical estimates for compound-nucleus radiation widths, and the inclusion of such a width in the estimate of capture from local levels can bring close agreement with the experimental cross-section data for all three transitions. This estimate of compound-nucleus strength is also supported by the study of a few transitions in capture by the carbon isotopes. Among these, only one [the 1.262-MeV transition of ^{12}C(n,γ)] appears unambiguously to be a direct transition.

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