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Unified description of πd-πd, πd-NN, and NN-NN reactions at intermediate energies

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The theory of the coupled πNN-NN systems is applied to the unified description of the πd-πd, πd-NN, and NN-NN reactions in the intermediate energy range. Off-shell modifications are introduced in the P_{33} and P_{11} πN channels and the effects of heavy meson exchange are investigated. In general, results for various observables are in agreement with experiments. We have assessed certain aspects upon which future work should improve.

I. INTRODUCTION

Recent years have witnessed an increasing amount of study in the theoretical investigation of the coupled πNN-NN problem.[1-11] This is due to (i) a continued effort to build a consistent theory of the nucleus with an explicit treatment of the pion and Δ degrees of freedom, for which the πNN-NN system is the fundamental testing ground;[12,13] and (ii) a large body of experiments done during the past several years in the $\pi d \rightarrow \pi d$, $\pi d \leftrightarrow NN$, and $NN \rightarrow NN$ channels in an attempt to search for the signals from the possible isospin-one dibaryon resonances.[14]

The ultimate goal of this πNN-NN theory is to attain a certain degree of successful description of the πd elastic, $\pi d \leftrightarrow NN$, and $NN \rightarrow NN$ processes (and furthermore of the processes with three-body final states: $NN \rightarrow \pi NN$ and $\pi d \rightarrow \pi NN$), or at least to isolate (or identify) its inherent deficiency in cases where the agreement with experiment may not be achieved. This then requires a careful selection and/or construction of the input first and a large scale numerical solution for the observables, which needs some extended effort in time. Since our last publication[3] we have improved our calculation in a number of ways, the results of which have been reported occasionally at various conferences and workshops as well as supplied to experimentalists working on related subjects, but which have never been written up for publication. We are still in the process of extending our models, and in the sense our work is still not completed. But since the bulk of our results has accumulated to a certain threshold, and since several calculations based upon sets of equations similar to our have appeared,[7,8,10,11,15] we have decided to put together a part of our results for publication.

The present body of results is concerned with the unified description of the $\pi d \rightarrow \pi d$, $\pi d \leftrightarrow NN$, and $NN \rightarrow NN$ observables for the energy range $T_{\rm lab}^{\rm N} \simeq 560$–$800$ MeV ($T_{\rm lab}^{\pi} \simeq 140$–$260$ MeV). One of the novel features of our calculation as compared with the previous one is the introduction of the heavy meson exchanges (HMX) in the two-body NN sector (not in the NN interaction in the πNN sector). This is expected to improve the result in the elastic NN and $NN \leftrightarrow \pi d$ channels at intermediate energies. For this objective we have employed one version of the one-boson-exchange (OBEP) Bonn potentials[16] replacing its one-pion-exchange (OPE) part by the one generated in our equation and readjusting the coupling strengths of σ, ω, and ρ exchanges, as the two-pion exchange NΔ box contribution generated by the equation accounts to a large extent for the intermediate range attraction in the OBEP. This adjustment has been done qualitatively but not quantitatively since we have restricted our aim to be rather modest in the present work, viz., to study the HMX effects principally in the $\pi d \rightarrow \pi d$ and $\pi d \leftrightarrow NN$ channels. A more extensive study (including the consideration of ρ exchange in the $N\Delta \leftrightarrow NN$ and $N\Delta \leftrightarrow \Delta N$ Born terms) to better account for the NN channel will be discussed in a separate publication. Another important ingredient which may require some exposé is the off-shell modification of the πN P_{33} and P_{11} t matrix in our two-body input. This procedure was already adopted in our previous publication.[3]

When compared with the recent semirelativistic calculation of Afnan and McLeod,[17] our present work may be regarded to some extent as complementary in the following sense: (i) they adhered to the P_{33} model of Thomas,[18] while we employed an off-shell modification to the P_{33} t matrix of Graz[19] and the modified Koch-Pietarinen model;[20] (ii) we have chosen, among our several P_{11} t matrices, the one which we judged to be best and applied an off-shell modification to it, whereas they employed and compared various P_{11} models that they generated; (iii) separable NN t matrices in all the S and P waves are implemented in their calculation in three-body sectors (where the pion remains as a spectator). Their claim is that those interactions affect the $\pi d \leftrightarrow NN$ and $NN \rightarrow NN$ observables[21] and that the ${}^3S_1$-${}^3D_1$ alone is not enough. We only included ${}^3S_1$-${}^3D_1$ partial waves in the (NN) $+\pi$ part of the three-body sector but, as has been mentioned, included the HMX in the NN sector.

The organization of the present article goes as follows: Section II gives a brief account of the equations as well as the input and the numerical procedure used in our calculation. We describe the procedure for the off-shell modification in the πN P_{33} and P_{11} amplitudes in Sec. III and give a first series of results for the πd-πd, πd-NN, and NN-NN reactions. Section IV is devoted to the heavy meson exchange potential in the NN sector and its readjustment in our equations. Then our results with HMX effects are presented for the above three processes. The discussion and conclusion appear in Sec. V, where each reaction is examined in relation to the other two, and, wherever appropriate, to other works.

II. THEORETICAL ASPECTS

A. Equations for the coupled πNN-NN systems

Assuming separable interactions for the πN and NN two-body subsystems, the equations for the coupled πNN-NN systems derived by Avishai and Mizutani[1] (AM), Afnan and Blankleider[8] (AB), and Rinat and Starkand[7] (RS) are similar to the usual Faddeev-Lovelace equations. In operator form they read

$$\begin{bmatrix} X_{\rm dd} \\ X_{\Delta \rm d} \\ X_{\rm Nd} \end{bmatrix} = \begin{bmatrix} 0 \\ Z_{\Delta \rm d} \\ Z_{\rm Nd} \end{bmatrix} + [K] \begin{bmatrix} X_{\rm dd} \\ X_{\Delta \rm d} \\ X_{\rm Nd} \end{bmatrix} \tag{1}$$

for the πd$\to\pi$d, and πd$\to$NN reactions where the kernel is given by

$$[K] = \begin{bmatrix} 0 & Z_{\rm d\Delta}R_\Delta & Z_{\rm dN}R_N \\ Z_{\Delta \rm d}R_{\rm d} & Z_{\Delta\Delta}R_\Delta & Z_{\Delta \rm N}R_{\rm N} \\ Z_{\rm Nd}R_{\rm d} & Z_{\rm N\Delta}R_\Delta & Z_{\rm NN}R_{\rm N} \end{bmatrix} . \tag{2}$$

The NN$\to\pi$d and NN$\to$NN reactions are obtained through a similar set of equations with the same kernel:

$$\begin{bmatrix} X_{\rm dN} \\ X_{\Delta \rm N} \\ X_{\rm NN} \end{bmatrix} = \begin{bmatrix} Z_{\rm dN} \\ Z_{\Delta \rm N} \\ Z_{\rm NN} \end{bmatrix} + [K] \begin{bmatrix} X_{\rm dN} \\ X_{\Delta \rm N} \\ X_{\rm NN} \end{bmatrix} . \tag{3}$$

In the above expressions X are the three-body amplitudes, Z the Born terms (or driving terms), and R the two-body propagators in three-body Hilbert space. The index d refers to the π(NN) three-body channel where the NN pair interacts in the ${}^3S_1$-${}^3D_1$ (deuteron) and eventually in any other partial wave, Δ corresponds to channels N(πN) with (πN) interacting pairs in the S and P partial waves (including the P_{11} nonpole part), and N refers to the N(N) channel where (N) is the (πN) P_{11} pole part. These are by now standard notations.

The $X_{\rm dd}$, $X_{\rm Nd}$, and $X_{\rm NN}$ terms evaluated on-shell correspond to the physical t matrices for the πd$\to\pi$d, πd$\to$NN, and NN$\to$NN reactions, respectively.

In the Afnan-Blankleider[8,17] (AB) approach, relativistic kinematics is used for the pion only, while the nucleons are considered as nonrelativistic particles [relativistic pion kinematics (RPK) approach]. In our approach, as well as in Rinat-Starkand[7] (RS), a fully relativistic treatment has been used. Following the Aaron-Amado-Young procedure,[22] the Blanckenbecler-Sugar reduction is used to integrate over the relative energy, so that the relativistic four-dimensional equations are reduced to a set of coupled three-dimensional integral equations which are Lorentz invariant and satisfy two- and three-body unitarity. After angular momentum decomposition, one obtains a set of coupled one-dimensional integral equations to be solved numerically.

B. Two-body input

The various partial waves of the NN and πN interactions used in the three-particle sectors are parametrized as rank-one separable potentials:

$$V(p,p') = \Lambda g(p) g(p') . \tag{4}$$

For the NN interaction, we retain only the ${}^3S_1$-${}^3D_1$ (d) channel. The $L=0$ and $L=2$ form factors are chosen as

$$g_L(p) = p^L(1+ap^2) \Big/ \prod_{i=1}^{L+2} (1+b_i p^2) . \tag{5}$$

The a and b_i parameters are fitted to the ${}^3S_1$ phase shift, binding energy, D-state probability (6.7%), quadrupole moment, and charge form factor of the deuteron.[23] Within the RPK approach we have tested the rank-two ${}^3S_1$-${}^3D_1$ interactions which also reproduced the ${}^3D_1$ phase shift.[24] No significant difference has been seen and so we thus decided to stay within the rank-one model.

For the πN interaction, we take into account all the S and P partial waves. For all channels, except for the P_{33} and P_{11}, which will be discussed separately, we take the parametrizations of Schwarz *et al.*[19] with the following form factors:

$$g_L(p) = p^L \left[\frac{A}{p^2+a^2} + \frac{B}{p^2+b^2} \right] , \tag{6}$$

the parameters of which are fitted to the phase shifts and scattering lengths (or volumes).

In the Δ (P_{33}) πN channel, we use a form factor like in Eq. (6) with $B=0$ and we take the strength Λ to be energy dependent ($\Lambda^{-1} = s - \tilde{M}_\Delta^2$) in order to reproduce the position of the resonance ($\tilde{M}_\Delta = 1322.4$ MeV; bare delta mass). See, for example, Woloshyn *et al.*[25]

The P_{11} πN channel where π absorption takes place is parametrized according to the method of Mizutani *et al.* described in Ref. 26. Namely, the total t matrix is written as $t = t_{\rm P} + t_{\rm NP}$, where $t_{\rm P}$ is the direct nucleon pole part and $t_{\rm NP}$ the remaining background (the nonpole part). The nonpole part is written as a separable form:

$$t_{\rm NP}(p,p';s) = g(p) R_{\rm NP}(s) g(p') , \tag{7}$$

and then, after taking into account the dressing of the vertex g by the virtual pions required by two-body unitarity, the pole part reads

$$t_{\rm P}(p,p';s) = h(p;s) R_{\rm N}(s) h(p';s) , \tag{8}$$

where the dressed πNN vertex h is expressed in terms of the bare πNN vertex and of the nonpole part, and the dressed nucleon propagator $R_{\rm N}$ is evaluated in terms of h [see Eqs. (2.29′) and (2.7′) of Ref. 26].

The parameters were fitted to the P_{11} phase shift up to $T_{\rm lab}^{\pi} \sim 350$ MeV, the scattering volume a_{11}, and the πNN coupling constant f^2, with the constraint that the renormalization constant of the nucleon wave function lies between 0 and 1.

It may be worth pointing out that a large variation ($\gtrsim 10\%$) observed in the πNN vertex function between the value of momentum squared 0 and the pion pole[17] has not been found in our vertex: The variation was identified as $\lesssim 4\%$, consistent with partial conservation of axial vector current (PCAC). From this we conclude that it is *not* the Yamaguchi type of form factor, in contrast to that of the Gaussian or spherical Bessel (appearing in chiral bag models), which makes the variation undesirably large. Rather, it must be the specific form of the parametrization adopted in Ref. 17 that has caused this problem. A related discussion will be given in the next section.

C. Numerical procedure

In order to avoid the moving singularities on the real axis due to the driving terms and propagators, we use the method of contour rotation. The partial wave projection of the driving terms and the propagators are evaluated off the real axis through Gauss-Legendre quadratures with 16 and 40 mesh points, respectively. The choice of the mesh points for solving the system of coupled integral equations needs more careful treatment. We have observed that, rather than using a single Gauss-Legendre quadrature of order N applied directly to $[0,+\infty]$, a better convergence was obtained with fewer mesh points by splitting the total interval into three parts, $[0,k_\pi]$, $[k_\pi,k_N]$, and $[k_N,+\infty]$ (k_π and k_N are the pion and nucleon on-shell momenta) and using Gauss-Legendre quadratures with N_1, N_2, and N_3 points in respective intervals. In practice, a good compromise between convergence, memory size, and CPU time is achieved by taking $N_1=N_2=6$ and $N_3=12$.

The inclusion of the NN and πN partial waves described in the preceding subsection leads to at most 19 coupled interval equations for a given three-body state J^π (J, total angular momentum; π, parity). Using 24 mesh points, we thus have to in principle invert a 456×456 complex matrix. The calculations presented here have been done on a CYBER-750 and an IBM-3081 by means of the diagonal Padé approximants technique. We have found that a [5/5] Padé (11 iterations) was necessary for $J\leq4$, while the first iterate was sufficient for $J>4$ up to $J=9$ (this maximum J value is enough to make the partial wave decomposition in the considered energy range convergent).

Besides the intrinsic convergence of each πd-πd, πd-NN, and NN-NN partial wave with respect to the number of mesh points, rotation angle, and the order of Padé, we have a very strict criterion for numerical accuracy by comparing the nondiagonal πd elastic scattering amplitudes $T^J_{J-1,J+1}$ and $T^J_{J+1,J-1}$. These amplitudes are obtained from separate equations: The former from the equation involving $T^J_{J-1,J-1}$ and $T^J_{J-1,J+1}$, and the latter from the one where only $T^J_{J+1,J+1}$ and $T^J_{J+1,J-1}$ appear. Since the kernel is the same and the driving terms have been checked to have good symmetry properties, we must have $T^J_{J-1,J+1}=T^J_{J+1,J-1}$. Similar criteria can be exploited for the off-diagonal $^3P_2$-$^3F_2$, $^3F_4$-$^3H_4$, . . . NN partial wave amplitudes as well as for the πd$\rightarrow$NN and NN$\rightarrow\pi$d partial wave amplitudes. Another check in the NN sector is that the inelasticity parameter must be 1 at low energies. In our calculation those criteria are satisfied generally up to the fourth digit and in the worst case up to the third digit.

III. THE OFF-SHELL MODIFICATION

With the above two-body NN and πN t matrices as input, the πNN-NN equations reproduce the experimental πd differential cross section fairly well in a wide range of incident pion energies ($T^\pi_{\rm lab}\lesssim300$ MeV) [see Fig. 2(a)]. On the contrary, it is found that the equations do rather poorly in both the NN$\rightarrow$NN and NN$\leftrightarrow\pi$d sectors: The calculated NN inelasticities as well as the NN$\rightarrow\pi$d cross sections are lower than experiment where the single pion production is the main source for the nonelastic NN processes. At $T^\pi_{\rm lab}=142$ MeV ($T^{\rm N}_{\rm lab}=570$ MeV) the calculated $^1D_2$ phase shift is too small (Re$\delta\simeq2.3°$), while a factor of ~4 is missing in the NN-πd cross section and the A_{y0} polarization does not reproduce the experimental data, as shown in Fig. 1 (dot-dashed curves). The same conclusions hold at higher energies. We note that similar observations were made by Araki *et al.* (the third article in Ref. 11) for the NN channel and by Rinat and Starkand[7] for the NN$\rightarrow\pi$d channel.

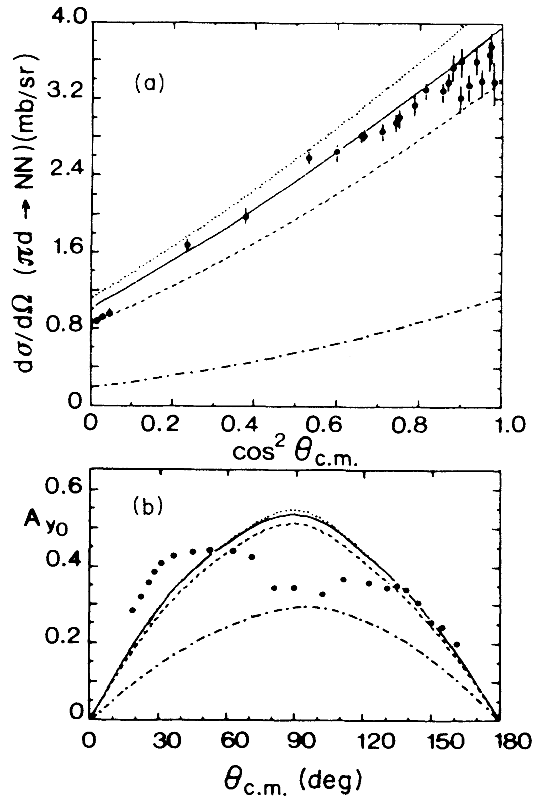

FIG. 1. (a) Differential cross section for the πd-NN reaction at $T^\pi_{\rm lab}=142$ MeV for different values of the off-shell parameters $(\gamma,\gamma_p)=(0.8,\ 1.2)$ GeV/c (full line), (0.7, 1.2) (dashed line), and (0.8, 1.5) (dotted line). The dot-dashed curves were obtained without off-shell modifications. Experimental data are from Ref. 38. (b) The same legend as for (a) for the $\vec{\rm p}$p$\rightarrow\pi^+$d asymmetry A_{y0}. The experimental data are from Ref. 40.

A. Off-shell modification of the P_{33} and P_{11} πN channels

Since the single pion production is known to proceed principally through the formation of the P_{33} (Δ) resonance, one is naturally led to reexamine the P_{33} and P_{11} πN amplitudes in our calculation, where the latter amplitude is responsible for the direct absorption and emission of the pion by a nucleon.

As explained in Sec. II B, a separable form has been adopted for our πN input. Specifically, the off-shell P_{33} t matrix takes the form

$$t(q',q;s)=g(q')R(s)g(q) , \tag{9}$$

with

$$R(s)^{-1}=s-\tilde{M}_{\Delta}^{2}-\Sigma(s) \tag{10}$$

and

$$\Sigma(s)=\frac{1}{(2\pi)^3}\int_0^\infty dk\frac{k^2(E_k+\omega_k)g^2(k)}{2E_k\omega_k[s^+-(E_k+\omega_k)^2]} . \tag{11}$$

In the above expressions, q and q' are the magnitudes of the off-shell momenta, $E_k=(m^2+k^2)^{1/2}$, $\omega_k=(m_\pi^2+k^2)^{1/2}$ (m, nucleon mass; m_π, pion mass), $\tilde{M}_\Delta$ is the bare delta mass, and $g(q)$ is the πNΔ vertex for the separable interaction which is determined by a fit to the P_{33} phase shift up to $T_{\rm lab}^{\pi}\sim 300$ MeV. This fit constrains $g(q)$ which, in terms of a monopole parametrization or its variants, obtains the cutoff mass $\sim 300\pm 50$ MeV/c.[9,19,23] When viewed as an off-shell form factor this gives a very strong cutoff, and it is not surprising that its use has resulted in the underestimation of the effect of pion production in NN$\to\pi$d and NN$\to$NN.

To improve upon this situation we first investigated more complicated forms of $g(q)$ as used in the work of Araki and Ueda[11] extended to implement the full relativistic kinematics. However, we found no substantial increase in the NN inelasticity, contrary to their claim. It may be worth mentioning also that with essentially the same P_{33} model as ours, Kloet and Silbar[27] found very large (more than experiment indicates) NN inelasticity. We employed the same P_{33} and P_{11} models as in their calculation but have not, up until now, reproduced their result. The origin of the discrepancy remains unknown.

As has correctly been pointed out by Oset *et al.*,[28] the πNΔ vertex in the separable P_{33} model is related to the Δ width:

$$\Gamma_\Delta(p)\propto {\rm Im}\Sigma(s)\propto g^2(p) , \tag{12}$$

where p is the c.m. momentum related to s; $\sqrt{s}=E_p+\omega_p$, but *has nothing to do* with the πNΔ form factor which appears, i.e., in the NN$\leftrightarrow$NΔ transition potentials[29] as a measure of the "off-shell-ness" of the pion. In fact, in an explicit K-matrix unitarization scheme for P_{33}, the width Γ [and thus $g(p)$] is constrained by the on-shell information (for π and N) alone.[28] This means that we have to supplement our P_{33} t matrix with an additional factor representing the off-shell pion. Note that this applies for other πN amplitudes as well. Now given this situation it is easy to understand why the πd elastic cross section is reproduced reasonably by the πNΔ vertex in the separable model: By far the dominant contribution to the elastic πd scattering at intermediate energies comes from the impulse term through the P_{33} πN scattering that is almost on shell due to the very weak deuteron binding (a very large angle scattering should of course be influenced by multiple scattering and thus the off-shell situation).

To implement an appropriate off-shell structure in the πN amplitudes and especially in the P_{33} channel, there exists a procedure based upon a dispersion approach for the half-shell extrapolation.[30] Here we shall employ a simpler method which has already been exploited before.[15,31] Let q and q' be the off-shell momenta and p be the on-shell momentum: $\sqrt{s}=E_p+\omega_p$. Then our off-shell extension reads

$$\tilde{t}(q',q;s)\equiv\frac{h(q')}{h(p)}t^{\rm on}(s)\frac{h(q)}{h(p)} , \tag{13}$$

where $t^{\rm on}$ is the on-shell t matrix which is supplemented by a new off-shell form factor to make up a new t matrix $\tilde{t}$. For simplicity we take a monopole form:

$$h(q)=\frac{1}{q^2+\gamma^2} . \tag{14}$$

In accordance with our three-dimensional reduction of the equation, q is taken as the magnitude of the three-vector (actually the magic vector in the three-body equations). It is not difficult to see that $\tilde{t}$ defined above conserves the unitarity of the equation. Regarding the value of γ, it may be usfeul to refer to the work of Reiner.[30] In this dispersion approach to the half-shell πN amplitudes, Reiner showed that the P_{33} channel can, to an excellent degree of accuracy, take a separable form and that in such an approximation γ should take a value around 650 MeV/c. We shall not stick strictly to this value but fix it later by a fit to some observables. It is interesting, however, that using this fit we found γ not too far from this value.

Now when a separable form is adopted for $t^{\rm on}(s)$ we find

$$\tilde{t}(q',q;s)=f(q',p)R(s)f(q,p) , \tag{15}$$

where the new form factor stands:

$$f(q,p)=h(q)\frac{g(p)}{h(p)} . \tag{16}$$

In many publications which deal with the Δ degrees of freedom this function f is taken as

$$f(q,p)\approx f(q,p_\Delta) \tag{17}$$

or

$$f(q,p)\approx h(q)\frac{g(p_\Delta)}{h(0)} , \tag{18}$$

or some four-vector extension therefore, where p_Δ is the πN c.m. momentum corresponding to the Δ mass, viz., $\sqrt{s}=1232$ MeV. In a strict sense this causes the unitarity violation. However, the violation of unitarity in practice turns out to be extremely small due solely to the fact that the Δ self-energy carries a substantial part of the pion production contribution as compared with the three-body states where the pion exchanged between two baryons becomes physical (on shell).

As stated before, the off-shell modification discussed in the context of the P_{33} wave should apply to all the rest of the πN partial waves. We shall employ this only with another partial wave, viz., the P_{11} channel since the influence from the remaining πN partial waves is weak. In order to contain the number of adjustable parameters, we have adopted the same form factor for both the pole (P) and nonpole (NP) parts. The modified vertices thus are

$$f_{\rm P}(q,p)=g_{\rm P}(p,s)\frac{h_{\rm P}(q)}{h_{\rm P}(p)}\ , \tag{19}$$

$$f_{\rm NP}(q,p)=g_{\rm NP}(p)\frac{h_{\rm P}(q)}{h_{\rm P}(p)}\ , \tag{20}$$

where $h_{\rm P}$ is assumed to have a monopole form with a cutoff $\gamma_{\rm P}$. For our purpose $\gamma_{\rm P}$ takes the value 1.0–1.2 GeV/c. This will be discussed somewhat more in detail later. One important point worthy of remark is that our off-shell modification avoids the problem encountered in Ref. 17 where the πNN vertex (viz., the vertex for the pole part) was found to extrapolate too rapidly from the pion on-shell point ($\simeq m_\pi^2$) to zero. The fact is that although this extrapolation is meant for the off-shell pion, the πNN vertex obtained in Ref. 17 involves only the on-shell pion leg (one of the nucleon legs may be off shell, though). Therefore the extrapolation seems a bit controversial.

B. Determination of the off-shell parameters

In order to fix the two cutoff masses γ and $\gamma_{\rm P}$ we have chosen to reproduce $d\sigma/d\Omega$ for NN$\rightarrow\pi$d for $T^{\pi}_{\rm lab}=142$ MeV, where experimental data are rich. We have not included the heavy meson exchanges in this calculation. Rather, our strategy is that the thus determined γ and $\gamma_{\rm P}$ will be used to constrain the heavy meson parameters (notably the σ and ω strengths) by selectively reproducing certain NN partial wave phase shifts.

As may be clear from Fig. 1(a), $d\sigma/d\Omega$ (πd$\rightarrow$NN) is affected sensitively by the value of γ but far less so by $\gamma_{\rm P}$. The optimal combination found was $\gamma=0.8$ GeV/c and $\gamma_{\rm P}=1.2$ GeV/c. These values are close to the ones used in our previous publication.[3] The asymmetry parameter A_{y0} is correctly reproduced at forward and backward angles when off-shell modification is employed, but a maximum is observed at around $\theta_{\rm c.m.}=90°$ instead of the dip in data. This situation is not improved by changing the values of γ and $\gamma_{\rm P}$. The same conclusion holds at higher energies.

C. Results without heavy meson exchange

To conclude this section, we show some typical results in which we have taken into account in an exact way the *full* two-body input described in Sec. II B [$^3S_1$-$^3D_1$ NN channel, P_{33}, P_{11} (pole plus nonpole), and all other S and P πN partial waves] with the above off-shell modifications ($\gamma=0.8$ GeV/c, $\gamma_{\rm P}=1.2$ GeV/c).

First, we give in Figs. 2(a)–(c) $d\sigma/d\Omega$, it_{11}, and t_{20} for the πd elastic scattering at $T^{\pi}_{\rm lab}=142$, 180, and 256 MeV ($T^{\rm N}_{\rm lab}=568$, 644, and 796 MeV). As a reference we show the results obtained without off-shell modifications. As expected, $d\sigma/d\Omega$ is only slightly affected by the off-shell effects, except at $T^{\pi}_{\rm lab}=180$ MeV, where a spectacular improvement is observed at backward angles. However, the situation at 256 MeV for $\theta_{\rm c.m.}\gtrsim 90°$ still remains to be improved. Concerning it_{11}, the off-shell effects make the maximum at $\theta_{\rm c.m.}\sim 80°$ wider, leading systematically to a better agreement with experiment. The tensor polarization t_{20} is sensitive to the off-shell modifications in the angular region $\theta_{\rm c.m.}\gtrsim 80°$, especially at 180 MeV. We note that, contrary to the experimental data from SIN,[35] the backward part of t_{20} at 142 MeV behaves smoothly and is too large compared with the LAMPF[36] and TRIUMF[37] data.

The quantities $d\sigma/d\Omega$ and A_{y0} for the πd$\rightarrow$NN reaction are shown in Figs. 3(a) and (b) at the three same energies as above. From examination of Fig. 3(a), it is clear that adjusting the off-shell parameters to reproduce $d\sigma/d\Omega$ at 142 MeV is not complete satisfactory because the theoretical curves are lower than experiment at higher energies. Moreover, the asymmetry A_{y0} systematically shows a maximum at $\theta_{\rm c.m.}\sim 90°$, in disagreement with the dip observed experimentally, and the overall magnitude becomes much too large as energy increases.

Finally, we show in Fig. 4 the $^1D_2$ and $^3F_3$ NN phase shifts [real part δ_R and inelasticity parameter $\eta=\exp(-2\,{\rm Im}\delta)$] calculated up to 1 GeV. As compared with the data, the real part is correctly reproduced up to ~ 300 MeV for $^1D_2$ and ~ 600 MeV for $^3F_3$. Above these energies, the lack of repulsion is manifest. The inelasticity parameters are correct up to ~ 600 MeV; beyond this energy, $\eta(^1D_2)$ shows a bold structure, in qualitative agreement with the Saclay data only, while the smooth variation of $\eta(^3F_3)$ with too large a value does not correspond to the experimental data. We point out that the lack of repulsion was also observed in other phase shifts like $^1S_0$ and $^3P_1$.

IV. INTRODUCTION OF HEAVY MESON EXCHANGES

From the above results concerning the $^1D_2$ and $^3F_3$ NN phase shifts, it is clear that heavy meson exchange effects must be incorporated in our model in order to introduce more repulsion at high energies.

A. Theoretical input

The HMX contributions are introduced at the level of the $Z_{\rm NN}$ Born terms as allowed by the AM theory; namely, we write

$$Z^J_{\rm NN}({\rm total})=Z^J_{\rm NN}(\pi\text{-three-body})+\sum_{\alpha\neq\pi} Z^J_{\rm NN}(\alpha)\ . \tag{21}$$

The first term is the three-body driving term for one-pion exchange, and $Z^J_{\rm NN}(\alpha)$ is the NN one boson exchange potential (OBEP) generated by the exchange of the meson α ($\alpha\neq\pi$).

For practical calculations we have used the OBEP formulation of Erkelenz[44] and Holinde.[16] The $Z^J_{\rm NN}(\alpha)$

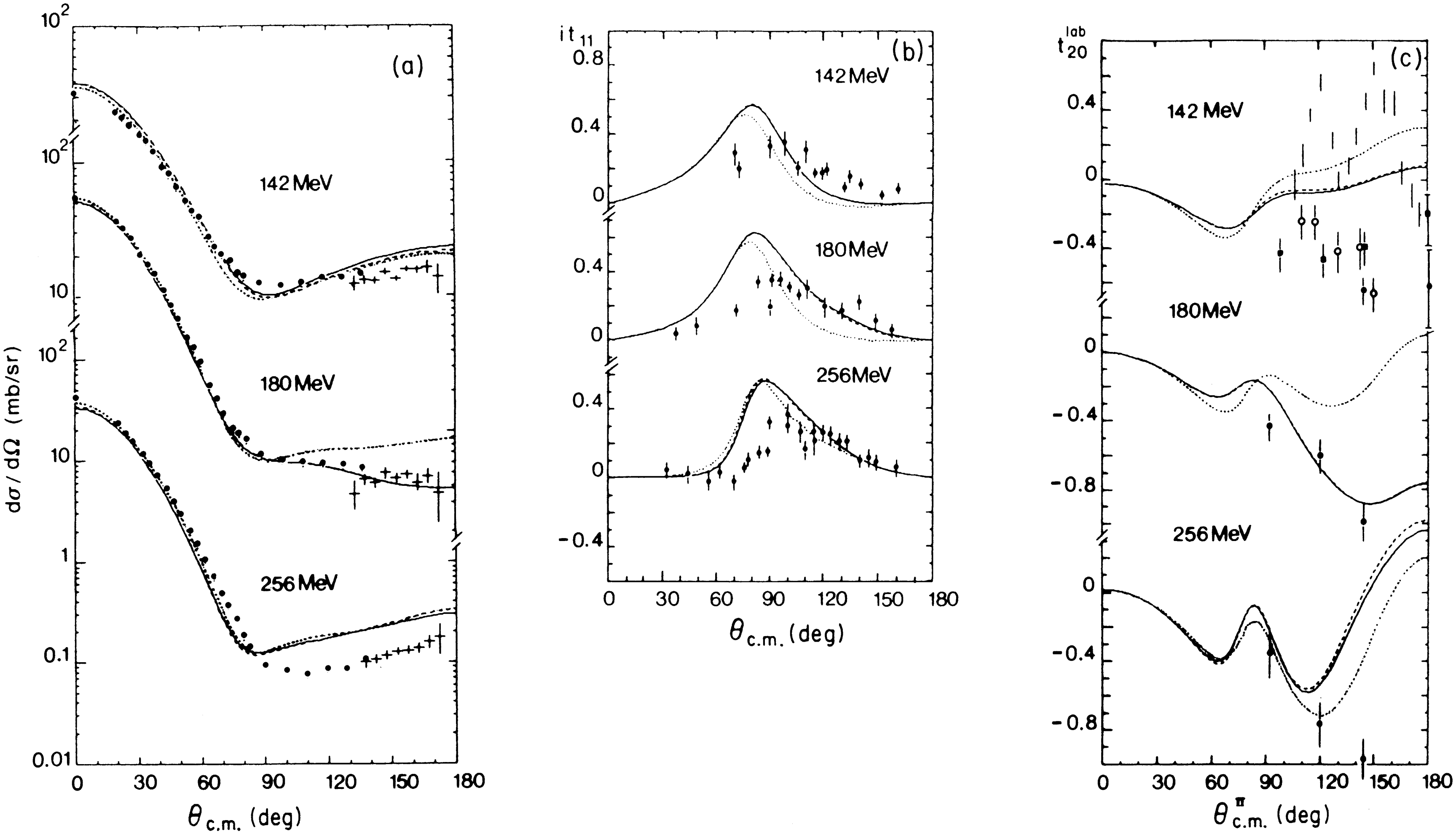

FIG. 2. (a) πd elastic differential cross section at $T^{\pi}_{\text{lab}}=142$, 180, and 256 MeV. Dotted line: no off-shell, no HM exchange; dashed line: off-shell (0.8, 1.2), no HMX; solid line: off shell + HMX (TAB4 parameters). The experimental data are from Refs. 32 and 33. (The dashed and solid curves are identical at 180 and 256 MeV.) (b) it_{11} vector polarization for πd elastic scattering. The legend is the same as for (a). The experimental data are from Ref. 34. (The dashed and solid curves are almost identical.) (c) t^{lab}_{20} tensor polarization for πd elastic scattering. The legend is the same as for (a). The experimental data are from Refs. 35 (1), (36) (■,●), and (37) (○). (The dashed and solid curves are identical at 180 MeV.)

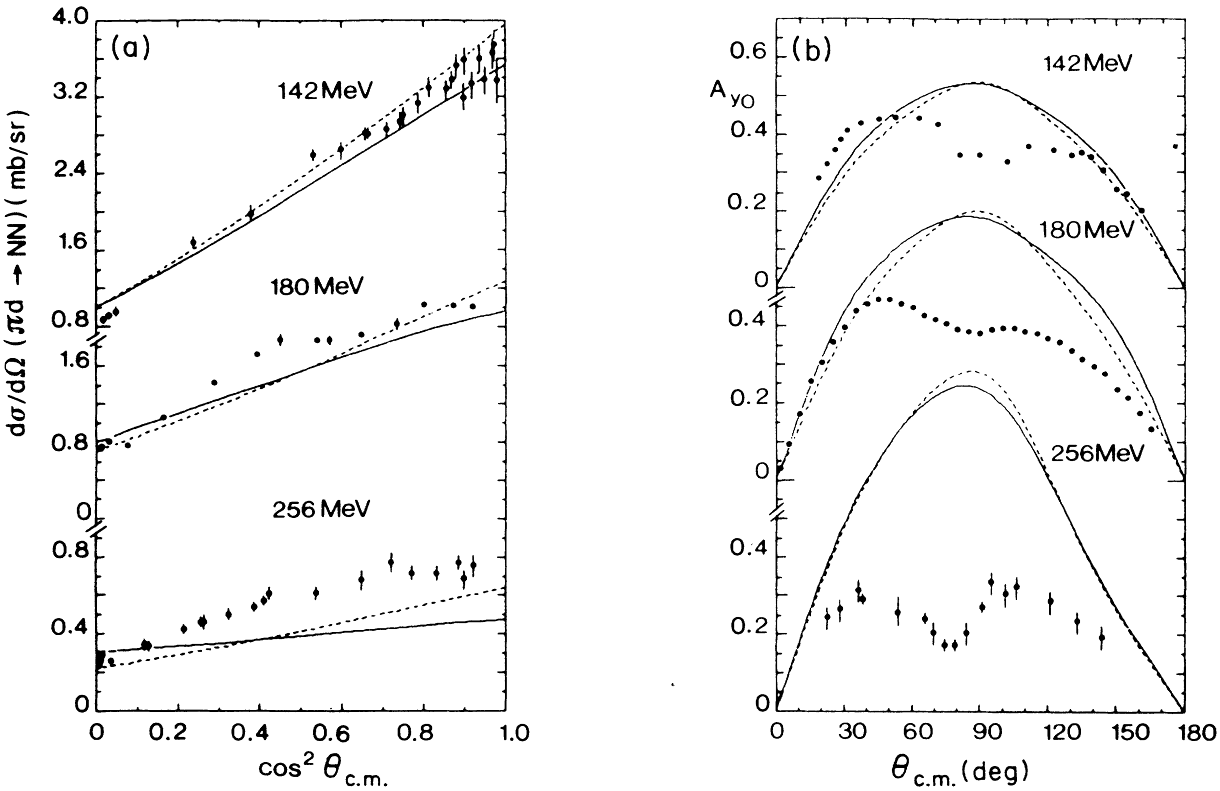

FIG. 3. (a) πd-NN differential cross section at $T^{\pi}_{\rm lab}=142$, 180, and 256 MeV. Off-shell modifications are included (0.8, 1.2). Dashed line: no HMX; full line: HMX (TAB4) included. The experimental data are from Refs. 38 and 39. (b) A_{y0} asymmetry parameters for the $\vec{p}p\rightarrow\pi^+d$ reaction. The legend is the same as for (a). The experimental data are from Refs. 40 and 41.

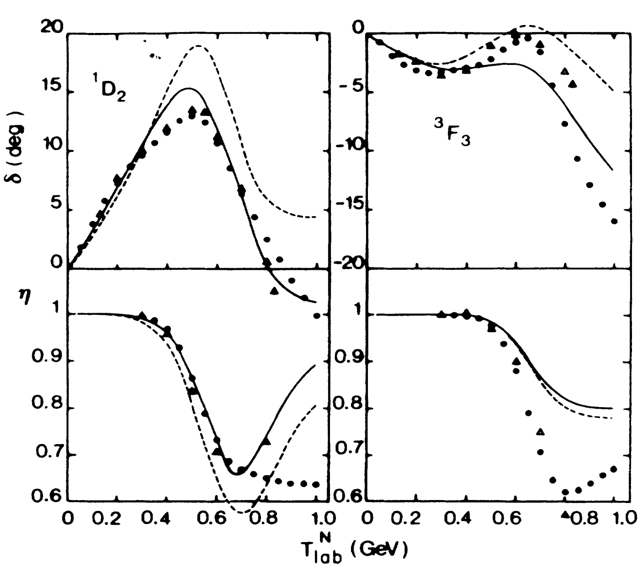

FIG. 4. $^1D_2$ and $^3F_3$ NN phase shifts and inelasticity parameters up to $T^{\rm N}_{\rm lab}=1$ GeV. Off-shell modifications are included (0.8, 1.2). Dashed line: no HMX; solid line: HMX (TAB4) included. The experimental data are from Refs. 42 (△) and 43 (●).

terms for $\alpha=\rho$, ω, σ, η, δ, and ϕ are evaluated in an approximation in which both the initial and final nucleons are put on mass shell. Their partial wave expressions in the $(l\Sigma)J$ three-body basis are obtained from the helicity state matrix elements given in Eqs. (2.11)–(2.17) of Ref. 44.

For convergence in the solution of the three-body equations, we introduce cutoff form factors at each nucleon-nucleon vertex as

$$[(\Lambda_\alpha^2-m_\alpha^2)/(\Lambda_\alpha^2-\Delta^2)]^{n_\alpha}\ ,$$

where Δ is the four-momentum transfer, m_α the mass of the exchanged meson, Λ_α the cutoff mass, and n_α a free parameter.

The πd-πd and πd-NN observables presented hereafter have been obtained with the HMX contributions included for all J values. In fact, these contributions must be taken into account at least up to $J=2$, their effect becoming negligible for higher J values, as might be guessed.

B. Determination of the heavy meson parameters

For each exchanged meson, we have the following parameters: one coupling constant g_α^2 for scalar or pseudoscalar mesons, two coupling constants for vector mesons (vector coupling g_α^2 and ratio of tensor to vector coupling f_α/g_α), the mass m_α, and the cutoff parameters Λ_α and n_α.

In conventional OBEP theories, these parameters are determined as follows: The masses of the well-established mesons are taken from the Particle Data Group tables, and other parameters (σ mass, cutoff masses, and coupling constants) are fitted to the NN phase shifts up to $T_{\rm lab}^{\rm N} \sim 0.4$ GeV, where the inelasticity is still negligible, with a constraint that they are consistent with the values known from experimental or theoretical investigations other than NN scattering.

Such values taken in the literature have been used as starting parameters in our Faddeev calculations. In principle, we then have to adjust some of these parameters to obtain a correct description of the NN phase shifts and also of the πd elastic and πd-NN observables. From the work of Kloet and Silbar,[45] we know that the ρ, ω, and σ coupling constants are key variables to obtain a fit to the NN phase shifts. After several tests, we adopted the following strategy: The $^1D_2$ NN phase shift is fitted up to 1 GeV by varying the ρ, ω, and σ coupling parameters within a *simplified* model where only the d-NN and $P_{33}+P_{11}$ (pole plus nonpole part) πN channels with off-shell modifications are retained. This is because the $^1D_2$ channel has a dominant contribution in the πd-NN cross section. Then, we make *full* runs (i,e., including the other S and P πN partial waves) at some characteristic energies to see if the remaining NN phase shifts as well as the π-d-πd and πd-NN cross sections are correctly reproduced. So, starting from the HM parameters given by Holinde (Table 4 of Ref. 16), we obtained the values shown in Table I (hereafter referred as TAB4). In what follows, we give some results to illustrate how this strategy was implemented.

First, we show in Fig. 5 the contributions to the $^1D_2$ and $^3F_3$ phase shifts from the various two-body channels. Considering the large effect of the d channel on δ_R ($^1D_2$) and η ($^1D_2$) at low energy, it is clear that at least the *simplified* model must be used to get meaningful results. This choice leads to reasonable computing time in the parameter search.

The contributions of the σ, ω, and ρ mesons to δ_R ($^1D_2$) are depicted in Fig. 6, which clearly shows that these three mesons must be included all together in order to combine their large individual attractive/repulsive effects. The total contribution from the other mesons η, δ, and ϕ was found to be small, so that Holinde's parameters were kept unchanged.

We give in Table II the variations of δ_R ($^1D_2$) and δ_R ($^3F_3$) relative to the σ, ω, and ρ coupling constants. In the considered range, these variations were found to be al-

TABLE I. Heavy meson parameters (TAB4). The values marked with an asterisk are from this work, and the others are from Table 4 (p. 183) of Ref. 16. m and Λ are given in MeV.

	$g^2(f/g)$	m	$\Lambda(n)$
π			
η	5.0	548.5	2000(1)
σ	6.4*	600	
δ	0.4	960	1300(1)
ω	20*	782.8	
ρ	0.5(6.6)*	776*	1650(3/2)
φ	5.4	1020	

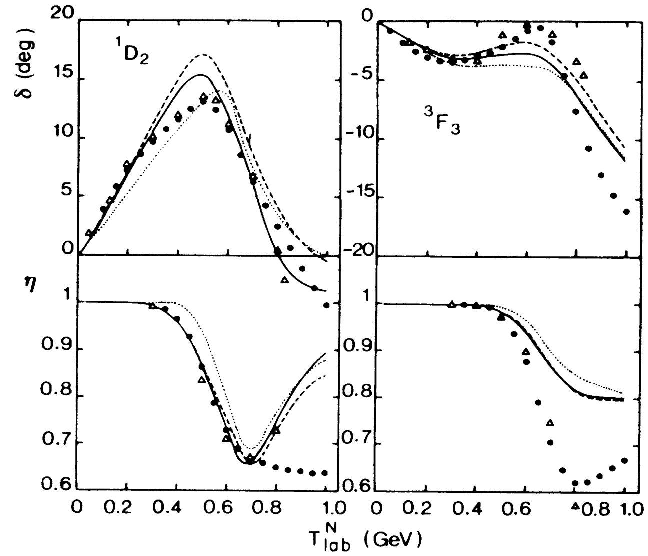

FIG. 5. Two-body channels contributions to the $^1D_2$ and $^3F_3$ NN phase shifts and inelasticity parameters. Off-shell modifications (0.8, 1.2) and HMX (TAB4) are included. Dotted line: only $P_{33}+P_{11}$ (P + NP) πN partial waves included; dashed line: the d channel is added (the *simplified* model); solid line: full calculation (the remaining S and P N partial waves are added). The experimental data are as in Fig. 4.

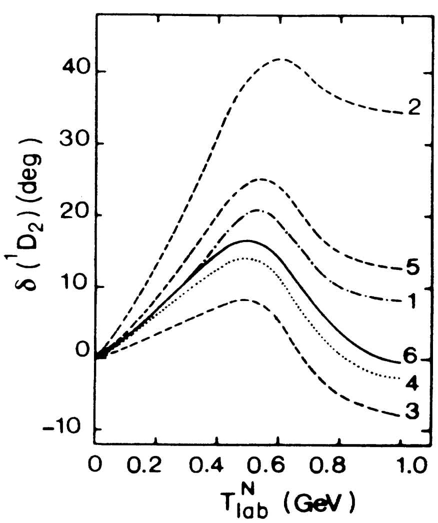

FIG. 6. Contributions of the σ, ω, and ρ mesons to the $^1D_2$ NN phase shift. The simplified model is used. The σ (curve 2), ω (curve 3), $\sigma+\omega$ (curve 4), ρ (curve 5), and $\sigma+\omega+\rho$ (curve 6) contributions are added to the pure three-body calculation (curve 1).

TABLE II. Variations of the real parts of the $^1D_2$ and $^3F_3$ NN phase shifts relative to the variations of the σ, ω, and ρ meson coupling constants [$5 \le g_\sigma^2 \le 8$, $18 \le g_\omega^2 \le 22$, $0.5 \le g_\rho^2 \le 1.0$, $4.0 \le (f/g)_\rho \le 6.6$].

$T_{\rm lab}^{\rm N}$ (MeV)	$\Delta\delta_R/\Delta g_\sigma^2$ $^1D_2$	$\Delta\delta_R/\Delta g_\sigma^2$ $^3F_3$	$\Delta\delta_R/\Delta g_\omega^2$ $^1D_2$	$\Delta\delta_R/\Delta g_\omega^2$ $^3F_3$	$\Delta\delta_R/\Delta g_\rho^2$ $^1D_2$	$\Delta\delta_R/\Delta g_\rho^2$ $^3F_3$	$\Delta\delta_R/\Delta(f/g)_\rho$ $^1D_2$	$\Delta\delta_R/\Delta(f/g)_\rho$ $^3F_3$
200	+0.46	+0.05	−0.16	−0.02	+2.1	0.0	+0.25	0.0
644	+1.1	+0.3	−0.55	−0.18	+3.8	+0.52	+0.70	+0.06
1000	+1.0	+0.46	−0.65	−0.35	+3.9	+1.2	+0.52	+0.15

most linear at a given energy, so we report the ratios $\Delta\delta_R/\Delta g^2$. From this table, we see how more repulsion can be put in the $^1D_2$ and $^3F_3$ channels either by increasing q_ω^2, or by decreasing the σ and/or ρ coupling constants.

We note that we kept the σ mass fixed in order to contain the number of free parameters, while the conventional value of g_σ^2 was reduced since the attractive contribution of σ is partly taken into account through the Δ box in the three-body equations.

Lastly, we must point out that we observed a large sensitivity of the $^1S_0$ and $^3P_1$ phase shifts to the σ, ω, and ρ coupling constants. We do not give any values concerning these variations since no constraint was put on these phase shifts in our search procedure. However, we will again consider this problem in the next subsection when the πd-NN reaction is discussed.

C. Results

In this subsection, we report a systematic of the πd-πd, NN-NN, and πd-NN, reactions. Using the *full* calculation (all two-body channels and off-shell modifications), we systematically compare the results obtained *without* and *with* heavy meson exchange contributions.

1. *πd elastic scattering*

The HMX contributions lead to rather small effects in the πd elastic scattering amplitudes and therefore in the πd observables, as shown in Fig. 2: The curves calculated without (dashed lines) and with (solid lines) HMX contributions are close, even in it_{11} and t_{20}, which are expected to be sensitive to any changes in minor partial wave amplitudes.

2. *NN phase shifts*

As a result of our procedure for adjusting the HM parameters, δ_R ($^1D_2$) is correctly reproduced (Fig. 4). The large repulsive contribution of HMX exchange at energies $\gtrsim 300$ MeV leads to the correct behavior at high energies, but the maximum around 500 MeV remains too high in comparison with the experimental data. The inelasticity parameter η ($^1D_2$) is also well reproduced up to 700 MeV. At this energy, the theoretical curve has a pronounced minimum, which agrees very well with the Saclay data[42] (available up to 830 MeV), but is inconsistent with the smoothly decreasing data of Arndt *et al.*[43]

In comparison with the calculation not including HMX, the result with HMX does not reproduce so well the structure observed experimentally in δ_R ($^3F_3$) around 600 MeV, but the high energy behavior is much better (Fig. 4). The η ($^3F_3$) inelasticity parameter is slightly affected by the HMX contribution and gives too small an inelasticity compared with the phase shift analysis of Arndt-Roper, especially at higher energies.

The phase shifts for low angular momentum ($l \le 1$) are poorly described: δ_R ($^1S_0$) is too repulsive, and δ_R ($^3P_1$) starts negative and decreases up to 300 MeV, in agreement with experiment, but then it becomes strongly attractive and takes positive values above 800 MeV. This situation could not be easily improved within our simplified procedure.

The peripheral phase shifts for total isospin 0 and 1 are well reproduced up to 1 GeV (Fig. 7), except for δ_R ($^3H_5$) and δ_R ($^1H_5$) which become, respectively, too repulsive and too attractive above 600 MeV.

3. *πd-NN reaction*

In Figs. 3(a) and (b), we note the non-negligible effect of HMX (solid curves) on $d\sigma/d\Omega$ and a smaller effect on A_{y0}. The HMX contribution leads to a better description of $d\sigma/d\Omega$ at 142 MeV and especially at 180 MeV, where the concavity observed experimentally is well reproduced. However, $d\sigma/d\Omega$ at 256 MeV becomes worse, and no structure is induced in A_{y0} with this HM parametrization (TAB4) in the whole energy range.

Considering the large variations of the low-J NN phase shifts relative to the σ, ω, and ρ coupling constants (see the end of the preceding subsection), it is relevant to ex-

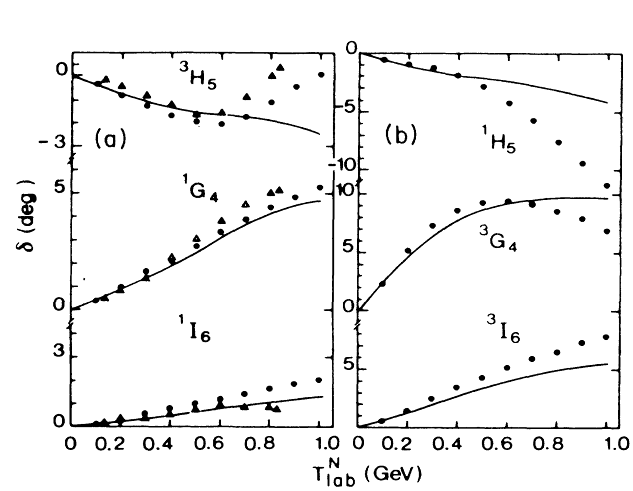

FIG. 7. Peripheral NN phase shifts for total isospin $I=1$ (a) and $I=0$ (b) up to 1 GeV. The experimental data are as in Fig. 4.

amine the sensitivity of the low πd-NN partial wave amplitudes to these parameters, concurrently with the variations induced in the A_{y0} asymmetry, the structure of which may arise from the difference among small amplitudes. By varying the $(f/g)_\rho$ and g_ρ^2 coupling constants, we have observed that the $J=0^+$ πd-NN amplitude is a very sensitive quantity, while the $J>0$ amplitudes were found to exhibit more regular variations, as shown in Table III. These variations become smaller for higher J values, as might be guessed. Similar effects are observed at 256 MeV. The resulting curves for $d\sigma/d\Omega$ and A_{y0} corresponding to the *full* calculation *with* all the HMX contributions are shown in Fig. 8. The solid curve is the same as in Fig. 3 (TAB4 parameters) and serves as a reference to the other curves obtained with TAB4 parameters for all mesons, except for the ρ-meson coupling constants. The variations of $d\sigma/d\Omega$ at 142 MeV are rather small, but we note a systematic improvement at 256 MeV, especially for the values $f/g=6.6$ and $g^2=1.0$, which give a very good description of the experimental shape, even if the magnitude remains too small (by a factor of ~ 1.2). The most interesting feature is the structure appearing in A_{y0} for the same combination ($f/g=6.6$, $g^2=1.0$). At 142 MeV, even if the structure observed around $\theta_{\text{c.m.}}=90°$ is not so pronounced as the experimental minimum, we note an overall improvement in comparison with the other curves which behave symmetrically around the maximum at $\theta_{\text{c.m.}}\sim 90°$. A similar tendency is observed at 256 MeV, but the calculated structure takes place at too large angles and the overall magnitude remains too large, especially at forward angles.

Finally, we give in Figs. 9 and 10 our results for other polarization observables which have been measured recently, namely the vector analyzing power iT_{11} in the $\pi\vec{d}\to pp$ reaction and the spin correlation coefficients A_{xx}, A_{yy}, A_{zz}, and A_{zx} in the $\vec{p}\,\vec{p}\to\pi d$ reaction. The theoretical curves correspond to *full* calculations, without HMX (dashed line), and with HMX contributions evaluated with the TAB4 parameters (solid line) or with the TAB4 plus ρ-modified parameters $f/g=6.6$, $g^2=1.0$ (dotted line). The magnitude of iT_{11} (Fig. 9) appears to be very sensitive to HMX as energy increases, the curves without HMX being in better agreement with the data at high energy. On the other hand, the ρ-meson parameters induce tremendous variations in this quantity: The data at 142 MeV are well reproduced with the TAB4 values ($f/g=6.6$, $g^2=0.5$) while the ρ-modified parameters give completely incorrect results, but situation is opposite at 256 MeV. Concerning the spin correlation coefficients (Fig. 10), neither the shape nor the magnitude of the experimental data for A_{xx} and A_{yy} are reproduced by our calculation, while A_{yy} and A_{zx} are well described, except that the theoretical curves are systematically lower than experiment, especially A_{zx}. These coefficients are moderately sensitive to the HMX contributions as to the ρ-meson parameters. However, the calculations including HMX have to be favored, in view of the magnitude of A_{yy} and A_{zx}.

V. DISCUSSION AND CONCLUSION

Based upon what we have presented in the preceding section, we shall discuss the physical content, merits, shortcomings, etc., of our present model calculation. We shall quote the results of other models of similar kind whenever some comparison may be useful.

A. πd elastic

The principle conclusion is that the inclusion of the heavy meson exchanges (HMX) in the NN sector has very little influence on the observables in this channel. We found the reason for this to be as follows: In the Δ resonance region this channel is dominated principally by the $J=2^+$ and to a lesser extent by the 3^-, partial waves, whereas the influence of the HMX is only noticeable in the 0^+ wave, which is not a dominant contribution, as the intermediate NΔ configuration coupled to this wave has the $l=2$ orbital angular momentum.

As for the off-shell modification of the input πN amplitudes in the P_{33} and P_{11} partial waves, one would naturally expect that it should show up at large angles (or at high momentum transfer). In our present model the effect manifests itself most eminently in the t_{20}, less so but still noticeably in the it_{11}, and least in the spin averaged $d\sigma/d\Omega$ except at $T_{\text{lab}}^{\pi}=180$ MeV. This insensitivity in the last physical quantity may be inferred from the unexpected success of the Glauber theory even at large angles;

TABLE III. πd-NN partial wave amplitudes obtained within *full* calculations for $J\le 2$ at $T_{\text{lab}}^{\pi}=142$ MeV with different ρ-meson coupling constants and the TAB4 parameters for the other mesons. The amplitudes $T^{J\pi}_{L(\text{NN}),L'(\pi d)}$ are dimensionless.

$(f/g)_\rho;g_\rho^2$ \ $J^\pi(L,L')$	0^+ (0,1)	1^- (1,2) / (1,0)	2^+ (2,3) / (2,1)	2^- (3,2) / (1,2)
(6.6;0.5)	$-36.6+i65.4$	$80.8+i51.1$	$5.14-i0.61$	$-11.6-i14.8$
(TAB4)		$99.9-i75.8$	$171.0+i630.4$	$-105.0-i56.3$
(6.6;1.0)	$120.7+i29.3$	$82.1+i52.0$	$5.93-i0.12$	$-16.9-i11.5$
		$101.7-i76.3$	$142.1-i637.9$	$-107.0-i84.1$
(4.0;1.0)	$-46.2+i84.8$	$80.9+i49.9$	$4.84-i0.76$	$-10.1-i15.5$
		$98.2-i76.9$	$181.6+i626.8$	$-106.5-i54.9$
(4.0;0.5)	$40.3+i111.2$	$80.5+i50.3$	$4.63-i0.85$	$-9.68-i15.1$
		$98.9-i75.6$	$189.4+i625.0$	$-93.2-i56.1$

see, for example, Kanai *et al.*[49] This apparent success indicates that eventually the πd scattering in the Δ region is strongly dominated by the impulse and double scattering processes through the πN P_{33} states which stay almost on-shell. On the other hand, at 180 MeV the off-shell modification substantially lowers the large angle cross section and attains remarkable agreement with the data [see Fig. 2(a)]. We do not understand exactly why it is so at this specific energy, but our guess is that this may be due to the fact that in the scattering at this energy the πN subsystem primarily sits at the Δ resonance energy. Then the dominant P_{33} amplitude becomes almost purely imaginary, which might eventually uncover the effect of the off-shell modification in the πN P_{11} amplitude.

The off-shell modification changes only mildly and improves the it_{11} in the backward hemisphere, but no change is observed in the forward hemisphere, where we need an improvement around 70°. This is somewhat unexpected, as we had anticipated it_{11} to be very sensitive to the input variation judging from its defining equation consisting of an interference between dominant and small partial waves. On the contrary, a large variation in t_{20} due to the off-shell modification is a little bit of a surprise to us. But anyhow, this latter observable appears controversial as (i) two almost mutually exclusive sets of data have been a matter of debate during the past few years[35,36] and (ii) model calculations with different treatments of the input P_{11} πN wave give considerably different results.[15,17] Concerning the experimental side, the new result from TRIUMF[37] favors the LAMPF data,[36] which are far smoother than the SIN data[35] and presumably do not require us to introduce any exotic ingredient like di-

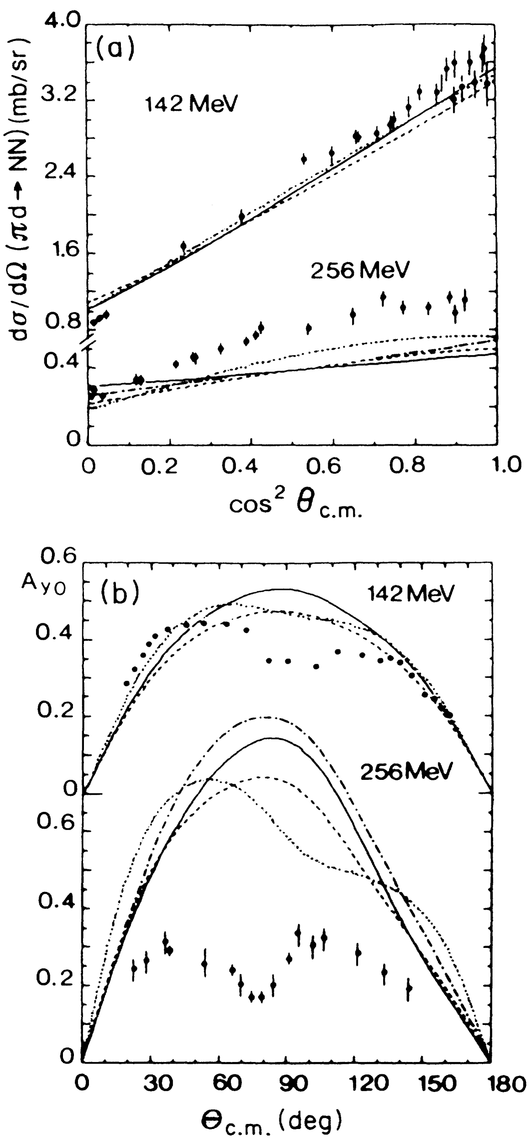

FIG. 8. (a) Effect of the ρ-meson coupling constants $(f/g;g^2)$ on the πd-NN differential cross section at $T^{\pi}_{\rm lab}=142$ and 256 MeV. Solid line: TAB4 parameters (6.6; 0.5); dashed line: (4.0; 0.5); dotted line: (6.6;1.0); dot-dashed line: (4.0; 1.0). At 142 MeV, the solid line and the dot-dashed line are almost identical. The experimental data are as in Fig. 3(a). (b) A_{y0} polarization for the $\vec{p}p\to\pi^+d$ reaction. The legend is the same as for (a). The experimental data are as in Fig. 3(b).

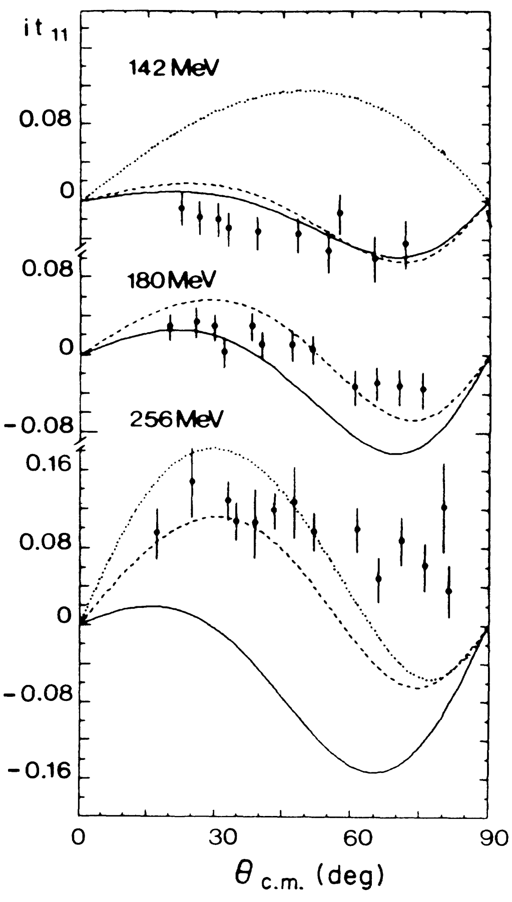

FIG. 9. it_{11} vector polarization in the $\pi\vec{d}\to$pp reaction at $T^{\pi}_{\rm lab}=142$, 180, and 256 MeV. Off-shell parameters: (0.8, 1.2). Dashed-line: no HMX; solid line: HMX (TAB4) included; dotted line: HMX (TAB4) with modified coupling constants for the ρ meson $(f/g=6.6;\ g^2=1.0)$. The experimental data are from Ref. 46.

baryons. Now concerning the theoretical side, the conventional nucleon pole type of πN P_{11} model, e.g., Refs. 10 and 27, or its variant[15] used in calculations seems to reproduce the result consistent with the LAMPF experiment, whereas more refined P_{11} models with the pole plus nonpole decomposition including our own[3–5,7,8,17] like the present one (when no off-shell modification is employed) give results not as good as those with the simpler P_{11} treatment. Based upon this, there appeared a repeated criticism[15] that the pole plus nonpole decomposition of the P_{11} amplitude and the subsequent separate use of those two parts in the πNN-NN coupled equations must be wrong. We shall here state our point of view: Unless one adopts the point of view of not dressing the nucleon by the pion, the Pauli principle forces one to separately consider the pole and nonpole parts of the P_{11} amplitude, and to independently take care of the NN channel (viz., not as the residue of the intermediate πNN state when the subenergy of the interacting πN pair is set equal to the nucleon mass). After all, this is precisely why the coupled πNN-NN equations are called for. Then the question still remains as to why these refined models were not in good agreement with the LAMPF data for t_{20} up until now. The answer is that the πN phase shift information alone is far from sufficient in tightly constraining the pole plus nonpole decomposition of the P_{11} amplitude; see Refs. 26, 50, and 51. Strictly speaking, one needs all the πN inelastic information at all energies. Therefore, from a practical viewpoint it is fair to say that rather the πd t_{20} data may be used for the time being as a constraint to be imposed on the P_{11} model amplitude. Note in this respect that our present off-shell modification indicates a direction for improvement, at least at 180 MeV. We want to point that good agreement with the existing t_{20} data at 256 MeV (Ref. 36) can be obtained with a certain version of the off-shell modification which, however, is incompatible with the observables in other channels. Therefore, this question should be settled eventually not only by the πd elastic process alone but by simultaneously studying other coupled channels like the NN$\leftrightarrow\pi$d and NN$\rightarrow$NN, as our effort has been directed.

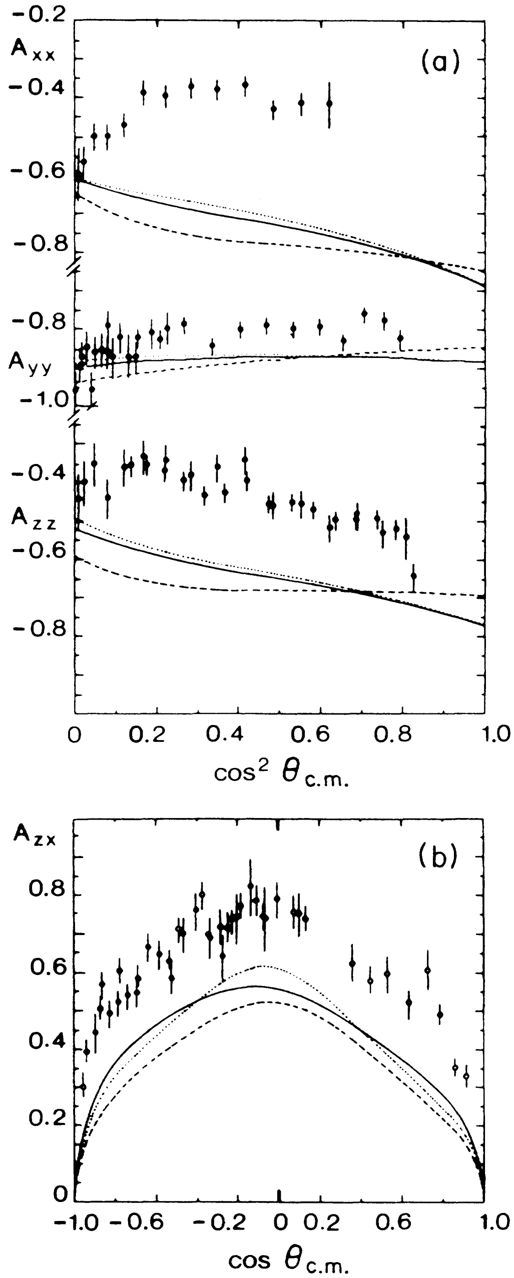

FIG. 10. (a) Spin correlation coefficients A_{xx}, A_{yy}, and A_{zz} of the $\vec{p}\,\vec{p}\rightarrow\pi d$ reaction calculated at $T_{\text{lab}}^{\pi}=142$ MeV ($T_{\text{lab}}^{\text{N}}=568$ MeV). The legend is the same as for Fig. 9. The experimental data at $T_{\text{lab}}^{\text{N}}=578$ MeV are from Ref. 47. (b) A_{zx} spin correlation coefficient of the $\vec{p}\,\vec{p}\rightarrow\pi d$ reaction. The legend is the same as for (a). The experimental data are from Ref. 47 (●) and Ref. 48 (○).

A recent report by Garcilazo[52] appeared to have finally realized the Pauli principle problem mentioned above and made a pole plus nonpole decomposition of the P_{11} amplitude. However, it should be pointed out that the claim therein of the nonpole part being negligibly small is based upon a decomposition which is not compatible with the unitarity of the three-body amplitude.

To close our discussion on the πd elastic sector, it is important to stress that all the existing models including our present one have problems reproducing (i) the large angle $d\sigma/d\Omega$ for $T_{\text{lab}}^{\pi}\approx 220$–$300$ MeV,[33] and (ii) the dip structure around $\theta_{\text{c.m.}}\sim 70°$ in it_{11} (Ref. 34) for a similar energy range. For the former a recent independent experiment[53] confirmed the first SIN-CERN data[32,33] so we should take the data as reliable. As for the latter, the problem is the development of the dip as a function of the scattering energy: The models give a slower energy dependence than the data show. In the context of our present study, it seems clear that the inclusion of HMX may not be able to cure these difficulties. There may still be some room for improvement within the variation in the P_{11} model or in the off-shell modification of the πN input that we have not explored yet. There is one point worthy of mention: Within the context of the three-body approach to πd scattering, Giraud *et al.*[23,24] pointed out the importance of including the minor πN partial waves in addition to the dominant P_{33} (and P_{11}) in order to explain it_{11}. It appears, however, that within an essentially nonrelativistic coupled channel model[10] one obtains it_{11} which is comparable to or in some cases quantitatively slightly better than the three-body results without including those small πN waves. It is thus useful to examine in detail the quantitative similarity and difference between this and πNN-NN (or three-body) models. In this connection it may be useful to notice that while the inclusion of the small πN waves appears indispensable for a qualitative agreement with data at ∼142 MeV, a better agreement with the data may be attained without them at higher energies where the discrepancies between models and data are more explicit. This might be inferred from Fig. 6 for it_{11} in Ref. 24. This point needs more attention.

B. πd↔NN

From its very construction the off-shell modification of our P_{33} and P_{11} πN amplitudes is fixed to set the correct scale for the spin averaged NN↔πd cross section $d\sigma/d\Omega$ at $T^{\pi}_{\rm lab}=142$ MeV (or $T^{\rm N}_{\rm lab}=568$ MeV). As described in Sec. III, this scale setting appears alright except at $T^{\pi}_{\rm lab}=256$ MeV ($T^{\rm N}_{\rm lab}=796$ MeV) where it somewhat underestimates the experimental cross section [Fig. 3(a)]. As will be discussed later, this tendency appears also in the NN inelasticity for the $^1D_2$ and $^3F_3$ partial waves (Figs. 4 and 5). Since NN→πd is of course a part of the inelasticity in the NN reactions, this trend is a simple manifestation that our model lacks the overall strength in the NN inelasticity above the Δ resonance region. For a comparison it is useful to refer to the result of Afnan-McLeod.[17] Within a model of relativistic kinematics only for the pion they corrected for the P_{11} amplitude of Blankleider and Afnan[8] which employed a nonunitarity decomposition into the pole and nonpole parts. The use of their preferred P_{11} model has resulted in underestimating $d\sigma/d\Omega$ at 142 MeV while at 256 MeV the model did reasonably well, which is just the opposite of what we have found in our present study. This and the results with various P_{11} models found in Ref. 17 indicate, as already mentioned in the preceding discussion on the πd elastic result, that in fact we still have a considerable degree of freedom in the pole plus nonpole decomposition of the P_{11} amplitude.

Other than changing the scale for $d\sigma/d\Omega$ and A_{y0}, the off-shell modification does not appear to modify the angular dependence of various πd↔NN spin observables. The effect of the heavy meson exchange in the NN sector affects the πd↔NN observable most sensitively through the ρ meson, as described in the preceding section. From Table III, it is clear that the ρ exchange influences mostly the $J^{\pi}=0^+$ partial wave amplitude, while the most dominant 2^+ amplitude is only moderately modified. With a suitable choice of the ρ parameter ($g^2=1.0$, $f/g=6.6$), which apparently gives a slightly stronger rho effect than in conventional OBEP models, A_{y0} tends to be improved, at least becoming closer to the dip structure observed in the data both at $T^{\pi}_{\rm lab}=142$ and 256 MeV, and $d\sigma/d\Omega$ at 256 MeV comes out with the right curvature. In order to study this point more closely, we have freely varied the values of our πd-NN partial wave amplitude at $T^{\pi}_{\rm lab}=142$ MeV and found that in fact the 0^+ partial wave is dominantly responsible for the shape of A_{y0}. In fact, by suitably choosing the value of this amplitude it was possible to make A_{y0} very close to the data. However, the situation at 256 MeV turned out not as simple as that at 142 MeV. That our overall scale for A_{y0} has come out too high may be directly related to the fact that our unpolarized $d\sigma/d\Omega$, which appears in the denominator of A_{y0}, is lower than the data. This, of course, cannot be cured by simply varying the 0^+ amplitude without further strengthening the dominant 2^+ and 3^+ amplitudes, specifically the latter at this energy. As for the numerator of A_{y0} which controls the dip and the asymmetric structure, it consists of the products of singlet-triplet as well as triplet-triplet amplitudes (in terms of the total spin of the NN channel). Thus it is natural that varying only the 0^+ partial wave (part of the singlet amplitude) could only improve the situation partially. We remind the reader of a work by Locher and Svarc[54] in which they observed that the dip in A_{y0} is largely controlled by the phase of the 0^+ amplitude. As pointed out by Saha *et al.*[41] in their analysis of experimental A_{y0}, it appears that existing theories lack the strength in the NN triplet related channels, and our present result is no exception. By comparing data on several NN→πd spin observables with their own relativistic model calculation, Grein *et al.*[55] also reached a similar conclusion. Also, a recent experiment on the A_{LL} and A_{SL} for $\vec{p}\,\vec{p}\to\pi d$ (Ref. 56) indicates the missing strength in the triplet channel above the Δ resonance region. This is in line with our small inelasticity in the NN $^3F_3$ partial wave above $T^{\rm N}_{\rm lab}\sim 650$ MeV, as will be discussed below. The origin of this missing triplet strength at higher energies appears to be a very serious problem in existing theoretical models, and requires a considerable theoretical endeavor.

Coming back to the effect of the ρ exchange in the NN channel on the πd↔NN spin observables, it is rather mild in almost all of them except it_{11}, where the strong ρ exchange worsens the agreement with the data[46] at $T^{\pi}_{\rm lab}=142$ MeV. Again, since the NN ρ exchange in our present model primarily modifies the 0^+ partial wave, this result indicates the relative importance (unimportance) of this wave in it_{11} (other spin observables). In fact, Locher and Svarc[54] found that the sign of it_{11} is controlled by the phase of the 0^+ amplitude, just like the dip structure of A_{y0}. Since our present objective is not to provide a good description of the central partial waves in the NN channel as stated in the Introduction, the outcome is not very surprising. However, the dominant influence from the ρ meson exchange as compared with others, e.g., σ and ω exchanges, is something we had least expected. A systematic study on this point is certainly needed.

Apparently it_{11} in our calculation is better described when no heavy meson exchange is included in the NN channel, as mentioned earlier. Yet as the energy increases, the agreement with the data gradually deteriorates, typically at $T^{\pi}_{\rm lab}\gtrsim 200$ MeV. This tendency is shared as well by $d\sigma/d\Omega$ and A_{y0} within the πd↔NN process, and our suspicion is that it may be related again to the missing triplet strength.

Before ending this subsection we want to make one remark. Contrary to the claim of Betz *et al.*,[57] we have not found that the A_{y0} is sensitive to the D-state probability of the deuteron. It should be remarked that while we have adopted a single deuteron model with varying D-state probability constrained by ordinary static properties and the change form factor of the deuteron,[23] they adopted different deuteron models for different D-state probabilities with no constraint on the charge form factor. Therefore, it is our opinion that their conclusion should be taken with some reservation.

C. NN→NN

As stated in the Introduction, we have only aimed at attaining a quantitative agreement with the data (phase shift analyses) for $l \gtrsim 2$ NN partial waves, although the dominant role of the heavy meson exchange exists in the central waves. Therefore, the $l \lesssim 1$ waves have come out only in qualitative agreement with the data or in some cases worse, which seems to be reflected in the NN-πd spin observables as discussed in the preceding subsection. The NΔ dominated partial waves, specifically the $^1D_2$ channel ($l_{N\Delta}=0$) has turned out to be reasonably well described up to $T^{N}_{lab} \sim 650$ MeV. The problem is that above this energy η increases rather rapidly, which is not observed in the analysis of Arndt *et al.*,[43] although it is consistent with the Saclay data.[42] Not only in our present result but also in all the existing theoretical models known to us[10,58,59] and in a more phenomenological coupled channel model of Lomon[60] a similar decrease or at least a saturation of the inelasticity in this partial wave has been observed while the data show larger inelasticity.[43] Also in this wave most theories underestimate the inelasticity from the inelastic (single π production) threshold up to ~ 550 MeV. As correctly pointed out by van Faassen and Tjon[61] and independently by us,[62] this should be due to the fact that most of the models do not include the coupling to the intermediate πd channel which strongly influences this wave. Our success in this energy range is simply because we have this coupling. On the other hand, authors of Ref. 10 apparently have succeeded in reproducing η ($^1D_2$) of Arndt *et al.*[43] in this energy range without this extra coupling. This means that once this πd intermediate state is implemented in their equation, the $\Delta(P_{33})$ parameters, etc., are to be readjusted to refit δ_R ($^1D_2$), η ($^1D_2$), etc. We shall discuss this problem in a separate publication.

As for the $^3F_3$ partial wave, we have not succeeded in obtaining enough inelasticity above 650 MeV. This problem is more serious than in the $^1D_2$ wave. We note that in an updated analysis of Arndt and Roper[63] this $^3F_3$ inelasticity has increased even more as compared with the previously published version.[43] So this lack of inelasticity seems to have become even more problematic. Again, to the best of our knowledge, no theoretical model to date has successfully reproduced this quantity compatible with the phase shift analysis. The lack of the NN spin triplet channel strength in the πd-NN channel observables as discussed in the preceding subsection seems to be due largely to this problem in $^3F_3$.

Another serious problem commonly shared by the existing models is the real $^3P_1$ phase shift δ_R ($^3P_1$). The result of the phase shift analysis shows that the phase shift at medium energies remains negative and becomes very large in magnitude as the energy increases.[43] Theoretical models, on the other hand, deviate from the data at ~ 600 MeV and stay less negative at higher energies, the cause of which is the strong NΔ attraction in the P wave intermediate state. This problem in models apparently causes the minimum in $\Delta\sigma_T$ and $\Delta\sigma_L$ centered at around ~ 800 MeV far less deep than seen in the data,[64] as observed by varying various partial wave amplitudes to reproduce those total spin cross-section differences.[65]

As a possible cure to the problems mentioned above it may not be difficult to think about introducing the ρ exchange in the NN↔NΔ transition interaction: In this way one may hope to selectively reduce (increase) the NΔ attraction for $^3P_1$ ($^1D_2$ and $^3F_3$) by increasing the πNΔ strength (in terms of the vertex form factor, etc.). We have been testing this procedure, but it has been up to now not very promising. In fact, many NN calculations[10,58,61] at medium energy already included the ρ exchange in the NΔ transition potential, yet still suffer from the problem. Other possible sources worth investigating in order to cure the problem are the form of the pion exchange propagator and (again) the vertex (form factor) for the meson-baryon coupling and the way the relativistic kinematics is implemented, i.e., on-mass-shell spectator approximation, partial relativistic kinematics, the forced mixture of the off-mass-shell/off-energy-shell concepts in almost all the existing models as correctly pointed out by Anastasio and Chemtob,[66] etc. Those are the subject of our current investigation and the result will be reported elsewhere.

Before closing, we want to note that our most recent study,[67] which includes the $^3P_0$, $^3P_1$, $^3P_2$, and $^1P_1$ NN channels in the (NN)$+\pi$ sector shows a considerable improvement in the $d\sigma/d\Omega$ (πd↔NN) and it_{11} ($\pi\vec{d}\to$pp) and some improvement in A_{y0}, which in part supports the observation of Afnan *et al.*[17]

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