Intruder-g.s. mixing in Cd nuclei

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Cross-section ratios in ^{112,114}Cd(t,p) and ^{114}Cd(p,t) are used to estimate mixing matrix element between rotational intruder 0^+ states and normal ground states.

The presence of low-lying intruder states in the Cd isotopes has been clearly demonstrated,[1-5] and the nature of the intruder states is reasonably well understood.[1,4,5] The normal states of even Cd nuclei represent perhaps the best vibrational spectra anywhere in the periodic table, with the two-phonon 0^+ level just above 1 MeV. The intruders are almost certainly of rotational character,[4,5] with the 0^+ band lead decreasing in excitation energy as one approaches the middle of the $N=50$–82 shell from either direction—apparently even becoming lower than the two-phonon 0^+ in ^{112,114}Cd.

However, the extent of mixing of these intruder states with the normal ones is still a matter of great uncertainty. Calculations[5] have been attempted for a mixing matrix element between normal ground state (ϕ_g) and rotational 0^+ state (ϕ_r) as small as 100 keV and as large as 500 keV. Other calculations[1] considered only mixing between the two-phonon 0^+ (ϕ_2) and ϕ_r. Values of $B(E2)$'s between 0^+ and 2^+ states just above 1 MeV exhibit dramatic A-dependent behavior.[1] This behavior has been interpreted as arising from mixing between rotational and two-phonon states.

Values of $E0$ matrix elements, however, are largest between the physical ground state and the predominantly rotational 0^+ state. As large $E0$'s are commonly associated with mixing between states of very different structure (e.g., radius), this pattern would imply mixing between ϕ_g and ϕ_r.

In a two-state model containing only ϕ_g and ϕ_r, the mixing potential matrix element between them, V_{gr}, must rigorously be less than about 0.567 MeV (one-half the minimum separation of ground and predominantly rotational 0^+ states). This matrix element is normally[1,5] taken to be the same in all even Cd nuclei. A similar argument involving only ϕ_2 and ϕ_r implies an upper limit on V_{2r} of 48 keV (one-half the 0_2-0_3 splitting in ^{116}Cd).

An attempt[6] to observe effects of $\phi_g-\phi_r$ mixing by investigation of the A dependence of g.s. Cd(p,t) cross sections did not succeed—primarily because the ground states so dominate the (p,t) spectra that no reasonable amount of mixing would produce observable effects in the absolute ground-state cross sections.

However, the effects of such mixing should be readily apparent in 2n-transfer *ratios* for predominantly rotational and ground states. The destructive interference expected between the two dominant amplitudes for the excited 0^+ state will enhance the observability. In ^{112}Cd(t,p)^{114}Cd, (Ref. 7), it is 0_2^+ that is stronger, while in ^{114}Cd(t,p)^{116}Cd (Ref. 8) it is 0_3^+ (see Table I). These are just the states that have been assigned[5,1] predominantly rotational character from totally independent evidence.

In this paper we ignore mixing between ϕ_2 and ϕ_r and attempt to use the (t,p) ratios to estimate the mixing between ϕ_g and ϕ_r. Hence,

$$\psi_A(g.s.)=\alpha_A\phi_g+\beta_A\phi_r\ ,$$
$$\psi_A(0^{+'})=-\beta_A\phi_g+\alpha_A\phi_r\ ,$$

with $0^{+'}$ being 0_2^+ in ^{114}Cd, but 0_3^+ in ^{116}Cd. If β_A/α_A can be determined from the 2n-transfer data, then the mixing matrix element V_{gr} can be determined approximately from the expression $-V=\alpha_A\beta_A E_A$, where E_A is the observed g.s.-$0^{+'}$ splitting in nucleus A. Of course, mixing between ϕ_2 and ϕ_r will have moved E_A somewhat from the position it would have had in the absence of ϕ_2, but this effect on E_A is only about 48 keV or less (see above)—translating into at most a few percent effect on V_{gr}.

To determine wave functions from (t,p) ratios, we need the 2n-transfer amplitudes connecting the basis states. The interacting boson approximation (IBA) amplitude in the vibrational limit[9] is identical to Yoshida's expression[10] within a shell of degenerate orbitals, all of which fill at the same rate. Hence, for $\phi_{gA}\rightarrow\phi_{gA+2}$, we use the amplitude $f_{gg}=\alpha_\nu(N_\nu)[(N_\nu+1)(\Omega_\nu-N_\nu)]^{1/2}$, where $2N_\nu=N-50$ and $\Omega_\nu=16$. Yoshida's amplitudes allow us to calculate the N_ν dependence of $\alpha_\nu^2(N_\nu)$ if we needed it—but we do not need it here.

For $\phi_{rA}\rightarrow\phi_{r,A+2}$ we use the IBA amplitude in the rotational limit:[9,11]

$$f_{rr}=\alpha_\nu(N_\nu)\left[(N_\nu+1)\left[\frac{2N_0+3}{3(2N_0+1)}\right]\times\left[\Omega_\nu-N_\nu-\frac{4(N_0-1)}{3(2N_0-1)}N_\nu\right]\right]^{1/2}.$$

Here, N_0 is the total number of active pairs (protons plus neutrons). If the rotational state is (as is usually[5,1] assumed) a two proton particle-hole excitation across $Z=50$, then $N_0=N_\nu+3$. The A dependence of these two processes is depicted in Fig. 1.

If the transfer from g to r vanishes, then the (t,p) cross-section ratios are

TABLE I. Two-neutron transfer ratios in even Cd isotopes.

Reaction	E_x (MeV)	$10^3\sigma(0^{+'})/\sigma$(g.s.)
^{112}Cd(t,p)^{114}Cd[a]	1.134	24.1±1.2
	1.305	$\lesssim 1$
^{114}Cd(t,p)^{116}Cd[b]	1.282	$\lesssim 1$
	1.378	20.7±1.0
^{114}Cd(p,t)^{112}Cd[c]	(1.224)	(12±2)

[a]Reference 7.
[b]Reference 8.
[c]Reference 13.

$$\frac{\sigma(0^{+'})}{\sigma(\text{g.s.})} = \left[\frac{-\alpha_A\beta_{A+2}f_{gg}(A)+\beta_A\alpha_{A+2}f_{rr}(A)}{\alpha_A\alpha_{A+2}f_{gg}(A)+\beta_A\beta_{A+2}f_{rr}(A)}\right]^2 = \left[\frac{-x_{A+2}+x_A R_A}{1+x_A x_{A+2}R_A}\right]^2,$$

where $x_A=\beta_A/\alpha_A$ and $R_A=f_{rr}(A)/f_{gg}(A)$, in an obvious notation.

For ^{114}Cd(t,p) and ^{112}Cd(t,p), the R_A values are equal, because above ^{114}Cd N_v is the number of neutron pair holes in $N=82$. The two excited-state to g.s. cross section ratios in these two reactions (we use 0_2^+ in ^{114}Cd and 0_3^+ in ^{116}Cd) provide a relationship between x_{112}, x_{114}, x_{116} depicted graphically in Fig. 2. Without further data, or an additional assumption, we cannot determine the three x's uniquely—only the relationship of Fig. 2. Of course, if (p,t) ratios were known between these same two pairs of nuclei, i.e., ^{116}Cd(p,t) and ^{114}Cd(p,t), then the problem would be overdetermined—allowing a consistency check on the model. However, in the available (p,t) data[12,13] it is not always clear which excited 0^+ state is populated. Also, in (p,t), the excited-state angular-distribution shape changes with A—implying[13] the presence of at least two competing reaction mechanisms. In (t,p), the angular distributions all look alike, perhaps indicating only one process. [The absence of strong excited-state cross sections in (p,t), however, requires all x's to be small, $x<0.4$.]

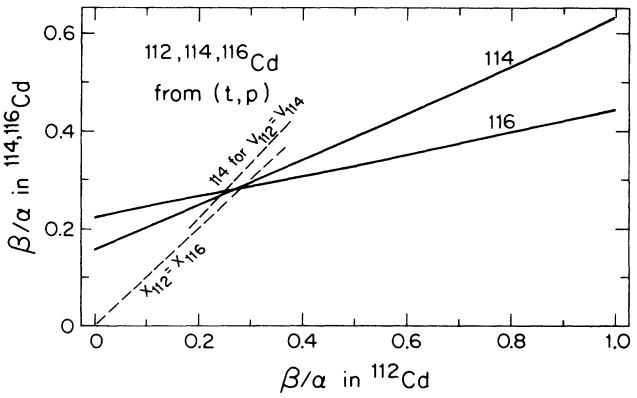

FIG. 2. Wave-function mixing amplitudes $x=\beta/\alpha$ for ^{114}Cd and ^{116}Cd in terms of that for ^{112}Cd, using only cross-section ratios from ^{112,114}Cd(t,p)^{114,116}Cd.

For now, we consider, separately, two reasonable assumptions. First, we assume symmetry about $N=66$ (i.e., about ^{114}Cd) in the mixing amplitudes, giving the dashed line labeled $x_{112}=x_{116}$ in Fig. 2. Next, we assume equal mixing matrix elements in ^{112,114}Cd, and use the average $0_2^+,0_3^+$ excitation energies to relate x_{114} to x_{112}, giving for x_{114} the dashed curve labeled $V_{112}=V_{114}$ in Fig. 2. Both

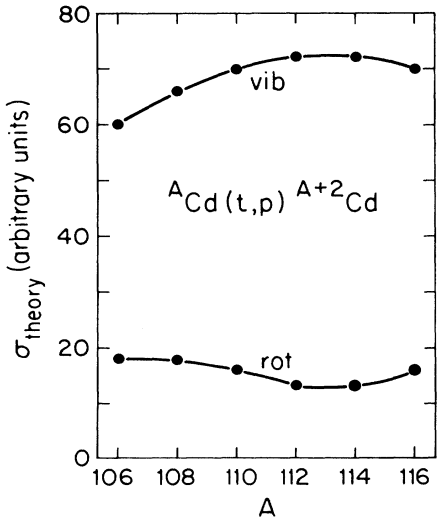

FIG. 1. Theoretical (t,p) cross sections vs A between spherical, vibrational states σ_{gg} and between deformed, rotational states σ_{rr} in Cd nuclei.

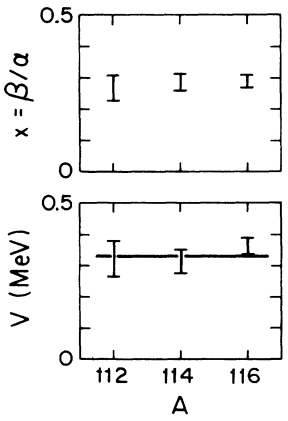

FIG. 3. In the top half is plotted vs A the estimated range of allowed values of the wave function admixture amplitude in ^{112,114,116}Cd. The bottom half displays the range of allowed values of the mixing potential matrix element resulting from these wave functions and the relation $|V_A|=|\alpha_A\beta_A|E_A$.

dashed lines intersect the solid lines at about the same place (i.e., for $0.23 < x_{112} < 0.28$).

If we assume the excited state for which data are given in Ref. 13 for ^{114}Cd(p,t)^{112}Cd is 0_2^+, then the (p,t) ratio also favors $x_{112} \sim 0.25$. We summarize the range of allowed x_A values in the top half of Fig. 3. In the bottom half we plot the allowed range of the mixing matrix elements arising from $V_A = -\alpha_A \beta_A E_A$. For the latter we have included the estimated uncertainty caused by mixing between ϕ_2 and ϕ_r. The "average" value of V is 330 keV.

It thus appears that the 2n-transfer cross sections to the predominantly rotational intruder states can be understood as arising from mixing of the intruder and normal ground states. The potential mixing matrix element $\langle \phi_g \mid V \mid \phi_r \rangle$ needed to understand the data is about 330 keV.

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