Nuclear structure dependence of the (p,π^+) reaction

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The relative cross sections for exciting particular states in the (p,π^+) reaction near threshold are calculated with the shell model assuming dominance of the elementary $p+p\rightarrow d+\pi^+$ reaction. Results from the plane-wave approximation show that strongly excited states for ^{12}C and ^{13}C targets can be accounted for in this manner.

I. INTRODUCTION

There is a considerable body of data for the (p,π^+) reaction near threshold including angular distribution and more recently analyzing power. There are detailed spectra of excited states for a number of light nuclei[1-3] and measurements of the analyzing power with polarized protons. The latter indicate[4] that the analyzing power for strongly excited states does not depend on nuclear structure but closely resembles that of the free $\vec{p}+p\rightarrow d+\pi^+$ reaction. There is a thorough treatment[5] of the theoretical aspects of the problem which has indicated the general features to be expected in the nuclear (p,π^+) reaction.

The purpose of this paper is to give a detailed look at the nuclear structure effects which determine the relative transition strengths to particular states, in some cases for which there are fairly well-tested shell-model wave functions, and to see whether including only the $p+p\rightarrow d+\pi^+$ channel leads to results resembling the observations.

II. PROCEDURE

The calculation was carried out with the following assumptions and approximations:

1. The only channel included is $p+p\rightarrow d+\pi^+$, which is known to be dominant at low energy in the free nucleon case. Embedding this reaction in the nucleus has been done previously,[6] but mainly to treat higher energy incident protons. A further restriction here is to treat the deuteron as being in a ^3S state, neglecting the D state admixture.
2. The targets are assumed to be $(0s)^4(1p)^{A-4}$ configurations and final states either $(0s)^4(1p)^{A-3}$ or $(0s)^4(1p)^{A-4}(2sd)^1$ depending on the parity. Harmonic oscillator wave functions are assumed in order to facilitate cluster expansions.
3. The embedded reaction is assumed to be of short range, and only terms first order in ζ are kept, where ζ is the relative separation of the protons.
4. Plane waves are used for the incoming proton and the outgoing π^+. Comparison will be made with data for protons of about 200 MeV impinging on carbon targets.

The first step in calculating the t matrix element is to expand the target wave function $\psi(Z,A)$ in a set of states of the form proton times $\psi(Z-1,A-1)$ and to expand[7] the final state wave function into a set of the form deuteron times $\psi(Z-1,A-1)$. The matrix element can then be written as

$$\langle \psi_{M_f}^{J_f\alpha_f}(Z,A+1)\chi(\mathbf{k}_\pi) \mid T(\mathrm{pp}\rightarrow \mathrm{d}\pi^+) \mid \psi_{M_0}^{J_0\alpha_0}(Z,A)\chi_{\sigma_0}(\mathbf{k}_0)\rangle$$
$$= \sum_{q_f l_f, ql, L} C(q_f,q,A)U_L(l_f,l) \sum_{L_z S_z m\sigma} \langle \psi_{M_f}^{J_f\alpha_f} \mid \chi_{L_z S_z}^{L\,10\dagger}(q_f l_f, ql) a_{m\sigma}^{ql}(\mathrm{p}) \mid \psi_{M_0}^{J_0\alpha_0}\rangle$$
$$\times \langle \psi_{L_z}^{QL}(\boldsymbol{\rho}_{\mathrm{d},A-1})\phi_{S_z}^{^3S_1}(\xi_\mathrm{d})\chi(\mathbf{k}_\pi) \mid T(\mathrm{pp}\rightarrow \mathrm{d}\pi^+) \mid \phi_{m\sigma}^{ql}(\boldsymbol{\rho}_{A-1})\chi_{\sigma_0}(\mathbf{k}_0)\rangle , \quad (1)$$

where C contains the cluster expansion coefficients and depends only on the mass number and the number of oscillator quanta q and q_f. Because of the assumed nature of the nuclear wave functions, the only allowed values are $ql=1p$ or $0s$ for the proton annihilation operator and $q_f l_f=1p$ for normal parity final states or $q_f l_f=2s$ or $2d$ for non-normal parity final states. The two integrals in Eq. (1) are one containing the nuclear structure information and one involving the interaction. If $ql=0s$, the nuclear structure integral reduces to a one-body neutron creation matrix element since we neglect the small amount of excitation of the $(0s)^4$ core. For $ql=1p$, the spatially symmetric deuteron state restricts the L values to $L=0,2$ for $q_f=1$ and $L=1,3$ for $q_f=2$. The factor $U_L(l_f,l)$ which also arises from the cluster expansion differs from unity only when $L=1$, $q=1=l$, and $q_f=2$, where it has the value $(5/9)^{1/2}$ for $l_f=0$ and $(4/9)^{1/2}$ for $l_f=2$. The excitation quanta, $Q=q_f+q$, all lie in the center-of-mass motion of the deuteron cluster with respect to the $A-1$ core as represented by Φ^{QL}. The motion of the proton in the target nucleus is represented by the oscillator function ϕ^{ql}.

The second integral in Eq. (1), the interaction integral, is evaluated by taking a free interaction of the form used by Koltun and Reitan[8] to treat production of S-wave pions near threshold. For our short-range approximation we only need the general form

$$\mathcal{T}(1\rightarrow 0)\sum_{\mu}\sigma_{\mu}^{1}V_{\mu}^{1^*}(\xi,\mathbf{k}_{\xi}) ,$$

where $\mathcal{T}(1\rightarrow 0)$ changes the isospin wave function from $T=1$ to $T=0$ and σ^1 is the sum of the spin of the two nucleons. Since this interaction requires that initially the two protons are in a state of total spin $S=1$, one can apply the Eckart-Wigner theorem to the σ^1 matrix element, and insert plane waves for the pion and incoming proton so that the interaction integral becomes

$$\sqrt{8}\sum_{\mu,\mathcal{S}_z}(1/2\,1/2\,\sigma\sigma_0\,|\,1\mathcal{S}_z)(11S_z\mu\,|\,1\mathcal{S}_z)\langle\Phi_{L_z}^{QL}(\boldsymbol{\rho}_{\mathrm{d},A-1})\phi^{0s}(\xi)e^{-\mathbf{k}_\pi\cdot\boldsymbol{\rho}_{\pi,A+1}}\,|\,V_{\mu}^{1^*}(\xi,k_{\xi})\,|\,\phi_{m}^{ql}(\boldsymbol{\rho}_{A-1})e^{i\mathbf{k}_0\cdot\boldsymbol{\rho}_{0,A}}\rangle .$$

Under the assumption of a short-range V the plane waves and proton oscillator wave function on the right-hand side can be expanded so that one can extract a common factor $\langle\phi^{0s}(\xi)\,|\,V^{1^*}(\xi,k_\xi)\,|\,\xi Y^1(\hat{\xi})\rangle$ from the interaction integral. The remaining integral then becomes a sum of terms involving angular momentum coupling factors and radial integrals containing oscillator functions and spherical Bessel functions. These are

$$I(QL,j_n,Q_0L_0)\equiv(L_0n\,00\,|\,L\,0)\times\int_0^\infty R_{QL}(\rho)j_n(\tilde{q}\rho)R_{Q_0L_0}(\gamma\rho)\rho^2d\rho , \tag{2}$$

where the Clebsch-Gordan coefficient limits the values of n, $\tilde{q}$ is proportional to the momentum transfer in the c.m. system, and γ is a coordinate scaling constant. These integrals can be evaluated analytically as functions of a parameter

$$g\equiv 6\beta/\tilde{q}^2 , \tag{3}$$

where β is the oscillator parameter in the exponent, $\exp(-\beta\rho^2/2)$, of $R_{QL}(\rho)$.

The nuclear structure integral contains a two-particle—one-hole operator which was evaluated in two different ways in order to provide a numerical check. The non-normal parity states are usually well represented by weak coupling of a $2sd$ nucleon to the low-lying normal-parity states of the $(A-1)$ nucleus. This suggests that it may be useful to recouple the operator in the nuclear integral into the form

$$a^{q_fl_f^\dagger}(t_3)\times(a^{ql^\dagger}(-t_3)\times\tilde{a}^{ql}(\mathrm{p}))^{\mathcal{J}(\mathcal{L}\mathcal{S})} ,$$

where $\mathcal{L}$ and $\mathcal{S}$ are the orbital and spin quantum numbers of the particle-hole operator which couple to a resultant $\mathcal{J}$. Here the hole operator with proper transformation properties is

$$\tilde{a}_{-m-\sigma}{}^{ql}=(-1)^{l+1/2-m-\sigma}a_{m\sigma}^{ql} .$$

Upon insertion of a complete set of $(0s)^4(1p)^{A-4}$ states of the target nucleus before the particle-hole operator, the nuclear integral becomes a sum of products of transition density matrix elements in the nucleus A times spectroscopic amplitudes to the nucleus $A+1$. In particular, when t_3 is a neutron and $\mathcal{L}=0=\mathcal{S}$, the particle-hole operator is just $(4l+2)^{-1/2}$ times the number operator $n_{ql}(\mathrm{p})$. This is the only term that can contribute when $ql=0s$ due to our configuration assumptions. For $ql=1p$ there are ten possible values for $\mathcal{J}(\mathcal{L}\mathcal{S})$, but the largest contributions usually come from 0(00) and 2(20), and we shall show how good an approximation is obtained by keeping just these two terms.

The sums over magnetic quantum numbers can be carried out with the results contained in $6j$ and $9j$ coefficients. The t matrix element of Eq. (1) becomes

$$\begin{aligned}
&\langle\psi_{M_f}^{J_f}\chi(\mathbf{k}_\pi)\,|\,T(\mathrm{pp}\rightarrow d\pi^+)\,|\,\psi_{M_0}^{J_0}\chi_{\sigma_0}(\mathbf{k}_0)\rangle\\
&\quad=\sum_{q_fl_f,ql}F(q_f,A,V^1)\Bigg\{\delta_{ql,0s}\sum_{\lambda j_f}Y_{(j_f)}^{\lambda}\langle\psi^{J_f}||a^{j_f^\dagger}(\mathrm{n})||\psi^{J_0}\rangle D_\lambda(j_f,g)\\
&\qquad+\delta_{ql,1p}\delta_{q_fl_f,1p}\sum_{\lambda J}Y^\lambda(J)\sum_{j_p,\mathcal{J}\mathcal{L}\mathcal{S}}D_\lambda(J,j_p,\mathcal{J}\mathcal{L}\mathcal{S},g)\\
&\qquad\qquad\times\langle\psi^{J_f}||[a^{j_p^\dagger}(\mathrm{n})\times[a^{1p^\dagger}(\mathrm{p})\times\tilde{a}^{1p}(\mathrm{p})]^{\mathcal{J}\mathcal{L}\mathcal{S}}]^J||\psi^{J_0}\rangle\\
&\qquad+\delta_{ql,1p}\delta_{q_fl_f,2l_d}\sum_{\lambda J}Y^\lambda(J)\sum_{j_d,\mathcal{J}\mathcal{L}\mathcal{S}}D_\lambda(J,j_d,\mathcal{J}\mathcal{L}\mathcal{S},g)\\
&\qquad\qquad\times\left\langle\psi^{J_f}\left|\left|\sum_{t_3}2t_3[a^{j_d^\dagger}(t_3)\times[a^{1p^\dagger}(-t_3)\times\tilde{a}^{1p}(\mathrm{p})]^{\mathcal{J}\mathcal{L}\mathcal{S}}]^J\right|\right|\psi^{J_0}\right\rangle\Bigg\} .
\end{aligned} \tag{1'}$$

In Eq. (1 ′) the factor F contains the factor C of Eq. (1) and the common integral of V^1 from the interaction integral. Otherwise F depends only on the mass number A and whether the final state has normal parity ($q_f l_f = 1p$) or non-normal parity ($q_f l_f = 2sd$). The Y^λ factor contains the remaining magnetic quantum number dependence and a spherical harmonic,

$$Y^\lambda(J) \equiv \sum_{\lambda_z, M} Y^{\lambda *}_{\lambda_z}(\hat{\mathbf{q}}_{\text{c.m.}})(J_0 J M_0 M \mid J_f M_f)(\lambda \tfrac{1}{2} \lambda_z \sigma_0 \mid JM) \; . \qquad (1'')$$

The D_λ factors are linear combinations of the radial integrals of Eq. (2) with the coefficients determined by the results of angular momentum couplings. They are functions of the parameter g in Eq. (3). The possible values of λ are restricted by the symmetries so that in the first term of Eq. (1′) $\lambda = q_f \pm 1$ only, while in the other two $\lambda = q_f + 3$ is allowed as well as $\lambda = q_f \pm 1$. The convention used in Eq. (1′) is that $t_3 = +1/2$ for a neutron. The reduced matrix elements in the last two terms of Eq. (1′) can be evaluated by introducing a complete set of states of the configuration $(0s)^4(1p)^{A-4}$ between the nuclear creation operator and the $\mathscr{J}\mathscr{L}\mathscr{S}$-coupled particle-hole operator.

The nuclear structure information was obtained from shell model calculations for the carbon isotopes in a complete $(0+1)\,\hbar\omega$ space and spurious states were eliminated. The procedure and results for ^{13}C have been published.[9] The t matrix element was then evaluated with a SPEAKEASY language[10] program which incorporated the nuclear structure information and also printed out the contribution of each $(j_f \mathscr{J}\mathscr{L}\mathscr{S})$ term separately. The quantity which will be compared to observation is $P(g)$, where

$$(2J_0+1)P(g) = \sum_{M_0, M_f, \sigma_0} \left| \langle \psi^{J_f}_{M_f} \chi(\mathbf{k}_\pi) \mid T(\text{pp} \rightarrow \text{d}\pi^+) \mid \psi^{J_0}_{M_0} \chi_{\sigma_0}(\mathbf{k}_0) \rangle \right|^2 . \qquad (4)$$

III. RESULTS

Numerical results were obtained for ^{12}C and ^{13}C targets, and in Fig. 1 the ^{12}C case is compared to experimental results[3,11] obtained with incident proton energy of about 200 MeV. The data cover a range of momentum transfer, $q_{\text{c.m.}}$ of about 480 to 700 MeV/c corresponding to a range of 0.42 to 0.20 for the parameter g of Eq. (3). Since one can only expect to obtain relative cross sections in a plane wave calculation, the calculated values, $P(g)$ of Eq. (4), were multiplied by the same constant for all final states to produce the calculated cross section curves of Fig. 1.

The five specific states plus the unresolved complex at 7.5 MeV represent six of the seven strong transitions that are observed in the reaction. Data for two similar energies are given for the $J_f = 1/2$ states to indicate the sensitivity to incident energy. The angular distribution for the 7.5 MeV complex has not been published, but it is seen to be a strong state in the spectrum at $\theta = 25°$ in Ref. 1. Another possible similarity is the lowest $J_f = 7/2^-$ state which is calculated to have a shape like the $J_f = 5/2_a^-$ state of Fig. 1 but only 0.4 the magnitude of that state. There is such a peak near the known $7/2^-$ state at 10.75 MeV seen in preliminary data.

While there is rough similarity to observation in the comparisons of Fig. 1, the $J_f = 3/2_a^-$ shows an obvious discrepancy in that the calculation does not give the rise at higher momentum transfer. The other major discrepancy is for the $J_f = 5/2^+$ state at 6.86 MeV which is observed to be a strong transition with a shape similar to that of the $5/2^+$ state at 3.85 MeV in Fig. 1. The calculated cross section is relatively flat and never exceeds 30 nb/sr. Aside from these serious discrepancies, other states are calculated to be much weaker than those of Fig. 1, in agreement with observation.

The calculations for ^{13}C(p,π^+)^{14}C give a strong transition to the lowest $J_f = 5^-$ at 14.8 MeV with a dependence on $q_{\text{c.m.}}$ like that of the $9/2_a^+$ state of Fig. 1. Flat states with about half the strength of the 5_a^- are calculated for the lowest $J_f = 4^-$ near 12 MeV and lowest 2^+ at 7.0 MeV. Three other strong states with sharp minima in their curves are calculated for the lowest $J_f = 1^-$ (6.1

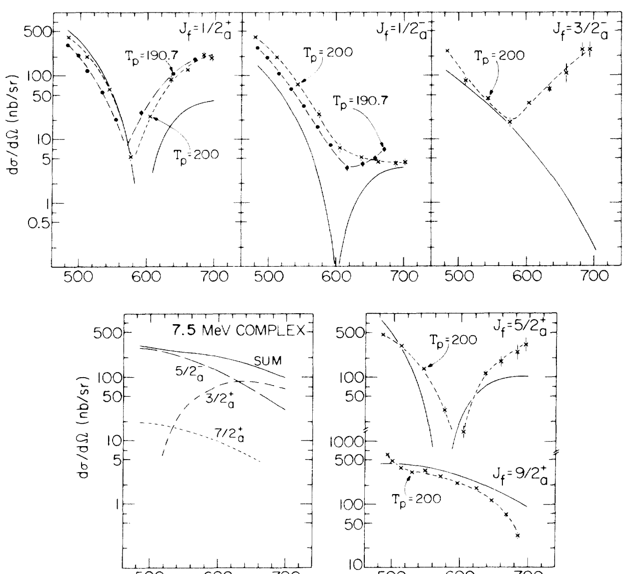

FIG. 1. Differential cross sections for ^{12}C(p,π^+)^{13}C compared to calculation. The observed excitation energies in MeV are $1/2_a^+$ (3.09), $1/2_a^-$ (0.00), $3/2_a^-$ (3.68) $5/2_a^+$ (3.85), and $9/2_a^+$ (9.50). The data with 200 MeV protons are from Ref. 3 (crosses) and with 190.7 MeV protons from Ref. 11 (solid dots). There is a strong transition at about 7.5 MeV excitation. The calculated curves (solid lines) are obtained by multiplying the quantity $P(g)$ of Eq. (4) by a single normalization constant for all cases.

TABLE I. Degree of approximation obtained by keeping only the one-body terms plus the terms resulting from $\mathscr{J}\mathscr{L}\mathscr{S}=220$ excitation of the first excited state of ^{12}C. Also given in the second column is R2B, the ratio of the total two-body contribution to the total t-matrix element. The variation is over the range of $q_{\mathrm{c.m.}}$ values of Fig. 1.

J_f	R2B	$d\sigma(\text{approx})/d\sigma(\text{total})$
$9/2_a^+$	1.00	1.13
$7/2_a^+$	1.00	1.60 to 1.79
$5/2_a^+$	−0.06 to 0.31	1.20 to 0.90
$5/2_b^+$	3.00 to 0.88	2.41 to 1.06
$3/2_a^+$	−3.06 to 0.71	1.01 to 0.97
$1/2_a^+$	0.02 to 0.12	0.97 to 1.05
$7/2_a^-$	1.00	0.51
$5/2_a^-$	1.00	0.42
$3/2_a^-$	0.42 to 7.70	1.13 to 7.80
$1/2_a^-$	0.36 to −0.98	1.53 to 0.02

MeV), $J_f=2^-$ (7.3 MeV), and $J_f=3^-$ (6.7 MeV). These are the only states calculated to be strong, which is in reasonable agreement with the spectrum at $\theta=25°$ in Ref. 2 and other unpublished preliminary results.

It is often a good approximation to the complete calculation in ^{13}C to include only the terms resulting from parentage to the ground state and first excited 2^+ state of ^{12}C. This is indicated in the last column of Table I for the states we have discussed. It is clear that for the strong positive parity states this is a very good approximation, often within 10%. For the negative parity states the approximation is not so good, often missing by a factor of 2 or more. This different behavior arises because the positive parity states are extremely well represented by weak coupling a j_d neutron to the low states of ^{12}C, whereas because of the Pauli principle such weak coupling is not good for the normal parity states.

IV. DISCUSSION

There is a good correlation between transitions found to be strong in the calculation and those observed to be strong in ^{12}C(p,π^+)^{13}C. This supports the main assumption that the pp $\rightarrow$dπ^+ channel is dominant and shows the role of nuclear structure in determining the relative cross sections. There are, of course, effects of distortion which have been omitted. The minima in calculated cross sections of Fig. 1 arise from cancellations among the one-body $0s$ and $1p$ contributions (which are generally in phase only for the low values of $q_{\mathrm{c.m.}}$) and the two-body contribution. Such effects can be seen in the second column of Table I where the ratio of the two-body contribution to the total t matrix element is given. It is not surprising that the location of the minima differs from observation since we assumed oscillator radial functions and plane waves.

The discrepancy with the observed strong transition to the second $5/2^+$ state as 6.86 MeV is puzzling. The state is not seen in inelastic pion scattering on ^{13}C, which agrees with calculation.[9] In the ^{12}C(d,p)^{13}C reaction the extracted[12] spectroscopic factor is less than 0.05 of the spectroscopic factor to the first $5/2^+$ state, consistent with the calculated weakness. Without a strong one-body contribution, the present calculation is not likely to produce a strong transition of the observed shape.

Nevertheless, the overall degree of agreement with observed relative cross sections for both ^{12}C and ^{13}C targets shows that our basic assumptions are reasonable. The simple approximation of keeping only a few of the many terms is seen to be a good way to estimate cross sections for the states of non-normal parity.

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