Dilepton radiation from high temperature nuclear matter

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The general features of electron-positron emission from high temperature nuclear matter are discussed. Estimates are made of the production rates arising from incoherent nucleon-nucleon scattering and from two-pion annihilation. It may be possible to infer the pion dispersion relation in hot and dense nuclear matter by measuring the invariant mass distribution of back-to-back electrons and positrons in the center of mass frame in high energy nucleus-nucleus collisions.

I. INTRODUCTION

Colliding heavy nuclei at high energy allows us to study the properties of nuclear matter at finite temperature and at densities away from normal density. So far, these studies have concentrated primarily on multifragmentation, pion production, and collective flow.[1] Recently, much attention has been drawn to the observation of copious direct photon production.[2] Depending on the energy and angle of the emitted photons, this should provide information on nuclear stopping power[3] and on the dynamics of baryon-baryon cascading in hot and dense nuclear matter.[4,5]

The theoretical study of dilepton production in nucleus-nucleus collisions was initiated with the suggestion that it may serve as a thermometer of quark-gluon plasma formed at ultrarelativistic energies.[6] However, dileptons should be produced at lower energies as well, where no quark-gluon plasma formation is expected. The argument is simply that whenever charged objects collide they radiate real and virtual photons, the virtual photons decaying into dileptons. The advantage of observing electromagnetic signals from a strongly interacting many-body system is that they travel relatively unscathed from the production point to the detector. Since production rates are rapidly increasing functions of temperature and density, these electromagnetic signals are good probes of the early high temperature and density stage of heavy ion collisions. A relevant analogy can be made with neutrino emission from the sun or from a cooling neutron star.

The purpose of this paper is to provide an exploratory study of dilepton radiation from finite temperature and density nuclear matter, below the critical temperatures and densities at which a phase transition to quark-gluon plasma occur.[7] A rigorous study is extremely difficult due to the strongly interacting nature of nuclear matter: For example, there are complications from two and three particle correlations, three body collisions, identification of the relevant degrees of freedom (nucleons, baryonic resonances, mesons, collective degrees of excitation, quarks, etc.), and finite temperature and density corrections to form factors, widths, and so on. Therefore our analysis will at times be quite phenomenological, and our study will be based primarily on relativistic kinetic theory.

The plan of the paper is as follows. In Sec. II we present a general discussion of the sources of dileptons, distinguishing between coherent and incoherent virtual bremsstrahlung, separating hard from soft processes, and noting annihilation processes. In Secs. III and IV we make explicit estimates of the rates due to virtual bremsstrahlung from baryon-baryon cascading and from $\pi^+\pi^-$ annihilation. The rates for the invariant mass distribution integrated over pair momentum, and for zero total pair momentum (back-to-back e^+e^-), are both evaluated numerically and analyzed in Sec. V. The rate for $\pi^+\pi^-$ annihilation depends dramatically on the pion dispersion relation $\omega(k)$ in nuclear matter, thereby allowing the possibility of measuring it in heavy ion collisions. We make some concluding remarks, and mention avenues for further theoretical research in Sec. VI.

II. GENERAL DISCUSSION OF DILEPTON PRODUCTION

To begin the discussion, let us first consider bremmstrahlung in hadron-hadron collisions. For example, in the reaction np$\rightarrow$np, photons (real and virtual) can be radiated since charged particles are accelerated. In the soft photon approximation photons are radiated only from the initial or from the final charged lines.[8] None come from the strong interaction blob. See Fig. 1. This approximation is valid if the energy carried by the photon is less than the inverse of the strong interaction collision time. The latter is usually estimated as about 1–2 fm/c, so that this mechanism is dominant if $E_\gamma < \tau_{\rm NN}^{-1} = 100$–$200$ MeV. For hard photons we must, in addition, look inside the strong interaction blob. One contribution is shown in Fig. 2. It represents radiation from the exchange of a charged pion.[9] Radiation from the internal pion line will be important for energies $E_\gamma > \tau_{\rm NN}^{-1} = 100$–$200$ MeV. Of course, the separation into soft and hard photons is not sharp. The relative contribution of radiation from internal and external lines, in general, depends not only on the average collision time, but also on other variables, such as the momentum transfer to the charged particles in the final state, on the angle of emission of the photon, and on the available center of mass energy.

In the soft photon approximation one retains only radi-

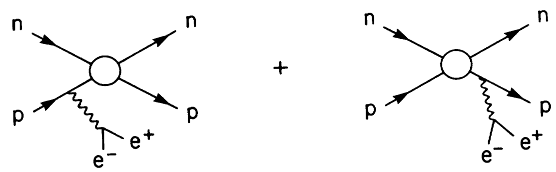


FIG. 1. Radiation from the external proton lines in np scattering.

ation from the external charged lines and in addition treats the strong interaction blob as being on shell. The invariant cross section for real photons of four momentum q^μ is[8,10]

$$q_0 = \frac{d^4\sigma^\gamma}{d^3q\, dX} = \frac{\alpha}{4\pi^2}\left[\sum_{\substack{\text{pol}\\ \lambda}} J\cdot\epsilon_\lambda J\cdot\epsilon_\lambda\right]\frac{d\sigma}{dX} \ . \tag{1}$$

Here, $d\sigma/dX$ is the strong interaction cross section for the reaction $a+b\to$n particles, ϵ_λ is the polarization of the emitted photon, and

$$J^\mu = -Q_a\frac{p_a^\mu}{p_a\cdot q} - Q_b\frac{p_b^\mu}{p_b\cdot q} + \sum_{i=1}^{n} Q_i\frac{p_i^\mu}{p_i\cdot q} \tag{2}$$

is the current, the Q's and p's representing the charges and four-momenta of the particles. Equation (1) is exact in the limit $q\to 0$, but can be off by a factor of 2–4 for hard photons.[5,9]

For virtual photons, that is, for dilepton production, it is possible to extrapolate the soft photon predictions from $q^2=0$ to $q^2=M^2$, M being the invariant mass of the e^+e^- pair. The result is[11] ($q^\mu = p_+^\mu + p_-^\mu$)

$$E_+E_-\frac{d^6\sigma^{e^+e^-}}{d^3p_+d^3p_-} = \frac{\alpha}{2\pi^2}\frac{1}{q^2}q_0\frac{d^3\sigma^\gamma}{d^3q} \ . \tag{3}$$

For hard virtual photons this extrapolation is, in general, not possible since the off-shell continuation of the strong interaction blob is different for $q^2=0$ and $q^2>0$. Note that real photon production is of order α, while dilepton production is of order α^2.

In nucleus-nucleus collisions the possibilities are even more interesting. Consider a collision between nucleus A and nucleus B leading to some distribution of particles in the final state. A and B may radiate a photon before impact, or the charged particles may radiate a photon in the final state. See Fig. 3. If the photon energy is less than

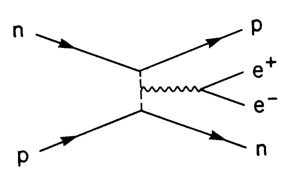


FIG. 2. Radiation from an internal charged pion line in np scattering.

the inverse nuclear collision time, estimated to be 10–20 fm/c, $E_\gamma < \tau_{AB}^{-1} = 10$–20 MeV, the charges will act coherently over the whole extent of the nucleus. In that case the cross section for real photons is proportional to $Z^2\alpha$, and for dileptons proportional to $Z^2\alpha^2$. At high incident energy the cross sections peak at angles near 0° and 180° in the c.m. frame (although they vanish at 0° and 180°) due to the sudden deceleration along the beam axis.[3]

It is known from computer simulations that the nucleons cascade during the collision.[1] In order to resolve the finer features of the nucleus-nucleus collision, such as the cascading, we must look at higher energy photons. Thus for 10–20 MeV$=\tau_{AB}^{-1} < E_\gamma < \tau_{\text{NN}}^{-1} = 100$–200 MeV, we can see the individual baryons cascade.[4,5] This radiation will be incoherent, proportional to Z, but will also increase with the number of collisions per baryon roughly as $Z^{1/3}$. At even higher photon energies, radiation from internal NN blobs will become important, as in Fig. 2.

The π^0 lifetime is 10^{-16} s. It decays outside the reaction zone in a heavy ion collision. The branching ratio into $\gamma\gamma$ is 98.85%, and into γe^+e^- (Daliz decay) it is 1.15%. The Dalitz decay may be viewed as a background to other e^+e^- sources. Fortunately, phase space restricts the invariant e^+e^- mass M to less than $m_{\pi^0} = 135$ MeV. In fact, 90% of the pairs have $M<15$ MeV, and 99% have $M<65$ MeV.[12] Still, the Dalitz pairs can be approximately subtracted out by taking the π^0 spectrum to be the average of the π^+ and π^- spectra and folding in the decay distribution. Also, the η meson has a decay mode γe^+e^-, but its branching ratio is only 0.5%, and besides the η should not be abundantly produced at the temperatures $T=40$–100 MeV of interest to us here.

In the nuclear many-body environment there will be a profusion of microprocesses involving nucleons, pions, and deltas. If a charged particle is involved it will radiate. A detailed comparison with experiment will require a complete enumeration of all these processes and their inclusion in Boltzmann-Uehling-Uhlenbeck (BUU) or cascade computer simulation calculations.[13] This will allow

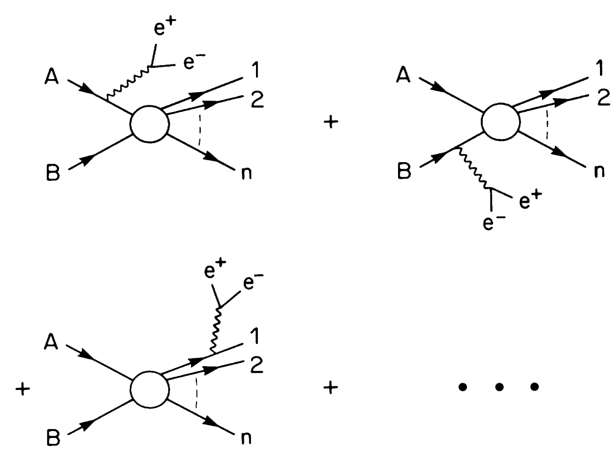


FIG. 3. Soft radiation from nucleus-nucleus collisions is a coherent superposition of radiation from the initial nuclei, A and B, and from the charged particles in the final state.

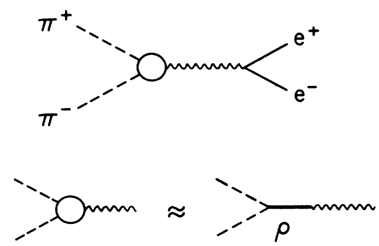


FIG. 4. Charged pion annihilation proceeds dominantly through the rho meson which subsequently decays into a virtual photon by vector dominance.

us to test our understanding of collision processes in hot and dense nuclear matter.

So far, our discussion has not distinguished between real and virtual photons. In fact, for measuring bremsstrahlung radiation real photons are favored over dileptons since the dilepton production rate is smaller by a factor of α. The real advantage of dileptons over photons arises when focusing on annihilation processes. Kinematically, annihilation of two massive particles into a single real photon is not allowed, and $\gamma\gamma$ correlation measurements are more difficult than e^+e^- measurements. The most important[6] annihilation process is $\pi^+\pi^- \rightarrow \rho \rightarrow \gamma^* \rightarrow e^+e^-$. See Fig. 4. Annihilation of hadrons always leads to a minimum energy, in this case $E^2 \geq q^2 = M^2 \geq 4m_\pi^2$. Thus in high temperature nuclear matter, and in high energy heavy ion collisions, we expect the rate for production of high mass e^+e^- pairs, $M > 2m_\pi$, to be dominated by pion annihilation. That region of invariant mass should provide information on the abundance of pions in the hot and dense medium since the production rate is proportional to the square of the pion number density.

An example of a semiannihilation process, one which is neither pure annihilation nor bremsstrahlung, is shown in Fig. 5. Ten or so events of this kind have been observed[14] in π^-p reactions at 16 GeV/c. Compared to $\pi^+\pi^-$ annihilation this is a small rate.[6]

We view annihilation processes to be of most interest from the point of view of dilepton production. Bremsstrahlung from cascading baryons and mesons is best studied via real photons, and from the perspective of annihilations forms a background. In the next two sections we make explicit estimates of the dilepton rates from cascading baryons and from pion annihilations, and compare them in Sec. V.

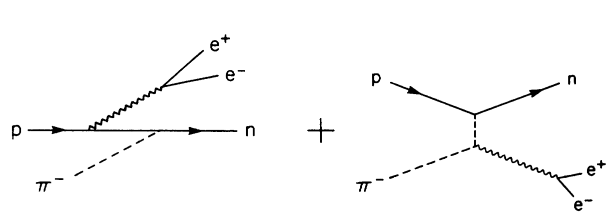


FIG. 5. Examples of pion annihilation on a nucleon to form a dilepton.

III. PROTON-NEUTRON VIRTUAL BREMSSTRAHLUNG

In this section we make a rough estimate of the production of dileptons by the cascading baryons. Consider a two-body reaction $a + b \rightarrow c + d$, where a, b, c, and d may be either a nucleon or a delta. These are the primary reactions in current cascade or BUU Monte Carlo simulations of heavy ion collisions.[13] In the soft-photon approximation a, b, c, and d may each radiate, depending on its charge. Among the three reactions pp→pp, np→np, and nn→nn, the most important are neutron-proton collisions.[4,5] This is because nn→nn has no charged particles in the initial or final states, and pp→pp is suppressed in the dipole limit and only makes a contribution due to relativistic effects; see (1). The suppression of pp compared to np, in fact, is seen experimentally.[15]

In high temperature nuclear matter the relative probability for a baryon to be found in a delta state of excitation as compared to the nucleonic ground state is approximately

$$2(m_\Delta/m_N)^{3/2}\exp[-(m_\Delta - m_N)/T] .$$

This is of order 10% for the temperatures of interest, $T = 40$–100 MeV. We neglect the delta excitation of the nucleon from the point of view of virtual bremsstrahlung in the baryon-baryon cascade not only for this reason but also for the following reasons. Baryon number is conserved, so by acknowledging the delta the rate of baryon-baryon collisions would not change by much. Furthermore, creation of a delta costs energy, so the mean temperature of the nuclear matter would decrease, which decreases the rate of (virtual) bremsstrahlung. Finally, since a delta is more massive than a nucleon, it will radiate less, other factors being equal. These considerations suggest that to obtain a first, order of magnitude estimate of the rate of e^+e^- production in the baryon-baryon cascade, we ignore the delta excitation and calculate the radiation only from proton-neutron collisions.

From (1)–(3) we can write the cross-section for an np collision to make an e^+e^- pair with invariant mass M as

$$\frac{d\sigma_{\rm np}^{e^+e^-}}{dM^2} = \frac{\alpha^2}{8\pi^4}\frac{1}{M^2}\int |\epsilon\cdot J|^2 \frac{d\sigma_{\rm np}}{dt}\delta(M^2-(p_+ + p_-)^2) \times \frac{d^3p_+ d^3p_-}{E_+E_-} dt , \tag{4}$$

where t is the four-momentum transfer in the np collision. The electron mass is essentially set to zero. For $|t| < 4m_N^2$, a fair approximation is[8]

$$|\epsilon\cdot J|^2 = \frac{2}{3}\frac{1}{q_0^2}\left[\frac{-t}{m_N^2}\right] . \tag{5}$$

The problem now is how to extrapolate from $q^2 = q_0^2 - \mathbf{q}^2 = 0$ to $q^2 = (p_+ + p_-)^2 = M^2$. We do so by replacing q_0^2 in (5) with the symmetrized combination $E(E^2 - M^2)^{1/2}$, where $E = E_+ + E_- = |\mathbf{p}_+| + |\mathbf{p}_-|$.

(Usage of E^2 or E^2-M^2 leads to qualitatively the same result.) Carrying out the resulting integration is then straightforward, with the answer

$$\frac{d\sigma_{\rm np}^{\rm e^+e^-}}{dM^2}=\frac{\alpha^2}{3\pi^2}\frac{\bar{\sigma}(s)}{M^2}\ln\left[\frac{s^{1/2}-2m_{\rm N}}{M}\right], \tag{6}$$

$$\bar{\sigma}(s)=\int_{-(s-4m_{\rm N}^2)}^{0}(-t/m_{\rm N}^2)\frac{d\sigma_{\rm np}}{dt}dt\ . \tag{7}$$

$\bar{\sigma}(s)$ is the momentum transfer weighted cross section. The logarithmic factor in (6) arises in the small M limit no matter which of the three substitutions one uses for q_0^2.

The np differential cross section is isotropic at low energy. It develops a strong forward peak at high energy, typical of hadron-hadron collisions. However, it also develops a strong backward peak due to the charge exchange force. Approximately, the differential cross section is symmetric about $\theta_{\rm c.m.}=90°$, or equivalently stated, $d\sigma_{\rm np}/dt$ is a symmetric function of t and u. It then follows that

$$\bar{\sigma}(s)=2\sigma_{\rm el}^{\rm np}(s)\left[\frac{s}{4m_{\rm N}^2}-1\right]. \tag{8}$$

For the np cross section we adopt the parametrization

$$\sigma_{\rm el}^{\rm np}(s)=\frac{18({\rm mb\,GeV})m_{\rm N}}{s-4m_{\rm N}^2}+10\ {\rm mb}\ . \tag{9}$$

In the independent particle approximation of kinetic theory the rate (number of reactions of the specified kind per unit time per unit volume) is computed as[16]

$$\frac{dR_{\rm np}^{\rm e^+e^-}}{dM^2}=\int\frac{d^3k_1}{(2\pi)^3}f_{\rm n}(\mathbf{k}_1)\times\int\frac{d^3k_2}{(2\pi)^3}f_{\rm p}(\mathbf{k}_2)\frac{d\sigma_{\rm np}^{\rm e^+e^-}}{dM^2}(s,M^2)v_{\rm rel}\ ,$$

$$v_{\rm rel}=\frac{[(k_1\cdot k_2)^2-m_{\rm N}^4]^{1/2}}{E_1E_2}\ . \tag{10}$$

Here the f's are the occupation probabilities in momentum space. For our purposes we use a relativistic Boltzmann distribution and neglect the Pauli blocking in the final state:

$$f_i(\mathbf{k})=2e^{\mu_i/T}e^{-E/T}\ , \tag{11}$$

where $E=(\mathbf{k}^2+m_{\rm N}^2)^{1/2}$, μ is the chemical potential, i refers to proton or neutron, and the 2 is a spin factor. Integrating over five of the six variables in (10) yields[16]

$$\frac{dR_{\rm np}^{\rm e^+e^-}}{dM^2}=\frac{T^6}{4\pi^4}\int_{z_{\min}}^{\infty}dz\,z^2\left[z^2-\frac{4m_{\rm N}^2}{T^2}\right]\times K_1(z)\frac{d\sigma_{\rm np}^{\rm e^+e^-}}{dM^2}(z)e^{(\mu_{\rm p}+\mu_{\rm n})/T}\ ,$$

$$z=s^{1/2}/T,\quad z_{\min}=(2m_{\rm N}+M)T\ . \tag{12}$$

The chemical potentials may be eliminated in favor of the densities by

$$n_{\rm p}=\frac{m_{\rm N}^2T}{\pi^2}K_2(m_{\rm N}/T)e^{\mu_{\rm p}/T}\ , \tag{13}$$

and similarly for the neutrons.

It will be useful later, for comparison with the rate from $\pi^+\pi^-$ annihilation, to know the rate for producing an e^+e^- pair with total momentum $\mathbf{q}=0$ in the nuclear matter rest frame. Recalling (2) we take $q=(M,\mathbf{0})$. This leads to the cross section

$$\left.\frac{d^4\sigma_{\rm np}^{\rm e^+e^-}}{d^3q\,dM}\right|_{\mathbf{q}=0}=\frac{\alpha^2}{6\pi^3}\frac{\bar{\sigma}(s)}{M^4}\ , \tag{14}$$

where we have made the approximation $t=-m_{\rm N}^2(\Delta\mathbf{v})^2$. The rate $d^4R_{\rm np}^{\rm e^+e^-}/d^3qdM\,|_{\mathbf{q}=0}$ is then obtained by replacing $d\sigma/dM^2$ with $d^4\sigma/d^3q\,dM$ in (12).

The formulae obtained in this section are only approximations to very complicated reactions, but they have the advantage of simplicity. There is no doubt that improvements will come in the near future. Probably the most important diagrams for masses of the order $M=100$ MeV will be those of the type illustrated in Fig. 2. Nevertheless, the estimates made here should serve as an order of magnitude estimate of dileptons originating from baryon-baryon cascading, which we consider as background to the process discussed in the next section.

IV. TWO PION ANNIHILATION

The most important annihilation process leading to dilepton production in nuclear matter is[6] $\pi^+\pi^-\to e^+e^-$. This reaction proceeds through the ρ meson, which decays into a virtual photon by vector dominance.[17] See Fig. 4. For quark-gluon plasma formation at ultrarelativistic energies this is considered a background. However, in nuclear matter we view it as the process of prime interest since it can provide direct information on pion dynamics.

The pion annihilation cross section is well known to be

$$\sigma_{\pi\pi}^{\rm e^+e^-}(M)=\frac{4\pi}{3}\frac{\alpha^2}{M^2}(1-4m_\pi^2/M^2)^{1/2}|F_\pi(M)|^2\ , \tag{15}$$

where $F_\pi(M)$ is the form factor. We use

$$|F_\pi(M)|^2=\frac{m_\rho^4}{(M^2-m_\rho'^2)^2+m_\rho^2\Gamma_\rho^2}\ ,$$

$$m_\rho=775\ {\rm MeV},\quad m_\rho'=761\ {\rm MeV},\quad \Gamma_\rho=118\ {\rm MeV}\ . \tag{16}$$

A relativistic Breit-Wigner form would have $m_\rho'=775$ MeV and $\Gamma_\rho=155$ MeV. The parametrization (16), though, is a good approximation to the more sophisticated Gounaris-Sakurai formula,[18] which itself provides an excellent fit to the data. See Fig. 6. The annihilation rate is then computed analogously to (10) with the result

$$\frac{dR_{\pi\pi}^{\rm e^+e^-}}{dM^2}=\frac{\sigma_{\pi\pi}^{\rm e^+e^-}(M)}{2(2\pi)^4}M^3TK_1(M/T)(1-4m_\pi^2/M^2)\ . \tag{17}$$

The pion dispersion relation refers to the function relat-

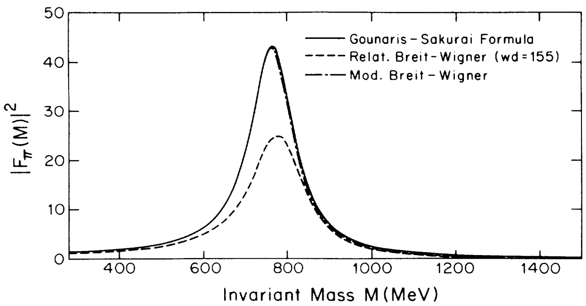


FIG. 6. The square of the pion electromagnetic form factor as a function of M. Shown are the relativistic Breit-Wigner formula with parameters taken from the Particle Data Tables, the Gounaris-Sakurai formula which represents the data well, and a modified Breit-Wigner formula, (16).

ing the energy $\omega(k)$ of a pion moving through nuclear matter to its momentum k. This relation depends both on temperature T and baryon density n. It is conjectured that the pion dispersion relation becomes softer with increasing density; that is, $\omega(k)$ is reduced at fixed k, due to the attractive p-wave interaction between pions and nucleons.[19] In fact, $\omega(k)$ may even have a minimum at some $k=k_0>0$ at finite density if the interaction is strong enough.[19-21] This may occur as a precursor to pion condensation, or it may occur even if pion condensation never sets in. It would seem that a softening of the dispersion relation would increase the number of pions and hence also the rate of dilepton emission. The number of real pions escaping from the reaction zone may in fact not increase at all because these pions are not "on the mass shell" at high density and are absorbed as the nuclear matter expands. Therefore the dilepton spectrum may be the only signal we have of the pion dispersion relation in dense nuclear matter.

The most direct connection between the pion dispersion relation and the dilepton spectrum occurs when the pions have equal but opposite momenta k in the nuclear matter rest frame.[22] Then the e^+ and e^- emerge back to back ($\mathbf{q}=0$) and their invariant mass is $M=2\omega(k)$. The total invariant mass spectrum, integrated over $\mathbf{q}$, is less sensitive to the form of $\omega(k)$ because there is a continuum of pion momenta which all have the same invariant mass M.

To compute the rate of production of dileptons with $\mathbf{q}=0$ and with an arbitrary pion dispersion relation, we recall the basic equation from relativistic kinetic theory:

$$\left.\frac{d^4R_{\pi\pi}^{e^+e^-}}{d^3q\,dM}\right|_{\mathbf{q}=0}=\int\frac{d^3k_1}{2\omega_1(2\pi)^3}\frac{1}{e^{\omega_1/T}-1}\int\frac{d^3k_2}{2\omega_2(2\pi)^3}\frac{1}{e^{\omega_2/T}-1}$$
$$\times\int\frac{d^3p_+}{2E_+(2\pi)^3}\int\frac{d^3p_-}{2E_-(2\pi)^3}|\mathcal{M}|^2(2\pi)^4\delta(\omega_1+\omega_2-E_+-E_-)$$
$$\times\delta(\mathbf{k}_1+\mathbf{k}_2-\mathbf{p}_+-\mathbf{p}_-)\delta(\mathbf{k}_1+\mathbf{k}_2)\delta(M-\omega_1-\omega_2)\ , \quad (18)$$

$$|\mathcal{M}|^2=\frac{e^4}{(q^2)^2}(k_1-k_2)^\mu(k_1-k_2)^\nu\mathrm{Tr}(\not p_+\gamma_\mu\not p_-\gamma_\nu)\ . \quad (19)$$

[Not written explicitly is the form factor $|F_\pi(M)|^2$ which multiples (19).] In these equations we simply let $k^0=\omega=\omega(\mathbf{k})$ for each of the two pions.

The origin of the various terms in (18) and (19) are as follows.[23] The piece of the Lagrangian which contains the coupling between the photon field A^μ and the charged pion field Φ is

$$\mathcal{L}_{\pi\gamma}=(\partial_\mu-ieA_\mu)\Phi^*(\partial^\mu+ieA^\mu)\Phi\ . \quad (20)$$

The $\pi\pi\gamma$ vertex comes from the term $-eA^\mu j_\mu$, where

$$j_\mu=i(\Phi^*\partial_\mu\Phi-\Phi\partial_\mu\Phi^*) \quad (21)$$

is the current density due to the charged pions. Notice that j_μ is conserved irrespective of the specific pion wave function. A plane wave of the form

$$\exp[-i\omega(\mathbf{k})t+i\mathbf{k}\cdot\mathbf{x})]=\exp(-ik^\mu x_\mu)$$

leads to the $\pi\pi\gamma$ vertex in a Feynman diagram of $e(k_1-k_2)^\mu$ as appears in (19). The $1/q^2$ in (19) arises from the photon propagator. The factors of $1/2\omega$ in (18) originate in the normalization of the pion plane waves. Thus, for a plane wave to be normalized properly in a box of volume V, we need

$$\Phi=\frac{e^{-ik^\mu x_\mu}}{\sqrt{2\omega V}}$$

to have

$$\int_V d^3x\,j_0=1\ .$$

These are independent of any assumed relation between ω and $\mathbf{k}$. The Bose-Einstein distribution for the pion is written only in terms of the pion energy irrespective of its momentum. The integration over pion momenta is also independent of the pion dispersion relation as it follows only from the box normalization of plane waves. The terms involving the electron and positron are, of course, not affected by the nuclear matter or by the pion dispersion relation. The various Dirac delta functions simply conserve energy and momentum.

We use the vacuum pion form factor in this paper. On one hand, we expect the pions to annihilate within one pion charge radius, 0.6 fm, of each other. This is a rather short-distance phenomenon in which many-body effects should not play a significant role. On the other hand, the rho mass and width are expected to change with increasing temperature and density reflecting the approach to chiral symmetry restoration.[24] We leave this as an open problem.

Expression (18) can now be evaluated with the final simple form

$$\left.\frac{d^4R_{\pi\pi}^{e^+e^+}}{d^3q\,dM}\right|_{q=0}=\frac{\alpha^2}{3(2\pi)^4}\frac{|F_\pi(M)|^2}{(e^{\omega/T}-1)^2}\times\sum_{\substack{k\\ \text{such that}\\ 2\omega(k)=M}}\frac{k^4}{\omega^4}\left|\frac{d\omega}{dk}\right|^{-1}. \tag{22}$$

Notice that this rate is inversely proportional to the group velocity of the pion, and to the fourth power of k/ω. The group velocity originates in the Jacobian of the transformation from momentum to energy.

The pion dispersion relation in nuclear matter is not known. For purposes of illustration we take the functional form

$$\omega(k)=[(k-k_0)^2+m_0^2]^{1/2}-U\ . \tag{23}$$

This is suggested by the following considerations: (i) The group velocity $d\omega/dk$ should not exceed the velocity of light. (ii) At high momenta, corresponding to short distances, many-body effects should be of secondary importance so that $\omega\to k+\cdots$ as $k\to\infty$. (iii) The energy should first decrease as the momentum increases, corresponding to the strong p-wave attraction, reach a minimum, and then increase with increasing momentum, following (ii).

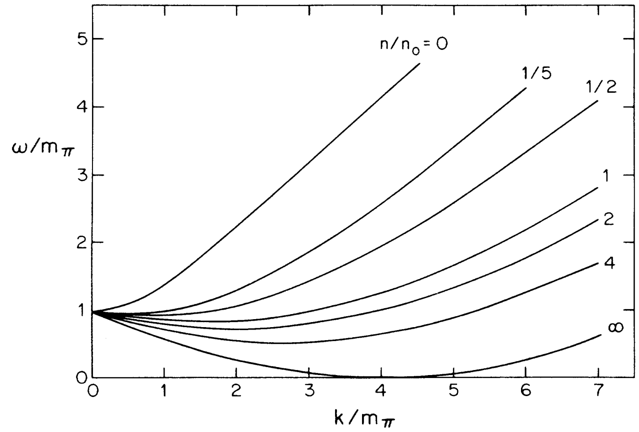


FIG. 7. A possible pion dispersion relation in nuclear matter. From Eqs. (23)–(25).

In general, k_0, m_0, and U are all temperature and density dependent. The strongest dependence should come from the baryon density, since it is the baryons which are primarily altering the pion dispersion relation. The s-wave interaction between pions and nucleons is very weak in comparison with the p-wave interaction. This means that the energy of a pion at rest in nuclear matter should not be much different than m_π. Therefore we restrict

$$(k_0^2+m_0^2)^{1/2}-U=m_\pi\ . \tag{24}$$

In the limit $n\to 0$ we must require that $k_0\to 0$ and $U\to 0$. Studies of *pi*-mesic atoms[20] suggest that when $n=n_0$, normal nuclear matter density, $\omega(k=2m_\pi)-m_\pi=-20$ MeV. We further, and somewhat arbitrarily, impose the condition that $\omega(k=k_0)=0$ when $k_0=4m_\pi$ and $n=\infty$. These requirements are met by the following parametrization:

$$\begin{aligned} m_0/m_\pi&=1+6.5(1-x^{10})\ ,\\ k_0^2/m_\pi^2&=(1-x)^2+2(m_0/m_\pi)(1-x)\ ,\\ x&=e^{-\alpha(n/n_0)},\quad \alpha=0.154\ . \end{aligned} \tag{25}$$

The dispersion relation so obtained is plotted in Fig. 7 for various densities.

V. NUMERICAL ESTIMATES OF THE RATES

Now we shall numerically evaluate the expressions derived in the preceding two sections. The simplest quantity

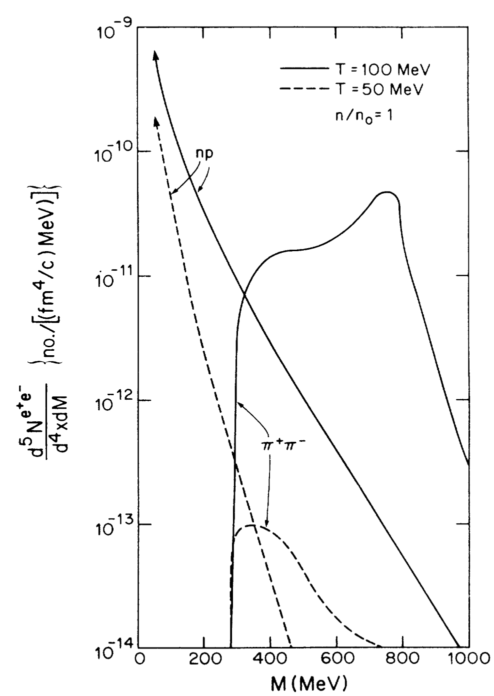
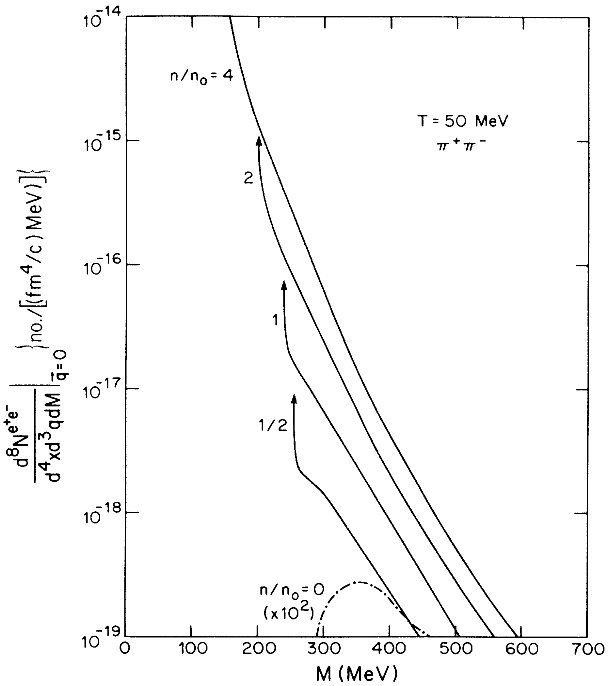


FIG. 8. The thermal rate for producing e^+e^- pairs of invariant mass M at normal nuclear density and at two different temperatures. Contributions from np collisions and from $\pi^+\pi^-$ annihilation (the pions have their free space dispersion relation) are shown separately.



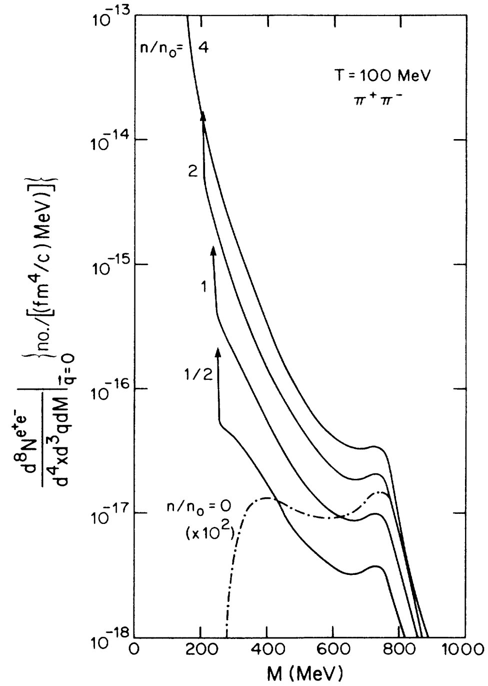


FIG. 9. The thermal rate for producing e^+e^- pairs of invariant mass M and zero total momentum (back-to-back e^+e^-) coming from $\pi^+\pi^-$ annihilation. The pions have the dispersion relation shown in Fig. 7. Note that the rate is a rapidly increasing function of baryon density n, and that the rate for the free space dispersion relation (denoted $n/n_0=0$) is multiplied by 100.

is the invariant mass distribution dR/dM. The contributions from the baryon-baryon cascade and from pion-pion annihilation are shown separately in Fig. 8, for $T=50$ and 100 MeV and $n=n_0$. In this figure the free space pion dispersion relation is used. It is apparent that $\pi^+\pi^-$ annihilation is a rapidly increasing function of T. To see this contribution in a heavy ion collision, clearly it is advantageous to use a high beam energy, perhaps the 1.7—2.1 GeV/nucleon available at the Bevalac or the 10—14 GeV/nucleon available at BNL. The best mass range is $400 < M < 850$ MeV. Note that the background from baryon-baryon cascading scales with density like n^2. To predict more accurately what will actually be seen in a heavy ion collision requires a detailed model for the dynamical evolution of the nuclear matter. A rough estimate would be to take the maximum volume-averaged temperature and density in a hydrodynamic or cascade simulation and multiply by a typical space-time volume of 1000 fm^4/c.

In order to test the impact of the pion dispersion relation on dilepton emission, it is best to examine the mass distribution of back-to-back e^+e^- pairs, $d^4R/d^3q\,dM$ at $\mathbf{q}=0$. We show results in Figs. 9(a) and 9(b) for $T=50$ and 100 MeV, respectively, for a sequence of baryon densities. The most noticeable aspect of these curves is the divergence at the threshold for annihilation. This is a consequence of the inverse relationship between the rate and the group velocity in (22). The minimum pion energy occurs at $k=k_0>0$ when $n>0$, at which point $d\omega/dk=0$ and $\omega(k_0)<m_\pi$, with our parametrization. The threshold mass shrinks with increasing density, as displayed in Fig. 10. The rate for densities $1 \le n/n_0 \le 4$ and for masses $M \le 800$ MeV is 2 or more orders of magnitude greater than for noninteracting pions.

The rate for $\pi^+\pi^-$ annihilation far exceeds the rate for baryon-baryon cascading for $\mathbf{q}=0$ pairs and with the pion dispersion relation of Fig. 9. This is shown in Figs. 11(a) and 11(b). Even if we underestimated the background due to baryon-baryon cascading or other sources by an order of magnitude the signal from $\pi^+\pi^-$ annihilation would

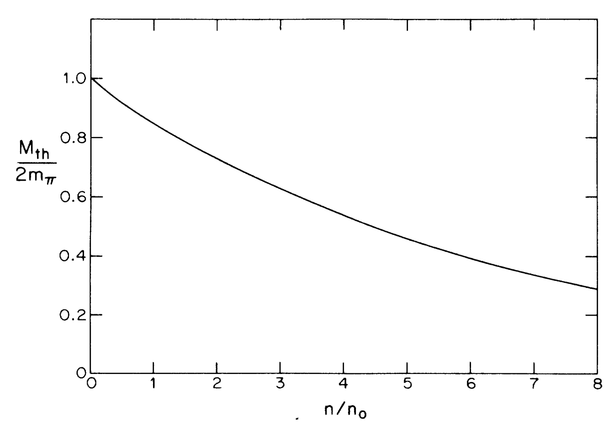
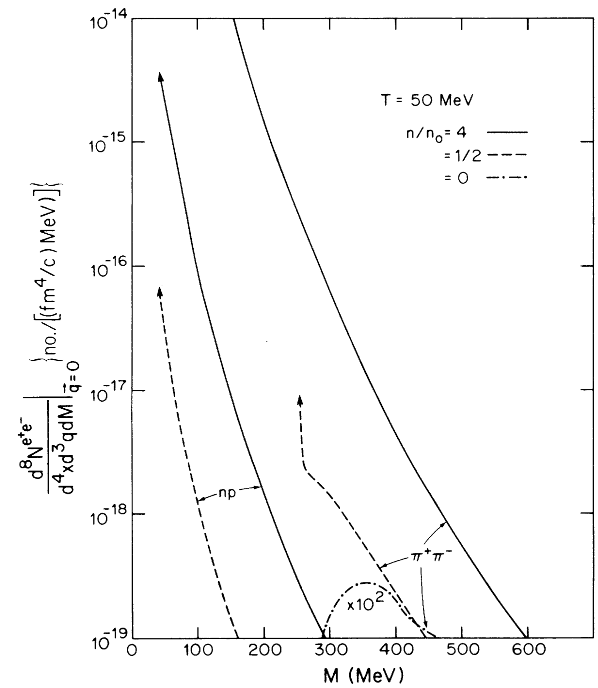


FIG. 10. The threshold mass for dilepton production as a function of baryon density for the pion dispersion relation shown in Fig. 7.



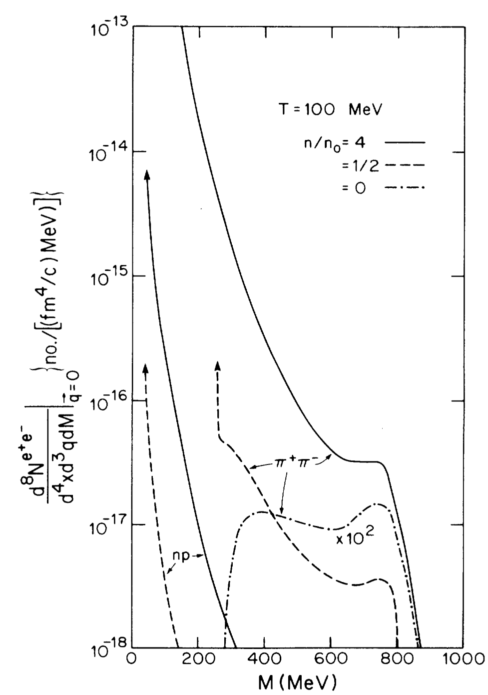


FIG. 11. Comparison of the thermal rates for producing e^+e^- pairs of invariant mass M and zero total momentum coming from np collisions and from $\pi^+\pi^-$ annihilation when the pions have the dispersion relation shown in Fig. 7.

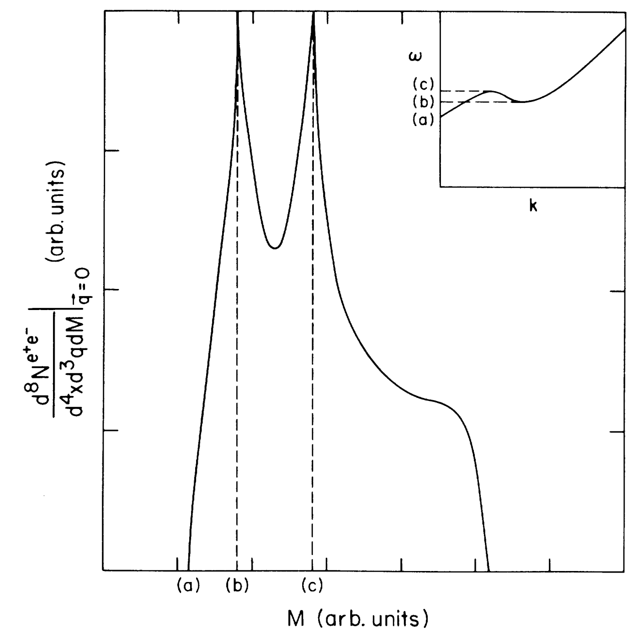


FIG. 12. If pions have the dispersion relation as qualitatively illustrated in the upper right-hand corner, then the rate for producing e^+e^- pairs with mass M and zero total momentum will have two peaks, corresponding to the local minimum and maximum in the dispersion relation.

still shine through. If the pion dispersion relation does have a dip at finite momentum it should be possible to detect it from measurements of the e^+e^- mass distribution at $\mathbf{q}=0$. This is a general conclusion independent of the specific parametrization used in Fig. 7. For example, $\omega(k)$ may have the form sketched in the upper right corner of Fig. 12. Then the mass distribution would have two peaks corresponding to the local minimum and maximum of $\omega(k)$. However, we must realize that in actual heavy ion collisions the peaks will be broadened due to finite system size effects and due to the time evolution of the baryon density.

VI. CONCLUSION

In this paper we have given an introductory account of the rate of dilepton emission from high temperature nuclear matter. Our estimates are based on relativistic kinetic theory. The dominant sources of this type of radiation seem to be two-body elastic and inelastic baryon-baryon collisions and two-pion annihilation. If the pion dispersion relation in nuclear matter has a dip in it due to the strong p-wave attraction to nucleons, then there will be a peak in the mass distribution of back-to-back electrons and positrons located at $M=2\omega_{\min}$ where $\omega_{\min}$ is the pion energy at the dip. If the pion dispersion relation in nuclear matter does not differ much from its free space form, then the dilepton mass distribution will still provide information on the pion to baryon ratio in dense, high temperature nuclear matter. In either case we can learn about pion dynamics in nuclear matter at high temperature and density, which apparently cannot be done any other way.

Improvements are needed in the computation of the cross sections for dilepton production in baryon-baryon collisions. Especially needed is a relaxation of the soft-photon approximation in emission from the external baryon lines, and a calculation for the process shown in Fig. 2. Also missing is a concrete estimate of the cross section for the process $\pi N \rightarrow N e^+ e^-$ illustrated in Fig. 5. Self-consistency should play an important role so as to avoid overcounting the available degrees of freedom in nuclear matter, especially with regards to the delta versus pion-nucleon degrees of freedom.[21]

When we are satisfied that we have adequate knowledge of the elementary processes then they can be used in conjunction with a dynamical calculation of the evolution of nuclear matter in a heavy ion collision (for example, the BUU) to predict absolute distributions. Soon, data should be available from the Dilepton Spectrometer at the Bevalac.[25]

In our calculations we have concentrated on e^+e^- production. The extension to muons poses no difficulty and can readily be done. Theoretically, muons would have an advantage over electrons in that np virtual bremsstrahlung is greatly suppressed, but would be at a definite disadvantage if $\omega(k)$ dropped below $m_\mu = 105$ MeV, so that a portion of the pion dispersion relation could not be seen. In any event electrons will be measured, but muons will not, in upcoming experiments.[25]

It is amusing to recall that a decade ago one of the major hopes was to discover pion condensation[19] in high energy heavy ion collisions.[26] Although it may not be possible to detect a condensate with dileptons either (condensation occurs in the spin-isospin sound branch, not the real pion branch), we will finally be able to study pion dynamics in dense, high temperature nuclear matter without fear that the signal will be absorbed upon expansion.

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