## Last members of the $K^{\pi} = 0^{+}_{4} \alpha$ -cluster rotational band in <sup>20</sup>Ne

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The location of the high spin members of the <sup>20</sup>Ne  $K^{\pi}=0^{4}_{4}$  rotational band is discussed by examining the low energy properties of the unique optical potential describing <sup>16</sup>O( $\alpha, \alpha$ ) elastic scattering on a broad range of energies and angles. In particular, the last  $J^{\pi}=10^{+}$  member is predicted to lie at about 29 MeV excitation energy and to be very broad, in contrast with the current expectations. It is suggested that the measurement of the  $\alpha + {}^{16}O$  fusion excitation function in the relevant energy range could provide a clear signature of the presence of this state. We discuss the implications of our results for the lower spin attributions to this band.

Although the rotational band structure of the <sup>20</sup>Ne nucleus has attracted considerable experimental and theoretical interest for many years, the attribution of experimental levels to some of these bands (for a recent compilation, see Ref. 1), particularly in the high spin region, is still a matter of controversy (see, e.g., Ref. 2 and references therein). The  $K^{\pi}=0^+_1$ ,  $0^-$ , and  $0^+_4$  bands are known to possess distinctive  $\alpha + {}^{16}O$  cluster structure. Beyond the specific spectroscopic interest of the identification of the high spin members of these bands, it is of value—as has recently been recalled by Bromley<sup>3</sup>—to search for these states to investigate the limits of existence of  $\alpha$ -cluster structure at high excitation energy: these states are expected to fragment into several partially overlapping components before they dissolve in the continuum.<sup>4</sup>

The situation is particularly intricate for the positive parity  $K^{\pi}=0_4^+$  band: This band, the members of which are interpreted as corresponding to a radial excitation of the intercluster degree of freedom,<sup>5</sup> and which is sometimes referred to as the "higher nodal" band, is composed of very broad states, the first two (or perhaps three) of which, with<sup>1</sup>  $J^{\pi}=0^+$  and  $E_x \sim 8.3$  MeV,  $J^{\pi}=2^+$  and  $E_x \sim 8.8$  MeV and  $J^{\pi}=4^+$  and  $E_x \sim 10.79$  MeV, are not a matter of controversy; these states manifest themselves in low energy <sup>16</sup>O( $\alpha, \alpha$ ) elastic scattering. As to the high spin members of this band with  $J^{\pi}=6^+$  and  $8^+$ , they are located at "as many excitation energies as there are authors writing on that subject,"<sup>6</sup> and the  $J^{\pi}=10^+$  member, which is expected to be the last member of this band in a cluster model description, has never been identified.

It is the purpose of this Brief Report to demonstrate that it is possible to make definite predictions on the location and width of the missing  $J^{\pi}=10^+$  member of the  $K^{\pi}=0^+_4$  band (as well as to revise the assignments retained in Ref. 1 for its  $J^{\pi}=6^+$  and  $8^+$  members) by examining the low energy properties of the unique optical potential describing  ${}^{16}O(\alpha,\alpha)$  elastic scattering on a broad energy and angular range,<sup>7</sup> and to suggest how this state could possibly be identified experimentally.

The potential extracted in Ref. 7, which is well determined in a wide radial range because of the particularly low absorption displayed by the  $\alpha + {}^{16}O$  system, provides a satisfactory description of the average angular distributions and excitation functions down to  $E_{\alpha} \simeq 15$  MeV. In particular, it reproduces nicely the broad, several MeV wide bump seen at large angles around  $E_{\alpha} \simeq 20$  MeV; this bump is actually split into several overlapping  $(\Gamma \sim 500 \text{ keV})$  in intermediate-width components the experimental excitation functions. The  ${}^{16}O({}^{6}Li,d){}^{20}Ne^{*}(\alpha){}^{16}O(g.s.)$  coincidence measurements of Artemov et al.<sup>4</sup> at  $E(^{6}Li) = 35$  MeV reveal a close similarity between the elastic scattering and d- $\alpha$  coincidence excitation functions; moreover, the analysis of the d- $\alpha$  angular correlation angular distributions for  $E(^{6}Li) = 45$ MeV shows<sup>8</sup> that several of these components share the same spin, three of the strongest components having  $J^{\pi} = 8^+$ .

The phase shifts<sup>7</sup> generated by the global potential indeed reveal that the broad bump predicted around 20 MeV is due to a broad l=8 resonance (confirming earlier findings by Ohkubo *et al.*<sup>9</sup>) which belongs to a quasirotational sequence of broad resonances with even l and principal quantum number N=10. In view of the structure of their wave functions, these states can be associated in a natural way with those of the  $K^{\pi}=0_{4}^{+}$  experimental band in <sup>20</sup>Ne; the first ones nearly coincide in absolute energy and width with their experimental counterparts. The potential predicts the  $K^{\pi}=0_{4}^{+}$  band termination at  $J^{\pi}=10^{+}$ , as a broad state located around 29 MeV excitation energy. However, the increase of absorption found between  $E_{\alpha}=20$  and 30 MeV makes its presence in the calculated elastic excitation function much less conspicu-

As a matter of fact, a rather broad ( $\Gamma \sim 700 \text{ keV}$ ) resonance with  $J^{\pi} = 10^+$  has been identified by Bergman and Hobbie<sup>10</sup> at  $E_{\alpha} = 29.4$  MeV ( $E_x = 28.2$  MeV). Independent evidence for the same state is provided by the d- $\alpha$  correlation measurements in the  ${}^{16}O({}^{6}Li,d){}^{20}Ne^{*}(\alpha){}^{16}O(g.s.)$  reaction performed by Artemov et al.<sup>11</sup> at  $E({}^{6}\text{Li}) = 57.8$  MeV; moreover, the same reaction leads to the observation of a whole series of rather wide, overlapping states in the excitation energy range  $E_x = 29 - 36$  MeV, all of which share the same  $J^{\pi} = 10^+$ assignment: the d- $\alpha$  correlation spectrum displays a characteristic  $[P_{10}(\cos\theta)]^2$  behavior throughout this energy range, as does the combined correlation function for the entire spectrum in the same range.<sup>11</sup> Measurements carried out at still higher <sup>6</sup>Li energies<sup>12</sup> reveal no further evidence for  $\alpha$ -cluster structure above  $E_x = 35$  MeV. Quite recently, a preliminary analysis<sup>13</sup> of coincidence measurements in the <sup>12</sup>C(<sup>16</sup>O,<sup>20</sup>Ne)<sup>8</sup>Be transfer reaction has revealed the presence of  $J^{\pi} = 10^+$  strength around  $E_x \simeq 28$  MeV.

Still another possibility of obtaining positive evidence for this  $J^{\pi} = 10^+$  state could be provided by the measurement of the  $\alpha + {}^{16}O$  fusion excitation function in the relevant energy range. We have estimated the  $\alpha + {}^{16}O$ fusion cross section between  $E_{\alpha} = 15$  and 40 MeV in the spirit of the model of Hatogai et al.,<sup>14</sup> i.e., by reducing the radius of the imaginary part of the global  ${}^{16}O(\alpha,\alpha)$ optical potential; this type of calculation has recently been applied to the calculation of the  $\alpha + {}^{40}$ Ca fusion cross section<sup>15</sup> and proved instrumental in solving the long-standing problem of the  $\alpha$ -cluster structure of <sup>44</sup>Ti.<sup>16</sup> The calculations were carried out using as the value of the parameter controlling the real potential depth (denoted as  $\alpha$ in Ref. 7) the one fitting the 32.2 MeV elastic scattering data, i.e.,  $\alpha = 3.407$ . For elastic scattering and reaction cross section calculations, we used for the parameter describing the energy behavior of the imaginary potential (which is the imaginary radius  $R_w$ ) a quadratic interpolation of the values  $R_w = 2.000$ , 3.122, and 3.536 fm assumed by this parameter at  $E_{\alpha} = 21.5$ , 32.2, and 39.3 MeV, respectively. For fusion calculations the imaginary radius was decreased according to two different prescriptions, to investigate the dependence of the predictions on the details of the fustion absorptive potential. The first prescription consists in defining the fusion imaginary radius  $R_w^F$  by subtracting a constant amount from the optical model imaginary radius  $R_w$ , while  $R_w^F$  is taken as a constant fraction of  $R_w$  in the second prescription.

The results of these calculations, for several values of the parameter defining the fusion imaginary potential within each prescription, are presented in Fig. 1, together with the reaction cross section calculated with the original absorption. This figure shows that the oscillations, which are already present in the reaction cross section excitation function  $\sigma_R(E)$ , tend to be enhanced in the fusion excitation function  $\sigma_F(E)$  as a result of the lower absorption used in the fusion calculation. Examination of the energy behavior of the partial fusion excitation functions  $\sigma_F^I(E)$ reveals that the two bumps located at about 20 and 30 MeV incident energy are, as expected, due to l=8 and l = 10 resonant contributions, respectively. As—whatever the details of the absorption used for calculating it-the  $J^{\pi} = 10^+$  fusion excitation function bump has a much more favorable peak-to-valley ratio than was the case in the elastic channel, fusion cross section measurements thus appear to be an interesting alternative for searching for the missing member of the  $K^{\pi} = 0_4^{+20}$ Ne rotational band. Moreover, the fusion channel offers better prospects than the elastic channel since the fusion cross section appears as an incoherent sum over the incident angular momenta, whereas the elastic scattering cross section involves a coherent sum with stronger interference effects among the various l: The fusion excitation function is thus expected to be less sensitive to the presence of nonpotential resonances and to provide a clearer signature of the presence of the l = 10 resonance.

As mentioned in the Introduction, the present assignments to the  $K^{\pi}=0^+_4$  band disagree with those retained in the most recent compilation<sup>1</sup> beyond the  $J^{\pi}=4^+$  ( $E_x=10.79$  MeV) state. Indeed, whereas the  $J^{\pi}=4^+$ 



FIG. 1. (a) Reaction excitation function  $\sigma_R$  (thick line) and fusion excitation functions  $\sigma_F$  obtained by substracting a constant value  $\rho$  (given in fm) from the imaginary radius  $R_w$  used in the reaction cross section calculation (see text); (b) same as (a), but  $R_w$  is multiplied by the constant factor  $\kappa$  at each energy.

member of this band is predicted to lie near the experimental energy with a reasonable estimate for its width, the potential locates the  $J^{\pi}=6^+$  state around  $E_x \sim 14-15$ MeV and predicts a width ( $\Gamma \sim 1-2$  MeV) much larger than that of the  $E_x = 12.56$  MeV,  $\Gamma_{c.m.} = 88$  keV state re-tained in Ref. 1. As for the  $J^{\pi} = 8^+$  member, Ref. 1 locates it at  $E_x = 17.30$  MeV with  $\Gamma_{c.m.} \simeq 52$  keV, whereas our calculations predict this state at about  $E_x \simeq 21$  MeV with a width of some 2 MeV's. Our classification makes the moment of inertia of the  $K^{\pi} = 0_4^+$  band very similar to that of the  $K^{\pi} = 0^{-}$  band, in contrast with the attributions of Ref. 1, which imply a crossing of these bands and a moment of inertia for the  $K^{\pi} = 0_4^+$  band considerably larger than that of the  $0^-$  band (see Fig. 2). Extrapolation to  $J^{\pi} = 10^+$  of the band built from the states retained in Ref. 1 would predict this state to lie at about 23 MeV excitation energy<sup>3</sup> (cf. Fig. 2), i.e., about 6 MeV below our predictions; if our picture is the correct one, it is therefore not surprising that no state with the expected quantum numbers has ever been found in the vicinity of this excitation energy. We like to note that our conclusions are in essential agreement with those of most existing  $\alpha + {}^{16}O$ resonating group (and more sophisticated) calculations,<sup>5</sup> in that these calculations invariably predict widths of several MeV for the states of the <sup>20</sup>Ne  $K^{\pi} = 0_4^+$  band, as has been emphasized, e.g., by Fujiwara et al. (see Ref. 5, p. 129).

In conclusion, we have shown—on the basis of the low energy properties of the unique  ${}^{16}O(\alpha,\alpha)$  optical model potential of Ref. 7—that the  $J^{\pi}=10^+$  member of the  $K^{\pi}=0^+_4$  rotational band of  ${}^{20}$ Ne should be located at a higher excitation energy ( $E_x \sim 29$  MeV) than is generally expected, and have a width of several MeV; a measurement of the  $\alpha + {}^{16}O$  fusion excitation function in the

- <sup>1</sup>F. Ajzenberg-Selove, Nucl. Phys. A392, 1 (1983).
- <sup>2</sup>H. T. Richards, Phys. Rev. C 29, 276 (1984).
- <sup>3</sup>D. A. Bromley, in *Proceedings of the Fourth International Conference on Clustering Aspects of Nuclear Structure and Nuclear Reactions*, Chester, United Kingdom, edited by J. S. Lilley and M. A. Nagarajan (Reidel, Dordrecht, 1985), p. 1.
- <sup>4</sup>K. P. Artemov et al., Nucl. Phys. A320, 479 (1979).
- <sup>5</sup>Y. Fujiwara *et al.*, Prog. Theor. Phys. Suppl. **68**, 29 (1980), and references quoted therein.
- <sup>6</sup>M. W. Buser, Helv. Phys. Acta 54, 439 (1981).
- <sup>7</sup>F. Michel et al., Phys. Rev. C 28, 1904 (1983).
- <sup>8</sup>K. P. Artemov *et al.*, Yad. Fiz. **23**, 489 (1976) [Sov. J. Nucl. Phys. **23**, 257 (1976)].
- <sup>9</sup>S. Ohkubo, Y. Kondo, and S. Nagata, Prog. Theor. Phys. 57, 82 (1977).



FIG. 2.  $K^{\pi}=0_{1}^{+}$ ,  $0^{-}$  and  $0_{4}^{+}$  rotational bands in  ${}^{20}$ Ne. The states of the  $K^{\pi}=0_{1}^{+}$  and  $0^{-}$  bands, as well as the  $J^{\pi}=0^{+}$ ,  $2^{+}$  and  $4^{+}$  members of the  $K^{\pi}=0_{4}^{+}$  band, are those retained in Ref. 1 (the vertical bars represent their widths). The  $J^{\pi}=[6^{+}]$  and  $[8^{+}]$  states, taken from Ref. 1, are those whose attribution is questioned in the text; they are linked to the low spin members of the  $K^{\pi}=0_{4}^{+}$  band with a dashed line. The  $J^{\pi}=6^{+}$ ,  $8^{+}$  and  $10^{+}$  states, linked with a solid line, are our  $K^{\pi}=0_{4}^{+}$  candidates; the vertical bars provide an estimate of their widths.

relevant energy range offers good prospects for experimentally detecting this state. We have shown likewise that the  $J^{\pi} = 6^+$  and  $8^+$  members of the same band must be located higher in energy and have a width considerably larger than the experimental states retained in the current attribution.

- <sup>10</sup>C. Bergman and R. K. Hobbie, Phys. Rev. C 3, 1729 (1971).
- <sup>11</sup>K. P. Artemov *et al.*, Yad Fiz. **26**, 9 (1977) [Sov. J. Nucl. Phys. **26**, 4 (1977)].
- <sup>12</sup>K. P. Artemov *et al.*, Yad. Fiz. **36**, 1345 (1982) [Sov. J. Nucl. Phys. **36**, 779 (1982)].
- <sup>13</sup>S. C. Allcock *et al.*, in Proceedings of the International Nuclear Physics Conference, Harrogate, United Kingdom, 1986, Contributed Papers (unpublished), p. 46.
- <sup>14</sup>K. Hatogai, M. Ohta, and S. Okai, Prog. Theor. Phys. 68, 2014 (1982).
- <sup>15</sup>F. Michel, G. Reidemeister, and S. Ohkubo, Phys. Rev. C 34, 1248 (1986).
- <sup>16</sup>F. Michel, G. Reidemeister, and S. Ohkubo, Phys. Rev. Lett. 57, 1215 (1986).