Non-hedgehog ansatz and the role of the ω meson in the chiral soliton models

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We propose a non-"hedgehog" ansatz for the pion in two types of quark models, numbers I and II, which are based on the linear and nonlinear σ model, respectively. With the new ansatz, we find the equations of motion can be solved exactly in model I and approximately in model II. We show that by introducing the ω-quark interaction, the predictions of g_A, $g_{\pi NN}$, and μ_p are significantly improved in a case within model II.

At present, the quantum chromodynamics (QCD) is believed to be the basic theory of the strong interaction. The MIT bag model,[1] soliton model,[2] and Skyrme model[3] are amongst the best known phenomenological models of QCD. There have been some attempts[4,5] to reason or justify the models from QCD. The most prominent one may be the series of works by Cahill and his collaborators.[5] They have derived from QCD, by functional-integral methods, several phenomenological models of hadrons including the MIT bag model,[1] the soliton model of Friedberg and Lee,[2] the chiral soliton model of Birse and Banerjee,[6] and the Skyrme-Witten model.[3] One of the most interesting results of Cahill's is that all sorts of meson-quark interactions, e.g., the interactions with π, ρ, ω, etc., are self-contained in the theory. Therefore, it seems to be reasonable to study the chiral soliton models with various types of mesons in the sense that we are working with the models which are derived from QCD.

The chiral soliton models which have been studied by many groups[6-9] are based on the σ model.[10] In the quark model based on the σ model, the pion is described in two ways, as π_λ in the linear σ model (model I),[6-8,11] or ϕ_λ in the nonlinear σ model (model II).[8,9,12-16] Model I includes the chiral soliton[6,7] and bag models,[11] while model II includes the nonlinear chiral soliton,[8,9] bag,[14,15] and potential[16] models. To solve models I and II, the "hedgehog" pion of the form

$$\pi_\lambda(r),\ \ \phi_\lambda(r)=\hat{\mathbf{r}}_\lambda h(r)\ , \tag{1}$$

and the "hedgehog" baryon $|h_g\rangle$, which satisfies $(\sigma_\lambda+\tau_\lambda)|h_g\rangle=0$, have been exclusively used in the mean field approximation (MFA).[6,17] It has been proved[18] that form (1) is unique for $\phi_\lambda(r)$ in the nonlinear chiral bag model (when the baryon is assumed to be $|h_g\rangle$). In model II, another form of the pion,

$$\phi_\lambda(r)=\sum_{i=1}^{3}\sigma^i\cdot\hat{\mathbf{r}}\,\tau_\lambda^i h(r)\ , \tag{2}$$

has been used[12,13,15,16] by expanding the nonlinear pion-quark coupling terms perturbatively. In using form (2), the baryon $|B\rangle$ is assumed to be SU(4) symmetric in the spin-isospin space. Curiously, however, form (2) has never been applied to $\pi_\lambda(r)$. It seems to be generally believed that the chiral soliton or bag models cannot be solved exactly without the "hedgehog" ansatz [form (1)]. However, as will be shown later in Eq. (7), this is incorrect for model I.

There are threee advantages in using form (2). Firstly, the degeneracy of the nucleon (N) and $\Delta(1232)$ can easily be resolved. In the case of employing form (1), one needs further approximations and assumptions[19] in order to handle the degeneracy, and the result depends on the prescription.[19] Secondly, form (2) is also applicable to the SU(6) case, which includes the strange baryons, while form (1) is by definition valid only for the nonstrange baryons. Thirdly, the comparisons with the quantum theory of the chiral bag model are expected to be more direct because (a) the baryon retains the same spin-isospin symmetry, and (b) the quantum pion from a static source can be written in form (2). In the following discussions we take the linear and nonlinear chiral soliton models as examples of models I and II, respectively.

The linear chiral soliton model Lagrangian consists of two parts, i.e., $\mathscr{L}=\mathscr{L}_s+\mathscr{L}_b$. The chirally symmetric part is written as[6-8,10]

$$\mathscr{L}_s=\bar{\psi}[\tfrac{1}{2}i\overleftrightarrow{\partial}-g(\underline{\sigma}+i\underline{\pi}_\lambda\tau_\lambda\gamma_5)]\psi+\tfrac{1}{2}[(\partial_\mu\underline{\pi}_\lambda)^2+(\partial_\mu\underline{\sigma})^2]-U(\underline{\sigma},\underline{\pi}_\lambda)\ , \tag{3}$$

where ψ, $\underline{\sigma}$, and $\underline{\pi}_\lambda$ are the field operators, and $U(\underline{\sigma},\underline{\pi}_\lambda)$ is the potential. In this paper, the chiral symmetry of the fields around the hadron is assumed to be realized both inside and outside in the Wigner mode. Following the introduction of the chiral symmetry breaking term[6,10] $\mathscr{L}_b=-f_\pi m_\pi^2\underline{\sigma}$, the partially conserved axial vector current (PCAC) condition, $\partial_\mu A_\lambda^\mu=f_\pi m_\pi^2\underline{\pi}_\lambda$, is derived directly from the Lagrangian.

In the quasiclassical formulation,[2] one can decompose the fields $\underline{\sigma}$ and $\underline{\pi}_\lambda$ into two parts,

$$\underline{\sigma}(\mathbf{r},t)=\sigma(\mathbf{r})+\underline{\sigma}'(\mathbf{r},t)\ , \tag{4a}$$

$$\underline{\pi}_\lambda(\mathbf{r},t)=\pi_\lambda(\mathbf{r})+\underline{\pi}'_\lambda(\mathbf{r},t)\ , \tag{4b}$$

where $\sigma(\mathbf{r})$ and $\pi_\lambda(\mathbf{r})$ are the time independent and c-number functions that satisfy $\sigma\to0$ and $\pi_\lambda\to0$ as $r\to\infty$. In accordance with this decomposition, one can split the Hamiltonian into two parts:[2] $H=H_{\rm qcl}+H_{\rm corr}$, where

$H_{\rm qcl}$ denotes the quasiclassical part and $H_{\rm corr}$ the quantum correction. The simplest choice for σ and π_λ in Eq. (4) in the MFA is given by[6] the expectation values of $\underline{\sigma}$ and $\underline{\pi}_\lambda$ in $|h_g\rangle$:

$$\sigma(\mathbf{r})=\langle h_g\,|\,\underline{\sigma}(\mathbf{r},t)\,|\,h_g\rangle\ ,$$

$$\pi_\lambda(\mathbf{r})=\langle h_g\,|\,\underline{\pi}_\lambda(\mathbf{r},t)\,|\,h_g\rangle\ .$$

In this case π_λ becomes form (1). Therefore, the Hamiltonian can now be decomposed as follows: $H=H_{\rm MFA}+H'_{\rm corr}$, where $H_{\rm MFA}$ is derived by replacing $\underline{\sigma}$ and $\underline{\pi}_\lambda$ in H with σ and π_λ in the MFA.

Let us depart for a moment from form (1). A general way of solving model I with a nonspherical pion ansatz could be to adopt the relativistic Hartree-Fock (RHF) theory[20] that has been applied to nuclear systems. In the RHF theory the meson fields are eliminated, and thus one has to solve coupled nonlinear equations. In this paper we solve the equations of motion in a similar spirit, but without elmininating $\underline{\sigma}$ and $\underline{\pi}_\lambda$. In contrast to the above-mentioned quasiclassical approach,[2,6,8,17] we introduce a new decomposition for $\underline{\pi}_\lambda$:

$$\underline{\pi}_\lambda(\mathbf{r},t)=\underline{\pi}^0_\lambda(\mathbf{r})+\underline{\pi}''_\lambda(\mathbf{r},t)\ , \tag{5}$$

while $\underline{\sigma}$ is decomposed as in Eq. (4a). The time-independent and q-number function $\underline{\pi}^0_\lambda(\mathbf{r})$ may be derived by solving the equation of motion for $\underline{\pi}_\lambda$, which is subject to the bound state boundary condition. We can achieve this indirectly by keeping in mind the energy minimization which will be mentioned later. We use form (2) as a trial function for $\underline{\pi}^0_\lambda(\mathbf{r})$, where $h(r)\to 0$ as $r\to\infty$. Due to Eq. (5), the Hamiltonian can now be split into two parts: $H=H_s+H''_{\rm corr}$, where the static part H_s is derived by replacing $\underline{\sigma}$ and $\underline{\pi}_\lambda$ in H with σ and $\underline{\pi}^0_\lambda$.

It could be thought that form (2) be introduced within the MFA by (1) replacing $|h_g\rangle$ with $|B\rangle$ to define σ and π_λ, where $B=$ N or Δ, and (2) defining a time-independent c-number function

$$\pi_\lambda(r)=-\left\langle B\left|\sum_i\sigma_i\tau_i h(r)\right|B\right\rangle$$

[which becomes form (1) if $|B\rangle=|h_g\rangle$]. In the RHF theory[20] the term which corresponds to $\bar\psi\gamma_5\tau_\lambda\pi_\lambda\psi$ in Eq. (3) is the baryon self-energy. If one introduces π_λ in the above-mentioned way, then the intermediate baryon in the self-energy term is limited to the N when $B=$ N. Therefore the completeness in SU(4) is lost. Also, this is not the case for $\phi_\lambda(r)$ as defined in Refs. 13, 15, and 16. We shall therefore exclude this approach.

Some may wonder which ansatz [(1) or (2)] is preferred and whether this question could be answered from the least action principle. However, for the following reason, the answer is negative: $|h_g\rangle$ is not a physical baryon in the sense that it is not the eigenstate of the physical observable. To derive the physical observable, one has to go beyond the MFA. This means that the time-dependent $H'_{\rm corr}$ is responsible for the splitting of the N and Δ.[19] For the new ansatz, on the other hand, the N and Δ are the two nondegenerate eigenstates of H_s, which is time independent.

Taking the variation

$$\delta\left\langle B\left|\int d^3x(\mathscr{H}-E\psi^\dagger\psi)\right|B\right\rangle=0 \tag{6}$$

with respect to G, F, $\sigma(r)$, and $h(r)$, where G and F are the upper and lower components of the radial wave function of ψ, one derives the following set of four equations:

$$G'+g\left[\sigma F+\frac{\alpha}{3}hG\right]-EF=0\ , \tag{7a}$$

$$F'+\frac{2}{r}F+g\left[\sigma G-\frac{\alpha}{3}hF\right]+EG=0\ , \tag{7b}$$

$$\sigma''+\frac{2}{r}\sigma'=-3g(G^2-F^2)+\frac{\partial}{\partial\sigma}\widetilde{U}(\sigma,h)+f_\pi m_\pi^2\ , \tag{7c}$$

$$h''+\frac{2}{r}h'-\frac{2}{r}h-m_\pi^2 h=2\gamma gGF+\frac{1}{\alpha}\frac{\partial}{\partial h}\widetilde{U}(\sigma,h)-m_\pi^2 h\ . \tag{7d}$$

In the case of the "hedgehog" pion [form (1)], $\widetilde{U}(\sigma,h)=U(\sigma,h)$, $\gamma=3$, and $\alpha=1$. In the case of the non-"hedgehog" pion [form (2)], $\widetilde{U}(\sigma,h)=\langle B\,|\,U(\underline{\sigma},\underline{\pi}_\lambda)\,|\,B\rangle$, $\gamma=1$, and $\alpha=\Sigma/3$, where $\Sigma=57$ for the N and $\Sigma=33$ for the Δ.

In this model the axial vector coupling constant may be expressed as

$$g_A=\beta\left[\gamma\left[1-\tfrac{4}{3}\int dr\,v^2\right]+\frac{8\pi}{3}\int dr\,r^2\left[h\sigma'-\sigma h'-\frac{2\sigma h}{r}\right]\right]\ , \tag{8}$$

where $\beta=1$ and $\frac{5}{3}$ for forms (1) and (2), respectively, and γ is the same as in Eq. (7). In Eqs. (8) and (13)–(15), u and v are defined as $u(r)=\sqrt{4\pi}rG(r)$ and $v(r)=\sqrt{4\pi}rF(r)$. With the aid of Eq. (7d), the pion-nucleon coupling constant $g_{\pi\rm NN}$ can be written as

$$\frac{g_{\pi\rm NN}}{2m_{\rm N}}=\frac{4\pi\beta}{3}\int dr\,r^3\left[-h''-\frac{2}{r}h'+\frac{2}{r^2}h+m_\pi^2 h\right]\ . \tag{9}$$

Let us now look at the nonlinear chiral soliton model. Here, $\phi_\lambda(r)$ can be introduced by a transformation, e.g.,[8]

$$\pi_\lambda/f_\pi=\hat{\boldsymbol{\phi}}_\lambda\sin(\phi/f_\pi)\ ,\quad \hat{\boldsymbol{\phi}}_\lambda\equiv(\boldsymbol{\phi}/|\boldsymbol{\phi}|)_\lambda \tag{10a}$$

$$\sigma/f_\pi=\cos(\phi/f_\pi)\ , \tag{10b}$$

where $\phi=\sqrt{\boldsymbol{\phi}\cdot\boldsymbol{\phi}}$ and f_π is the pion decay constant. [This transformation is unique only when the topological winding number $Z=\phi(0)/(\pi f_\pi)$ is specified.[8]] By applying Eq. (10) to Eq. (3), one derives the following Lagrangian,

$$\mathscr{L}_s=\bar\psi[\tfrac{1}{2}i\overleftrightarrow{\partial}-gf_\pi\exp(i\gamma_5\tau_\lambda\phi_\lambda/f_\pi)]\psi+\tfrac{1}{2}(D_\mu\phi_\lambda)^2\ . \tag{11}$$

The covariant derivative[15] D_μ is related to the normal derivative as $D_\mu=\partial_\mu+0(\phi_\lambda^3)$. Correspondingly, the chiral symmetry breaking term and the PCAC condition become $\mathscr{L}_b=-m_\pi^2\phi_\lambda^2/2+O(\phi_\lambda^4)$ and $\partial_\mu A^\mu_\lambda=f_\pi m_\pi^2\phi_\lambda+O(\phi_\lambda^2)$, respectively.[15] Only with form (1) can model II be solved exactly.

Before describing the details of model II, let us review the compatibility of $g_{\pi\rm NN}$, g_A, and $\mu_{\rm p}$ in models I and II. Since the model Lagrangians are chirally invariant, one expects that the Goldberger-Treiman (GT) relation holds even in the MFA.[6] Thus, the ratio $g_A/g_{\pi NN}$ is compatible with the experimental value within a few 10%.[6,12–16,18] However, the absolute values of $g_{\pi NN}$ and g_A may vary according to the details of the model. For example, the surface effect, due to the different phases inside and outside the hadron in the nontopological ($Z=0$) chiral bag models[13,15] within model II, causes the nonzero pionic contribution to g_A, i.e., $g_A(\phi)\neq 0$. This leads to too large a g_A, resulting in too large a $g_{\pi NN}$ due to the GT relation. In the same types of models[12,25] one can let $g_A(\phi)=0$ by allowing the pion also inside the hadron. This may bring g_A down to a reasonable value. But, in general, $g_A(\pi/\phi)\neq 0$ even when the pion is allowed to be inside, as shown in the second term of Eq. (8). In fact, in the model of Birse and Banerjee[6] (where $Z=1$), $g_A(\pi)$ reaches about 40% of the total g_A, which is about 50% larger than its empirical value. It seems reasonable to speculate that g_A could become a reasonable value if one constructs a theory where $g_A(\pi/\phi)$ is negligible in comparison with g_A. Later, we are going to show that $g_A(\pi/\phi)$ can become smaller if the ω meson is introduced into the theory.

Another unwanted result of the chiral models of the N is too small a Magnetic Moment (MM). For example, the proton MM is about 70% and 50% of its experimental value in the static MIT bag model,[1] for the bag radius $R=1$ fm, and in the soliton model,[17] respectively. It is generally advocated that the defect can be improved by introducing the pionic correction and the center-of-mass (c.m.) correction. In the present semiclassical approach, $\mu_p(Q)/\mu_n(Q)=-\frac{3}{2}$ because of the SU(4) symmetry in the spin-isospin space. The total MM of the proton and the neutron are expressed as $\mu_p=\mu_p(Q)+\mu_\pi$ and $\mu_n=\mu_n(Q)-\mu_\pi$, where μ_π is the pionic current contribution. The above three equations, together with the experimental values of μ_p and μ_n, require that $\mu_p(Q)=2.64(e/2m_N)$, $\mu_n(Q)=-1.76(e/2m_N)$, and $\mu_\pi=0.15(e/2m_N)$. Therefore, the quark part should be dominant in the MM, provided the quantum effect is small. There is no unambiguous way of handling the c.m. correction.[21] It can be shown that, to the order of p/m_N, the c.m. correction of the MM, due to the projection method,[22] is already included[23] in the above calculation. However, the c.m. correction of the MM is shown to be negative[24] in a theory where the relativistic motion of the quark is taken into account. In the following discussion we will show that the too small a MM can be improved by introducing the ω meson into the theory.

To clearly see the effect of the ω meson in the MM, let us try the following model:[16]

$$\mathscr{L}=\bar{\psi}[\tfrac{1}{2}i\overleftrightarrow{\partial}-V_s\exp(i\gamma_5\tau_\lambda\phi_\lambda/f_\pi)-\gamma_0V_v]\psi+\tfrac{1}{2}(D_\mu\phi_\lambda)^2-\tfrac{1}{2}(m_\pi\phi_\lambda)^2 . \quad (12)$$

This Lagrangian can be obtained immediately from Eq. (11). If one replaces V_s with $\delta(r-R)/2$ and sets $V_v=0$, then Eq. (12) becomes the cloudy bag model (Ref. 12) Lagrangian. The V_v term may be attributed to the time component of the ω-quark interaction. The space component of the ω-quark interaction turns out to be zero because the source term is zero whether or not one uses $|h_g\rangle$ or $|B\rangle$. The time component of the ρ-quark interaction is zero if one uses $|h_g\rangle$. It should also be noted here that the time component of the Abelian piece of the gluon-quark interaction is zero[1] in the static approximation.

With the non-"hedgehog" ansatz [form (2)], this model cannot be solved exactly, due to the nonlinearity in the exponential and covariant terms. In our previous paper,[16] the ϕ and ϕ^2 terms were treated exactly, and the $\phi^n(n\geq 3)$ terms were treated approximately by averaging them in a certain manner. In that paper we found that the contribution of the ϕ^n ($n\geq 3$) terms was unimportant but that the ϕ and ϕ^2 terms drastically changed the result[25] which was obtained without the pion distortion. (We refer the readers who are interested in the quantitative results to Ref. 16.) In Eq. (12) we drop the $\phi^n(n\geq 3)$ terms altogether for the present purpose of qualitative discussions. Unlike the perturbative approaches of others,[12,13,15] we solve the equations of motion self-consistently. The equations of motion are derived from the variation of the expectation value of H, with respect to F, G, and h [in form (2)], as in Eq. (6).

In a similar manner exploited by Tegen *et al.*,[25] the quark part of the MM can be related to $g_A(Q)$ as

$$\mu_p(Q)=\frac{m_N}{2E}\left[1+\tfrac{3}{5}g_A(Q)+\frac{4\alpha}{3f_\pi}\int dr\, rV_s(u^2+v^2)-\tfrac{8}{3}\int dr\, rV_v uv\right] . \quad (13)$$

One can observe from Eq. (13) that the value of the proton MM has increased because of the following two reasons. Firstly, the quark eigenenergy E becomes smaller. This is due to the fact that the ω field energy is added to the pion field energy and $3E$ in order to equal the baryon mass. Secondly, the third and fourth terms in Eq. (13) are all positive, because V_s, V_v, and u are positive quantities while v is a negative quantity. In our previous paper[16] it was demonstrated numerically that the MM was significantly improved by including V_v, even though the ω field energy was not taken into account.

From the set of equations of motion for the quark, one can find the equality

$$\int dr\, rV_s uv=\int dr\, v^2-\frac{3}{4}+\frac{127}{6f_\pi^2}\int dr\, rV_s h^2 uv . \quad (14)$$

Using Eqs. (9) and (14) and the equation of motion for the pion, one can derive the following relation:

$$\frac{g_{\pi NN}}{m_N}=\frac{g_A(Q)}{f_\pi}+\frac{1270}{9f_\pi^2}\int dr\, rV_s h\left[\frac{u^2-v^2}{19}-\tfrac{1}{3}uvh\right] . \quad (15)$$

Note that $g_A(\phi)=0$ in this model. It may also be worthwhile here to note the following two points: (i) the vector coupling does not affect the GT relation; (ii) in Eq. (15), the term involving the integration is a positive quantity, and thus this extra term may serve to fill the small gap between the experimental result and the theoretical

GT relation. In our previous paper[16] we showed numerically that g_A and $g_{\pi NN}$ can be fitted exactly to the experimental values. This was done by choosing the forms V_s and V_v, even though Eqs. (13) and (15) and the direct relation of V_v to the ω field were not considered.

In this paper we regard the ω meson as a clue to help improve the chiral models which give too large a g_A and too small a MM. However, it may be worthwhile to note here that the ω meson has previously been introduced in order to stabilize the chiral bag[26] and chiral soliton models.[27] There are also some arguments[27,28] that the quartic term in the Skyrme model represents the degree of freedom of the ω meson. It should be noted that the relativistic quark model,[29] employing a potential of the form $(1+\gamma_0)V$, is able to give an overall agreement of g_A, r_p, and μ_p with their experimental values, although the form is chosen so as to facilitate the computation analytically. Together with the reasons noted before, we may deduce that the ω meson is a necessary and important ingredient in the quark model of the nucleon among the mesons suggested[5] by QCD.

Except for the chiral bag models, it is necessary to ensure that the quarks are confined in models I and II. This is achieved by solving the classical solutions self-consistently.[6–8,16] In the nonlinear chiral soliton (or σ) model, we do not have any measure to appreciate the convergence of the perturbative calculations, unlike the chiral bag model, where the bag radius is the measure.[15] As seen in Eq. (5), our pion ansatz is not purely classical (since our pion is not commutable). However, it is not our present aim to pursue the pure quantum effects such as the Dirac sea contribution.[7]

In this paper we proposed a non-"hedgehog" ansatz, i.e., form (2). With this new ansatz we showed that the equations of motion can be solved exactly in model I. This is what we want to emphasize most. We also showed that the equations can be solved approximately in model II. The "hedgehog" ansatz may be useful to study the differences between π_λ and ϕ_λ since, with the ansatz, the equations of motion can be solved exactly both in models I and II. However, as previously mentioned, the new ansatz has the three distinctive advantages. We showed that the overall agreement of g_A, $g_{\pi NN}$, and μ_p with their experimental values is possible in a case within model II by introducing the ω-quark interaction. We infer that this is commonly true in the chiral bag, soliton, and potential models independently of the topology. This idea is based on the following speculations. (1) The ω field does not affect the (theoretical) GT relation. Therefore the ratio $g_{\pi NN}/g_A$ is not altered. (2) The inclusion of the ω field reduces the share of the pion field energy in the baryon mass. Hence, the pion field itself is attenuated and the pionic contribution to g_A becomes smaller. (3) The ω-quark interaction produces a further contribution to μ_p, as seen in Eq. (13). Also, μ_p is lifted up because the quark field energy decreases as seen in the same equation.

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