

Energy levels of the nucleus ^{166}Ho

P. C. Sood

*Department of Physics, Banaras Hindu University, Varanasi 221005, India
and Florida State University, Tallahassee, Florida 32306*

R. K. Sheline

Florida State University, Tallahassee, Florida 32306

R. S. Ray

*Department of Physics, Banaras Hindu University, Varanasi 221005, India
and R. R. Campus, Tribhuvan University, Janakpur, Nepal*

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The complete low energy two-particle spectrum arising from the coupling of the single-particle orbitals with summed energy ($E_p + E_n$) up to 750 keV, and from the coupling of all known proton orbitals up to 1.5 MeV with the ground state neutron, is calculated for the doubly odd deformed nucleus ^{166}Ho . The bandhead energies, the Gallagher-Moszkowski splitting energies, and the nuclear moment of inertia parameters for deriving the rotational energies are calculated for a zero range residual interaction for each of the 34 possible bands for comparison with the available experimental results from radioactive decay, neutron capture, and particle transfer reaction studies wherein 21 bands have been postulated. Our calculations agree with the characterization of 11 of these bands. We further present evidence for selecting specific K^π and/or configurations for the remaining 10 bands. Predicted location and character of 13 bands not observed so far are given. Identification of 3 of these bands with known levels is proposed.

I. INTRODUCTION

The nucleus $^{166}_{67}\text{Ho}_{99}$ is possibly the best studied¹⁻¹⁵ odd-odd deformed nucleus. It is also unique in that it provides the only known exception to the Gallagher-Moszkowski¹⁶ (GM) rule in the energy ordering of the spin doublets in doubly odd deformed nuclei. The energy levels of this nucleus have been extensively investigated through β decay studies and through a variety of neutron-capture reactions, e.g., (n, e^-) , (n, γ) , average resonance capture (ARC), polarized neutron capture in polarized target, etc., as well as through (d, p) and (t, α) particle transfer reactions. In spite of this prodigious experimental effort, not only is the spectroscopic information incomplete (only 21 out of the expected 52 two-particle configurations from summed single particle orbitals energy up to 1 MeV have been tentatively identified so far), but the interpretation of the available experimental information provides a considerable challenge. The interpretation of the negative parity bands is particularly unsatisfactory. The problem is further complicated by the fact that the coupling of the ground state proton orbital $\frac{7}{2}^-$ [523] with the various excited neutron orbitals, and the coupling of the ground state neutron orbital $\frac{7}{2}^+$ [633] with the various excited proton orbitals, yield very similar GM band pairs with identical values of K^π . Although one would not expect much admixture in these bands with the same K^π in view of both the neutron and the proton orbitals being different, the transfer reaction populations and the decay patterns indicate the presence of significant mixing in several cases.

Whereas particle transfer reactions can generally indicate the configuration of the involved state, information from neutron capture reactions is much less definitive in this regard. Even though specific processes, e.g., conversion electron measurements,^{5,8} average resonance capture,⁹ lifetime measurements,¹¹ polarized n and polarized target experiments,¹² etc., may lead to reliable spin-parity assignments, the placement of these levels into specific K^π bands can often be attempted only through plausibility arguments. Even then, the configuration assignment of various K^π bands is left uncertain, to be decided on qualitative considerations.

Under these circumstances it is worthwhile to attempt a theoretical calculation of the *complete* two-particle spectrum for this nucleus based on a quantitative evaluation of the residual n-p interaction energy for the odd-odd nucleus under consideration. The residual interaction is explicitly spin dependent (contains a $\bar{\sigma}_p \cdot \bar{\sigma}_n$ term) in view of the observed validity of the GM rule;¹⁶ it also causes an odd-even shift of the levels in the $K=0$ bands, usually referred to as the Newby¹⁷ term. The residual interaction calculations^{7,17-21} so far had mainly concentrated on the evaluation of these two terms, i.e., the GM splitting energy E_{GM} and the Newby odd-even shift coefficient B , with a variety of interactions.²⁰ However, this information is not sufficient for a quantitative prediction of the bandhead energy since the residual interaction contains a spin-independent (Wigner) term in addition to the $\bar{\sigma}_p \cdot \bar{\sigma}_n$ term. A fit to E_{GM} and B cannot by itself yield a measure of the Wigner term. Recently, Sood and Singh²² have developed a formulation which, by combining the atomic

mass data with the GM splitting energies, yields a measure of the Wigner as well as the spin-dependent contributions for a zero-range residual interaction, and thus provides a quantitative prediction for the bandhead energies for each (Ω_p, Ω_n) configuration. The formulation has been successfully applied^{22–24} to the description and prediction of the energy levels of the doubly odd nuclei of the actinide region. It has also been found successful for confirming and/or deducing the configuration assignments to known levels²⁵ and for examining the level structures of the isotopic sequences of doubly odd nuclei of the rare earth region^{26,27} as well. Preliminary results from an application of this formulation to the energy levels of $_{67}\text{Ho}$ isotopes have been reported earlier.²⁷ Analyses of the levels in ^{158}Ho and ^{170}Ho , dealing primarily with the placement of the low-energy isomers, have been published earlier.^{28,29} Here we report the results of our first study of the complete two-particle spectrum of a doubly odd rare earth nucleus using this formulation. In Sec. II we give a brief outline of the formulation and the procedure for our calculations. The results are presented and discussed in comparison with the available experimental information in Sec. III. The concluding section includes the summary and conclusions of the present study.

II. FORMULATION AND PROCEDURE

The two-particle states in an odd-odd deformed nucleus are constructed by adding an odd proton (Ω_p) and an odd neutron (Ω_n) to a deformed even-even $(Z-1, A-2)$ inert core. Each (Ω_p, Ω_n) combination gives rise to two bands (usually referred to as a GM pair) with $K^\pm = |\Omega_p \pm \Omega_n|$. The bandhead energy is then written as

$$E_K(\Omega_p, \Omega_n) = E_p(\Omega_p) + E_n(\Omega_n) + E_{\text{rot}} + E_{\text{int}}, \quad (1)$$

where the rotational energy term is introduced to take into account the relation between the intrinsic and the observed bandhead energies given by

$$E_K(J) = E_0^K + \frac{\hbar^2}{2\mathcal{I}} [J(J+1) - K^2] \quad (2)$$

$$= E_0^K + \frac{\hbar^2}{2\mathcal{I}} K \quad \text{for } J=K, \quad (2a)$$

and the last term in Eq. (1) accounts for the residual neutron-proton energy for the two valence nucleons. Assuming the near equivalence of the moment of inertia parameters in the odd-mass and the odd-odd nuclei, we obtain

$$E_{\text{rot}} \approx \frac{\hbar^2}{2\mathcal{I}} [K - (\Omega_p + \Omega_n)] \quad (3a)$$

$$= \frac{-\hbar^2}{2\mathcal{I}} (2\Omega_<) \delta_{K,K-}. \quad (3b)$$

For the residual n-p interaction, we assume a zero range interaction with an explicit $\vec{\sigma}_p \cdot \vec{\sigma}_n$ term and obtain²²

$$E_{\text{int}} = W[(1-\alpha)A_0(K) + \alpha A_\sigma(K)] + (-1)^J B \delta_{K,0}, \quad (4)$$

where W and αW are the strengths of the spin-independent (Wigner) and the spin-dependent terms in the zero range interaction, A_0 and A_σ are the matrix ele-

ments of these two terms using the appropriate product-type Nilsson wave functions evaluated at the specific deformation for each configuration, and B is the measure of the Newby¹⁷ (odd-even shift) term for the $K=0$ bands. The details of the formulation and the analytical expressions for the interaction matrix elements have already been published.²² Here we present only the essential results and the outline of the procedure for calculation of the bandhead energies and the GM splitting energies. Combining Eqs. (1), (2), and (4), we obtain,²² for the bandhead energy in an odd-odd nucleus,

$$\begin{aligned} E_K(Z, A, \Omega_p, \Omega_n) &= E_p(Z, A-1, \Omega_p) + E_n(Z-1, A-1, \Omega_n) \\ &+ \frac{\hbar^2}{2\mathcal{I}} [K - (\Omega_p + \Omega_n)] \\ &+ W[(1-\alpha)A_0(K) + \alpha A_\sigma(K)] \\ &+ (-1)^J B \delta_{K,0}. \end{aligned} \quad (5)$$

The GM splitting energy contains only the contribution from the spin-dependent term in the interaction and is given by²²

$$E_{\text{GM}} = 2\alpha W |A_\sigma(K)| \quad (6)$$

since the magnitudes of A_0 and A_σ are the same for both the K^+ and K^- bands. Thus the strength parameter αW of the spin-dependent term in the interaction can be derived from the experimentally observed splitting energy after taken into account the rotational energy correction and Newby shift if relevant. The odd-even Newby¹⁷ term for the $K=0$ bands can be obtained²¹ from the lowest two observed levels in the band as

$$\begin{aligned} B &= \frac{1}{2} (-1)^J [E_0(J) - E_0(J+1)] \\ &+ \frac{\hbar^2}{2\mathcal{I}} [(-1)^J (J+1) + a_p a_n \delta_{\Omega_{n,1/2}}], \end{aligned} \quad (7)$$

where the last term contains the decoupling parameters a in the case in which $\Omega_p = \Omega_n = \frac{1}{2}$. Theoretical estimates of B (sometimes denoted N) are also available for a variety of interactions.^{20,21,37}

By using the fact that the ground state in each odd-mass as well as odd-odd nucleus is the observed bandhead for the respective configuration, we obtain,²² from Eq. (5), for the $K \neq 0$ case,

$$\begin{aligned} E_M &\equiv [M(Z, A) - M(Z, A-1) - M(Z-1, A-1) \\ &+ M(Z-1, A-2)] - E_{\text{rot}} \\ &= W A_0(K_G) - \alpha W [A_0(K_G) - A_\sigma(K_G)], \end{aligned} \quad (8)$$

where K_G denotes the ground state band number. Thus using the atomic mass data to evaluate E_M defined in Eq. (8) and the experimental splitting energy of the GM pair including K_G , we can evaluate both the interaction parameters W and αW . The interaction parameters are assumed to be configuration dependent and are the same for every (Ω_p, Ω_n) combination appearing in any nucleus. If the configuration under investigation is not observed as the ground state in any nucleus and/or GM splitting is known only in an excited GM pair, we obtain an approximate value for the interaction parameters by assuming an

average value for $\alpha=0.25$ and, using this value with the known E_M or E_{GM} , obtain W through Eq. (6) or (8). In case neither of the two quantities is experimentally known for the specific configuration, we adopt the ground state parameters derived for the nucleus under consideration or the nearest neighbor nucleus with similar orbitals.

Having obtained the interaction parameters and calculated the matrix elements at the appropriate deformation for each configuration, the bandhead energies and the GM splitting energies for each band are calculated using Eqs. (5) and (6).

The effective moment of inertia parameter for each band is calculated from the rotational parameters of the even-even core and of the odd-mass neighbors using the formula

$$\mathcal{I}_{o-o} = \mathcal{I}_{o-e} + \mathcal{I}_{e-o} - \mathcal{I}_{ee} . \quad (9)$$

The energies of the rotational levels in each band are then obtained by adding the $J(J+1)$ -dependent contribution to the bandhead energy.

III. RESULTS AND DISCUSSIONS

Our predicted locations of the bandheads for the various two-quasiparticle configurations expected in the low energy excitation spectrum of ^{166}Ho are shown in Figs. 1 and 2. We have also evaluated the moment of inertia for each of the bands and have calculated the positions of the associated rotational levels as well. However, considering the density of the levels, we do not show these levels in the figures for clarity and for ease in identification through comparison with the experimental results. Whenever considered relevant, we compare the calculated and the experimental values of these moments of inertia in the following discussion.

We include in Fig. 1 *all* the possible two-particle configurations arising from the superposition of the orbitals with the summed neutron and proton single particle energies (taken from the observed spectra of the $A-1$ isotone or isotope) up to 650 keV. This set is comprised of 24 two-particle bands, including 12 from Table I, four from Table II, and eight from Table III. The system of designating these bands is also presented in the tables. Our predicted bandhead energies for these 24 intrinsic states are shown in Fig. 1 in comparison with the 15 postulated bands based on the beta-decay, neutron-capture, and (d,p) reaction studies.¹⁻¹³ Figure 2 is a similar plot for the 16 predicted bandheads (the six bands of the three lowest GM pairs being common with Fig. 1) arising from the coupling of the $\frac{7}{2}^+[633]$ neutron orbital to the various proton orbitals listed in Table II compared to the 12 bands in this category populated through the (t, α) experiments.¹⁴

With respect to the experimental studies,¹⁻¹⁵ we note the following points. In common with the decays from the $I^\pi=0^+$ ground states of the neighboring even-even nucleus, the beta transitions populate only the low spin levels in the doubly odd daughter nucleus. However, in the present case Q_{β^-} is just 481 keV and studies^{1-4,8,13} of the gamma transitions following β decay yield definitive

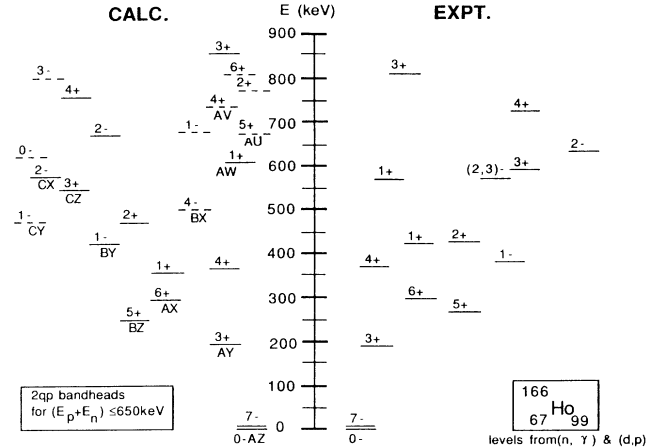


FIG. 1. Observed bandhead energies of the two-particle levels experimentally identified (Refs. 1–13) in the nucleus ^{166}Ho from the beta decay of ^{166}Dy , and the (n, e^-) (n,γ), (n_{res},γ), and (d,p) reactions compared with our calculated bandhead energies (shown on the left) for the various two-particle configurations. The labeling of the configurations is specified in Table I. The calculated level scheme includes *all* the possible two-particle excitations arising from the orbitals with summed single-particle energies [$E(\Omega_p)+E(\Omega_n)$] up to 650 keV. The levels not observed so far are shown by dashed lines.

information on the three lowest members of the ground state $K^\pi=0^-$ rotational band and deduce the existence of only two other levels, a $J^\pi=1^+$ level at 426 keV and a negative parity level with $J=0$ or 1 at 373 keV, configuration assignments in both cases being undecided. The (d,p) reaction spectroscopy^{7,8} yields information on only

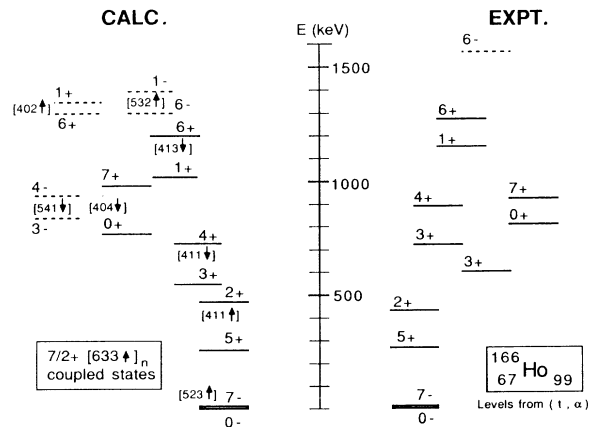


FIG. 2. Same as Fig. 1, but for the experimental levels identified in ^{166}Ho through the (t, α) reaction (Ref. 14). The corresponding calculated level schemes include bandhead energies only for the two-particle excitations involving a coupling of the $\frac{7}{2}^+[633]$ neutron orbital to the specified proton orbital. The labeling for the latter gives $[Nn_3\Lambda\Sigma]$, the corresponding $\Omega=\Lambda\pm\Sigma$ with + for \uparrow spin projection and - for \downarrow spin projection and the parity being equal to $(-1)^N$.

TABLE I. Two-quasiparticle states expected in ^{166}Ho from the coupling of various neutron states with the proton state $\frac{7}{2}^- [523\uparrow]$, labeled A , which is the ground state in ^{165}Ho . The column headed E_x lists the neutron excitation energies observed in the isotonic $Z - 1$ nucleus ^{165}Dy . A_0 and A_σ are the calculated matrix elements for the Wigner and the spin-dependent terms in the zero-range residual interaction for the deformation parameter $\delta = 0.28$.

Label	$\Omega_n^\pi [Nn_3\Lambda\Sigma]$	E_x (keV)	K_T^π	K_S^π	$ A_0 $ ($\times 10^3$)	$ A_\sigma $ ($\times 10^3$)	Observed ground state
AX	$\frac{7}{2}^+ [633\uparrow]$	0	7^-	0^-	132	91	^{166}Ho
AY	$\frac{1}{2}^- [521\downarrow]$	108	3^+	4^+	97	70	^{168}Ho
AX	$\frac{5}{2}^- [512\uparrow]$	184	6^+	1^+	86	40	^{170}Ho
AW	$\frac{5}{2}^- [523\downarrow]$	534	1^+	6^+	168	127	^{164}Ho
AV	$\frac{1}{2}^- [510\uparrow]$	570	4^+	3^+	71	48	
AU	$\frac{3}{2}^- [521\uparrow]$	574	5^+	2^+	112	66	

three intrinsic configurations in addition to the ground state GM pair. These five bands arise from the coupling of the $\frac{7}{2}^- [523]$ proton to the $\frac{7}{2}^+ [633]$, $\frac{1}{2}^- [521]$, and $\frac{5}{2}^- [512]$ neutron orbitals with $I^\pi K$ assignments given for the levels up to 470 keV excitation energy. A proton pickup study,¹⁴ which utilizes the $^{167}\text{Er}(t,\alpha)$ reaction, establishes eight bands involving four excited proton configurations, and suggests the possible existence of another high-lying ($E_x \cong 1560$ keV) $K^\pi = 6^-$ band.

Thus the charged particle spectroscopy has so far identified 14 bands in ^{166}Ho . As discussed below, our calculations confirm the dominant configuration experimentally deduced for most of these bands. We further predict, as shown in Figs. 1 and 2, the expected locations of 12 other bands of this category involving either a $\frac{7}{2}^- [523]$ proton or a $\frac{7}{2}^+ [633]$ neutron.

The energy levels in ^{166}Ho have also been studied^{5,6,8,9,11,12,15} through a variety of neutron-capture processes. Although the gamma transition energies following n capture have high precision and resolution, the place-

ment of these transitions to deduce energy levels is not a straightforward process. Information has been deduced on the excitation energies and the spin-parity assignments for a number of levels by combining the analyses of high and low energy neutron capture spectra, conversion electron data, ARC analyses, etc. However, grouping of these levels to deduce band structures and, further, to arrive at the configurations for the postulated bands from such studies alone, cannot lead to unambiguous conclusions. While the most recent study¹⁵ does not even attempt J^π assignments, Bollinger and Thomas⁹ had postulated 15 bands in this nucleus, shown on the right in Fig. 1. We find that several of these bands, which are not independently confirmed through particle transfer reaction studies, may have alternative, more acceptable characterization. The negative parity bands, in particular, require serious reconsideration.

In the following we discuss some of these bands individually based on our theoretical calculations and the observed experimental features.

TABLE II. Two-quasiparticle states expected in ^{166}Ho from the coupling of various proton states with the neutron state $\frac{7}{2}^+ [633\uparrow]$, labeled Z , which is the ground state in the $Z - 1$ isotone ^{165}Dy . The excitation energies E_x are the experimental values for the $A - 1$ isotope ^{165}Ho . Other columns are the same as in Table I.

Label	$\Omega_p^\pi [Nn_3\Lambda\Sigma]$	E_x (keV)	K_T^π	K_S^π	$ A_0 $ ($\times 10^3$)	$ A_\sigma $ ($\times 10^3$)	Observed ground state
AZ	$\frac{7}{2}^- [523\uparrow]$	0	7^-	0^-	132	91	^{166}Ho
BZ	$\frac{3}{2}^+ [411\uparrow]$	362	5^+	2^+	78	55	^{164}Tb
CZ	$\frac{1}{2}^+ [411\downarrow]$	429	3^+	4^+	80	56	^{168}Tm
DZ	$\frac{1}{2}^- [541\downarrow]$	680	3^-	4^-	62	22	
EZ	$\frac{7}{2}^+ [404\downarrow]$	715	0^+	7^+	87	58	^{170}Lu
FZ	$\frac{5}{2}^+ [413\downarrow]$	995	1^+	6^+	109	77	
GZ	$\frac{5}{2}^+ [402\uparrow]$	1056	6^+	1^+	55	36	
HZ	$\frac{3}{2}^- [532\uparrow]$	(1400)	6^-	1^-	158	155	

TABLE III. Two-quasiparticle states expected in ^{166}Ho arising from single particle states with $[E\Omega_p] + E(\Omega_n)$ up to 750 keV which are not listed in Tables I and II; all are seen to form negative parity bands. The relevant energies are listed in earlier tables and same notation is also followed for the various entries.

Label	$\Omega_p^\pi[Nn_3\Lambda\Sigma]$	$\Omega_n^\pi[Nn_3\Lambda\Sigma]$	E_T^π	K_S^π	$ A_0 (\times 10^3)$	$ A_\sigma (\times 10^3)$
BY	$\frac{3}{2}^+[411\uparrow]_p$	$\frac{1}{2}^-[521\downarrow]_n$	1 ⁻	2 ⁻	135	71
BX	$\frac{3}{2}^+[411\uparrow]_p$	$\frac{5}{2}^-[512\uparrow]_n$	4 ⁻	1 ⁻	154	143
CY ^a	$\frac{1}{2}^+[411\downarrow]_p$	$\frac{1}{2}^-[521\downarrow]_n$	1 ⁻	0 ⁻	142	79
CX ^a	$\frac{1}{2}^+[411\downarrow]_p$	$\frac{5}{2}^-[512\uparrow]_n$	2 ⁻	3 ⁻	147	130

^aThe configurations CY and CX appear, respectively, as ground states in the nuclei ^{170}Tm and ^{172}Tm .

A. Ground state $K^\pi=7^-$ and $K^\pi=0^-$ GM pair

The ground state configuration $\{\frac{7}{2}[523\uparrow]_p \pm \frac{7}{2}[633\uparrow]_n\}$ gives rise to the triplet $K_T^\pi=7^-$ and the singlet $K_S^\pi=0^-$ bands. The Gallagher-Moszkowski¹⁶ rule predicts the K_T to lie lower in energy than the K_S . However, experimentally the 7^- state has been established¹¹ at 5.98 keV excitation energy, with the 0^- state becoming the ground state in apparent violation of the GM rule. Struble and Rasmussen³⁰ suggested that a larger configuration mixing in the 0^- state than in the 7^- state, and a rotational zero point energy could push the former below the expected parallel spin $K^\pi=7^-$ state. However, the recent (t,α) studies¹⁴ found “no indication of significant configuration mixing in the 0^- band” based on the observation that, relative to the $K^\pi=7^-$ band, the $K^\pi=0^-$ band appears to contain all the theoretically predicted spectroscopic factors. It is possible to understand the apparent violation of the GM rule in this case employing the criteria for its applicability discussed by Sood and Singh.³¹ They had concluded that in the case in which the spin-antiparallel ($\Sigma=0$) state K_S is the $K^- = |\Omega_p - \Omega_n|$ state, the competition between E_{rot} and E_{int} decides its placement relative to the $K_T=K^+$ state. For the $K=0$ bands, E_{int} contains E_{GM} as well as the odd-even shift E_N . The relative magnitude of these terms for the present case are shown in Fig. 3. In this case the contributions from E_{rot} and E_N reinforce each other and the two together more than cancel out E_{GM} to push the $I^\pi K=0^-0$ below the $K^\pi=7^-$ bandhead.

B. Positive parity bands

1. $K^\pi=1^+$ and $K^\pi=6^+$ AX and AW bands

The AX configuration $\{\frac{7}{2}[523\uparrow]_p \pm \frac{5}{2}[512\uparrow]_n\}$ involving the 184 keV neutron orbital gives rise to the $(6^+, 1^+)$ pair of bands, while the AW configuration $\{\frac{7}{2}[523\uparrow]_p \mp \frac{5}{2}[523\downarrow]_n\}$ involving the 534 keV neutron orbital results in the $(1^+, 6^+)$ pair of bands. Experimentally, a $K^\pi=6^+$ bandhead^{7,8,11} at 295 keV and two $K^\pi=1^+$ bandheads^{8,9} at 426 and 567 keV, respectively, have been proposed.

The first order estimate for the $K^\pi=6^+$ AX bandhead energy, obtained by adding the ground state E_N and E_{rot} contributions from Fig. 3 to the summed orbital excita-

tion energy, is 279 keV. The inclusion of the relative residual interaction energy contribution brings the calculated value still closer to the observed 6^+ energy, thus confirming the 295 keV 6^+ level as the $K^\pi=6^+$ bandhead from the AX configuration.

The characterization of the two observed $K^\pi=1^+$ bandheads is not so straightforward. The 426 keV 1^+ level has been deduced¹⁻⁴ from the ^{166}Dy beta decay. However, the populating β transition has not been directly observed. Its intensity, deduced from the intensities of the connected gamma rays, has been quoted within the 1.1–2.8% range. The higher intensity value yields $\log ft=4.9$ for this transition. This consideration led Motz *et al.*⁸ to classify this transition as allowed-unhindered (a.u.) and hence to assign the $\{\frac{7}{2}[523]_p - \frac{5}{2}[523]_n\}$ configuration AW to the 426 keV level. Later, Bollinger and Thomas⁹ proposed a $K^\pi=1^+$ band with the 1^+ bandhead at 567 keV. Examining the available configurations giving rise to the $K^\pi=1^+$ band, aside from the one already used by Motz *et al.*⁸ for the 426 keV 1^+ level, they suggested the AX configuration for the 567 keV 1^+ level.

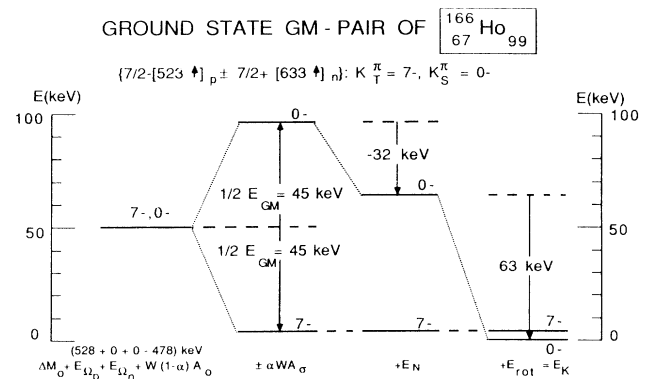


FIG. 3. The term-by-term bandhead energy calculation for the ground state GM pair in ^{166}Ho using the expression given in the bottom line revealing the factors responsible for the “apparent” violation of the GM rule, and for bringing the singlet state with $K_S^\pi=0^-$ below the expected ground state with $K_T^\pi=7^-$.

The above assignments rest solely on the a.u. classification for the β transition feeding the 426 keV level based on the assumed $\log ft = 4.9$. However, the nuclear data evaluators¹⁰ have since concluded $\log ft = 5.2$ for this transition. The selection rules for β decay framed by Alaga³² were applied to the deformed even mass nuclei by Gallagher.³³ Recently, Sood and Ray³⁴ undertook a comprehensive survey of the a.u. transitions and obtained $\log ft = 4.8 \pm 0.2$ for the 54 such transitions observed for the rare earth nuclei. The Nuclear Data Sheets—adopted value¹⁰ of $\log ft = 5.2$ puts this transition outside the a.u. class and nullifies the basis for the AW assignment to the 426 keV 1^+ level.

The Alaga³² rules were reexamined by Fujita *et al.*,³⁵ who concluded that the allowed transitions obeying the selection rule

$$\Delta N = 0, \quad |\Delta n_3| = |\Delta \Lambda| = 1,$$

can have $\log ft$ values only marginally higher than those for the a.u. transitions. Under this rule the $\log ft = 5.2$ connecting the 426 keV 1^+ level points to the AX configuration assignment for this level. This assignment is further supported by the bandhead energy calculations as well as the consideration of the GM splitting energy as discussed below.

The earlier assignment AX to the 567 keV 1^+ bandhead makes it the GM counterpart of the $K^\pi = 6^+$ AX bandhead at 295 keV. This leads to the “experimental” E_{GM} for the AX configuration ≈ 320 keV. A fit to this value using the matrix elements listed in Table I would require the unacceptably large value $\alpha W \approx 4$ MeV for the interaction parameters in comparison with the least squares fit value $\alpha W = 0.87$ MeV.^{20,21} The $K^\pi = 1^+$ AX assignment to the 426 keV 1^+ level is clearly more acceptable on this basis. The first order estimate for the $K^\pi = 1^+$ AW bandhead energy is 566 keV, whereas that for the $K^\pi = 1^+$ AX bandhead energy is around 400 keV.

Thus the bandhead energy calculations, the consideration of the GM splitting energy, and the beta-decay characteristics all support the AX configuration assignment for the 426 keV 1^+ level.

The bandhead energy calculations and an examination of all the possible $K^\pi = 1^+$ bands in the specified energy region lead to the AW configuration assignment to the 567 keV 1^+ level. Unfortunately, the Q value for β decay (482 keV) is too low to populate the 567 keV state which, according to the present assignments, would have the a.u. character. We predict the $K^\pi = 6^+$ bandhead for the AW configuration to lie around 825 keV.

2. $K^\pi = 3^+$ and $K^\pi = 4^+$ AY and CZ bands

The AY configuration $\{\frac{7}{2}[523\uparrow]_p \mp \frac{1}{2}[512\downarrow]_n\}$ involving the coupling of the ground state proton with the 108 keV neutron orbital, and the CZ configuration $\{\frac{1}{2}[511\downarrow]_p \mp \frac{7}{2}[633\uparrow]_n\}$ involving the coupling of the ground state neutron with the 429 keV proton orbital give rise to the two ($3^+, 4^+$) GM band pairs.

The 190.9 keV 3^+ and 372 keV 4^+ AY bands have been established in several studies^{7-9,11} and our calculations, as shown in Fig. 1, are in agreement with this assignment.

For the CZ pair of bands, Bollinger and Thomas⁹ had suggested the 592 keV based $K^\pi = 3^+$ band and the 719 keV based $K^\pi = 4^+$ band. On the other hand, Dewberry *et al.*¹⁴ identified their 718 keV peak with the $K^\pi = 3^+$ CZ bandhead and their 884 keV peak with its $K^\pi = 4^+$ GM counterpart; they left the moderately strong peak at 590 keV unassigned while accepting the $I^\pi = 3^+$ assignment for this level. On the basis of the total intensity of the transitions, Bollinger and Thomas⁹ conclude that the 592 keV 3^+ level is almost surely a bandhead with the 4^+ and 5^+ rotational levels located at 671 and 769 keV, respectively. Dewberry *et al.*¹⁴ constructed the CZ $K^\pi = 3^+$ rotational band by assigning the 721, 821, and 945, and 1090 keV levels as the $3^+, 4^+, 5^+,$ and 6^+ rotational members of this band yielding the moment of inertia parameter equal to 12.3 keV. If we accept the Bollinger-Thomas assignment of these levels as comprising the CZ $K^\pi = 4^+$ rotational band with spin parity $4^+, 5^+, 6^+,$ and 7^+ , an equally good $J(J+1)$ fit is obtained with the moment of inertia parameter equal to 10.3 keV. This latter value for the CZ $K^\pi = 4^+$ band is comparable to the corresponding value of 10 keV obtained for the CZ $K^\pi = 3^+$ band built on the 592 keV 3^+ bandhead by Bollinger and Thomas.

From the theoretical considerations, the first order estimate for the $K^\pi = 3^+$ CZ bandhead is obtained as 524 keV by adding the single particle energy (429 keV) from Table II and the E_N and E_{rot} for the ground state from Fig. 3. The CZ configuration has been experimentally observed as the ground state in the isotonic nucleus ^{168}Tm ; this provides us with a reliable basis for the evaluation of the interaction parameters as described in Sec. II. Further, as seen from Table II, the interaction matrix elements for the CZ configuration are appreciably smaller than those for the ground state configuration, thus causing the CZ triplet $K^\pi = 3^+$ bandhead to be pushed further up from its first order estimate to bring the calculated value into a close agreement with the observed 592 keV 3^+ level. No acceptable residual interaction parameters can push the pure CZ 3^+ bandhead energy to match the proposed 719 keV 3^+ assignment by Dewberry *et al.*¹⁴

A careful reinvestigation of the proton pickup reaction and a better determination of the rotational levels in the 592 keV based band are suggested to clarify the situation.

3. $K^\pi = 5^+$ and $K^\pi = 2^+$ AU and BZ bands

The BZ configuration $\{\frac{3}{2}[411\uparrow]_p \pm \frac{7}{2}[633\uparrow]_n\}$ involving the coupling of the ground state neutron with the 362 keV proton orbital, and the AU configuration $\{\frac{7}{2}[523\uparrow]_p \pm \frac{3}{2}[521\uparrow]_n\}$ involving the coupling of the ground state proton with the 574 keV neutron orbital, give rise to the two ($5^+, 2^+$) GM band pairs.

Motz *et al.*⁸ had identified a 5^+ level at 263.787 keV and a 2^+ level at 430.1 keV. On the basis of the energy considerations, the lower-lying ($5^+, 2^+$) pair should correspond to the excited proton configuration BZ ; however, the deexcitation of the 264 keV 5^+ and 430 keV 2^+ levels into the levels of the $K^\pi = 3^+$ and $K^\pi = 4^+$ (AY) bands suggests their assignment as the excited neutron configuration AU . Accordingly, Motz *et al.*⁸ concluded that the

observed ($5^+, 2^+$) bands may be admixtures of both these configurations. Bollinger and Thomas,⁹ as well as Schilling *et al.*,¹¹ simply list these ($5^+, 2^+$) states as the excited neutron configuration *AU*. Dewberry *et al.*¹⁴ found all related intense peaks in their (t, α) spectrum with angular distributions to be in good agreement with $l=2$ transfers, indicating the pickup of a $\frac{3}{2}^+$ [411] proton. Our calculations for the *BZ* configuration, as shown in Fig. 2, agree with the results of the (t, α) experiment. The calculated moment of inertia parameter is 9.0 keV, in comparison with the experimental values of 9.27 keV for the $K^\pi=5^+$ band and 8.5 keV for the $K^\pi=2^+$ band. However, as noted above, the depopulation of these two bands is exclusively to the levels of the *AY* configuration. If the ($5^+, 2^+$) bands had the pure *BZ* configuration and the connected ($3^+, 4^+$) bands had the pure *AY* configuration, such decays involving changes of both the neutron and the proton orbitals would be greatly hindered. The observed decay patterns can only be understood by band admixtures which, as discussed for the ($5^+, 2^+$) bands by Dewberry *et al.*,¹⁴ cannot be presently understood through residual interaction calculations. We only conclude that the 264 keV 5^+ and the 420 keV 2^+ bands have the predominantly excited proton configuration *BZ*.

The experimental location of the other $K^\pi=5^+$ and $K^\pi=2^+$ bands with dominant excited neutron configuration *AU* remains an open question. We search for the possible candidates for these levels based on our theoretical estimates of the bandhead energies for the *AU* configuration and an estimation of the still unassigned levels with $J^\pi=(2^+, 5^+)$ from the (\bar{n}, γ) results⁹ within the excitation energy range 550–800 keV. We expect the $K^\pi=5^+$ bandhead to lie around 650 ± 50 keV and the $K^\pi=2^+$ bandhead about 100 ± 20 keV above it. The relatively large range indicated for the predicted locations is partly due to the uncertainty in the residual interaction parameters (since this GM pair has not been identified in any nucleus so far) and partly to allow for the configuration mixing displacements. Looking at Table IV of Ref. 9 and Table VII of Ref. 8, the only unassigned $(2, 5)^+$ level from (\bar{n}, γ) , and also seen in high energy (n, γ) as well as in (d, p) reactions on the high energy side, is the 768.8 keV level. Bollinger and Thomas⁹ had tentatively placed it as the 5^+ member of the *CZ* excited proton $K^\pi=3^+$ band beginning at 592 keV. As discussed in the preceding subsection, the (t, α) reaction¹⁴ identifies the $K^\pi=3^+$ band of this configuration starting at 721 keV with 5^+ located at 945 keV. We suggest the experimentally observed 769 keV $(2, 5)^+$ level as the most likely $K^\pi=2^+$ bandhead for the *AU* configuration. Looking for a possible 5^+ bandhead, we note that Bollinger and Thomas⁹ had tentatively placed the 655 keV level as the $J^\pi K=5^+ 4$ state of their proposed $K^\pi=4^+$ band for the *AV* configuration starting at 558 keV. Our calculations, as discussed below, place the *AV* $K^\pi=4^+$ bandhead around 730 keV as the GM counterpart of the observed 814 keV $K^\pi=3^+$ band. Further, the 558 keV 4^+ level has since been reassigned as $4^+ 2$ by Bosman and Postma¹² based on their experiments on polarized n capture by polarized ^{165}Ho . Accordingly, we suggest the 655 keV level as the $K^\pi=5^+$ bandhead of the *AU* configuration. The 655 keV level has been seen in

the (d, p) study and is assigned J^π consistent with (\bar{n}, γ) analysis.

4. $K^\pi=4^+$ and $K^\pi=3^+$ *AV* bands

The *AV* configuration $\{\frac{7}{2}[523\uparrow]_p \pm \frac{1}{2}[510\uparrow]_n\}$ involving the coupling of the ground state proton with the 570 keV neutron orbital gives rise to the $(4^+, 3^+)$ GM band pair. Bollinger and Thomas⁹ had proposed the $K^\pi=4^+$ bandhead at 558 keV and the $K^\pi=3^+$ bandhead at 814 keV. The latter assignment agrees with our calculations for the 3^+ *AV* bandhead energy. Our calculated moment of inertia parameter is 9.8 keV, compared with 9.5 keV for the experimentally suggested band. However, we place the $K^\pi=4^+$ bandhead around 730 keV. The suggested⁹ assignment of the 558 keV level as the $K^\pi=4^+$ bandhead would yield an experimental GM splitting energy of 265 keV, which is over 3 times larger than the predicted²¹ value of 82 keV for this configuration. An examination of the available states in the neighborhood of the predicted bandhead energy reveals a state at 721 keV and another at 736 keV, both assigned $J^\pi=(3, 4)^+$ from (\bar{n}, γ) studies⁹ and both having been observed in the (d, p) spectrum.⁸ The 721 keV state has since been identified as the bandhead for the *CZ* configuration in (t, α) studies.¹⁴ We suggest the observed 736 keV $(3, 4)^+$ state as the $K^\pi=4^+$ bandhead for the *AV* configuration based on the calculated bandhead energy and the splitting energy from its 814 keV $K^\pi=3^+$ GM counterpart.

5. Other positive parity bands

In addition to the 14 positive parity bands discussed in the preceding subsections, six other such bands are expected in the spectrum of ^{166}Ho formed by the coupling of the ground state neutron with the excited proton orbitals; see Table II. The $(0^+, 7^+)$ GM band pair from the *EZ* configuration $\{\frac{7}{2}[404\downarrow]_p \mp \frac{7}{2}[633\uparrow]_n\}$ has been observed¹⁴ with the 0^+ bandhead placed at 803.36 keV and the 7^+ bandhead at 914.8 keV. The configuration appears as the ground state in the isotonic nucleus ^{170}Lu . Our bandhead energy calculations agree with this assignment. Our calculated $E(7^+ \rightarrow 0^+)$, including the calculated E_{GM} , E_N , and the zero point rotational energy, is about 180 keV; it is larger than the observed $7^+ - 0^+$ separation energy¹⁶ of 114 keV, but the difference is not as large as deduced by Dewberry *et al.*¹⁴ The $(1^+, 6^+)$ GM band pair from the *FZ* configuration $\{\frac{5}{2}[413\downarrow]_p \mp \frac{7}{2}[633\uparrow]_n\}$ has been observed with the $K^\pi=1^+$ bandhead placed at 1150 keV and the $K^\pi=6^+$ bandhead at 1282 keV. Our calculations again support these assignments, although, as shown in Fig. 2, the predicted bandhead energies are somewhat lower than the experimental values. We further predict the $(6^+, 1^+)$ bandheads for the *GZ* configuration $\{\frac{5}{2}[402\uparrow]_p \mp \frac{7}{2}[633\uparrow]_n\}$ around 1300 keV separated from each other by 50 ± 30 keV.

C. Negative parity bands

1. Bands involving the ground state neutron

In addition to the ground state GM pair $(7^-, 0^-)$ discussed earlier, negative parity bands are expected from the

coupling of the ground state $\frac{7}{2}^+[633]$ neutron to two other proton states, i.e., the $\frac{1}{2}^- [541]$ proton ($E_p = 680$ keV) yielding the $(3^-, 4^-)$ GM pair for the *DZ* configuration, and the $\frac{5}{2}^- [532]$ proton (estimated $E_p \sim 1400$ keV) yielding the $(6^-, 1^-)$ GM pair for the *HZ* configuration. While neither of the former bands has been identified, the recent (t, α) study¹⁴ proposed a tentative $K^\pi = 6^-$ band starting at 1560 keV with levels up to $j^\pi K = 9^- 6$ suggested on the basis of the rotational energy spacings. Our calculations place (see Fig. 2) the $K^\pi = 6^-$ bandhead from the *HZ* configuration around 1400 keV. The $K^\pi = 1^-$ bandhead is expected to lie $\gtrsim 200$ keV higher and would be very hard to identify considering the density of levels around that excitation energy.¹⁵

We calculate the bandhead energy for the $K^\pi = 3^-$ level of the *DZ* configuration as 830 ± 40 keV, with the $K^\pi = 4^-$ bandhead lying about 70 ± 20 keV above it. The (\bar{n}, γ) spectrum⁹ lists nine negative parity states with $J^\pi = 2^- 5$ within this energy range, but in the absence of any other evidence, it is not possible to assign specific $I^\pi K$ values to the individual states.

2. Band involving excited configurations

Eight other negative parity bands involving both neutron and proton excited configurations within the energy range $E_p + E_n$ up to 750 keV are listed in Table III. Our calculated bandhead energies for these eight bands are shown in Fig. 1. The experimental situation with respect to these excited negative parity bands is very uncertain. Particle transfer reactions obviously cannot provide any definitive results for the bands built on both the neutron and proton excited states. Radioactive decay studies indicate the presence of only one negative parity state (other than the three rotational states of the ground state $K^\pi = 0^-$ band) located at 373 keV with a spin value of 0 or 1. Average resonance neutron capture spectra generally indicate the parity of the states within the spin range 2–5, but further limitation of J^π to either $(2, 5)^-$ or $(3, 4)^-$ has been possible in only a few cases. Based on their ARC data, Bollinger and Thomas⁹ proposed, with some reservations, the existence of three bands corresponding to the configurations under discussion. However, due to an oversight, the quantum numbers K in two cases were deduced incorrectly by them. Their bands *G* and *K* (see their Fig. 19 and Table V) arise from the coupling $\{\frac{3}{2}^+[411\uparrow]_p \mp \frac{1}{2}^- [521\downarrow]_n\}$, which results in the bands with $K^\pi = 1^-$ and 2^- ; they erroneously listed the resulting bands as $K^\pi = 2^-$ and $K^\pi = 3^-$ and sought to find rotational levels fitting these bandheads. We have reanalyzed their data and the results are discussed in the following paragraphs.

The lowest negative parity excitation in ^{166}Ho is populated in the ^{166}Dy beta decay; it depopulates^{1,4,8} through the 290.6 keV *M1* transition to the 82.4 keV 1^- level of the ground state band. The β -decay characteristics for the 373 keV level restrict its possible J^π values to 0^- or 1^- . Motz *et al.*⁸ accordingly suggested either the parallel spin ($K^\pi = 1^-$) or antiparallel spin ($K^\pi = 0^-$) *CY* configuration to this level. Bollinger and Thomas⁹ tried incon-

clusively to fit the 373 keV state as the bandhead of a $K^\pi = 0^-$ band; they also suggested a $K^\pi = 2^-$ band with the $2^- 2$ bandhead at 416 keV as the spin-parallel member of the GM pair of the *BY* configuration. As noted in the preceding paragraph, the correct spin-parallel band for this configuration has $K^\pi = 1^-$ instead of 2^- . Later, Schilling *et al.*¹¹ constructed a $K^\pi = 1^-$ band with the 373 keV $1^- 1$ and the 416 keV $2^- 1$ members of this band, which they assigned to the *CY* configuration.

The other negative parity bands proposed by Bollinger and Thomas⁹ are a $K^\pi = 3^-$ band starting at 563 keV as the spin-antiparallel member of the *BY* GM pair and a $K^\pi = 2^-$ band starting at 638 keV assigned to the *CX* configuration. We note that the correct spin-antiparallel member of the *BY* configuration is $K^\pi = 2^-$ instead of 3^- . It so happens that the J^π deduced for the 563 keV state (see Table VI of Ref. 9) is $(2, 3)^-$, and it can be very well be accepted as the $K^\pi = 2^-$ bandhead.

Thus we have the experimental indication of three excited negative parity bands—a $K^\pi = 1^-$ band starting at 373 keV,¹¹ and two $K^\pi = 2^-$ bands, one starting at 563 keV and the other at 638 keV.⁹ The experiments on ^{166}Ho do not presently provide any basis for a preferred configuration assignment to any of these bands; however, four of these bands involving $\frac{1}{2}^+[411]_p$ have been identified in three Tm isotopes and the lower two involving $\frac{3}{2}^+[411]_p$ have been seen in ^{170}Tm . Our calculated spectra, as shown in Fig. 1, use interaction parameters derived from a fit to the observed energy levels in Tm isotopes. From our Table III and the corresponding results in Fig. 1, we conclude the following. The matrix elements of the residual interaction for these four configurations (eight bands) and the ground state bands, as well as the interaction parameters derived from the ground state parameters of the respective nuclei, wherever applicable, are very similar; as such, to a first approximation these negative parity bands are expected to be placed sequentially according to the summed single particle energies and in accordance with the GM rule. Thus the lower excited negative parity band is the $K^\pi = 1^-$ band; the $K^\pi = 0^-$ band, being a spin singlet, will lie above its counterpart spin-triplet $K^\pi = 1^-$ band. Accordingly, for the observed 373 keV $(0, 1)^-$ state, the $K^\pi = 1^-$ assignment of Schilling *et al.*¹¹ is the acceptable choice. Based on the summed single particle energies, 1^- (*BY*) is expected to be lower by 67 keV than the 1^- (*CY*) level. However, the interaction energy contribution cancels about 30 keV of this difference. Further, the $K^\pi = 1^-$ and $K^\pi = 0^-$ *CY* bands are expected to have significant Coriolis admixture; this is in evidence in the $(1^-, 0^-)$ pair in the ^{170}Tm spectrum³⁶ as the odd-even staggering observed for the $K^\pi = 1^-$ ground band levels. This admixture pushes the observed $K^\pi = 1^-$ (*CY*) bandhead below its unperturbed position. A 10% admixture brings the calculated bandhead energy of the 1^- (*CY*) band into agreement with the observed 373 keV level. Thus, presently it is not possible to choose between the unperturbed *BY* and the admixed *CY* configurations for the 373 keV 1^- bandhead.

For the two experimentally suggested $K^\pi = 2^-$ bands with bandheads at 563 and 638 keV, respectively, we have two candidates, namely the spin-antiparallel $K^\pi = 2^-$

member of the *BY* configuration and the spin-parallel $K^\pi=2^-$ member of the *CX* configuration. Our calculations suggest the *CX* as the main configuration for the 563 keV 2^- band and the *BY* as the main configuration for the 638 keV 2^- band. The $K^\pi=3^-$ member of the *CX* configuration is predicted to lie about 200 keV above its 2^- GM counterpart, as shown in Fig. 1. The observed (\bar{n}, γ) spectrum shows a number of negative parity states in this energy range; however, in the absence of any other pointer, it is not possible to make a specific identification presently.

IV. SUMMARY AND CONCLUSIONS

We have calculated the complete two-particle low energy excitation spectrum of the doubly odd nucleus $^{166}_{67}\text{Ho}_{99}$ built from the single particle orbitals with the summed neutron and proton energies up to 750 keV and also from the coupling of the ground state neutron to all the known proton orbitals with excitation energies up to 1500 keV. The bandhead energies for the 34 bands expected for these coupled configurations have been calculated based on a quantitative evaluation of the zero range residual neutron-proton interaction energy contribution with the interaction parameters evaluated from the atomic mass data and the ground state GM splitting energies of specific configurations from the nuclei of the region. Thus the input for our calculations includes no experimental information from this nucleus except the ground state data. The moment of inertia parameters and the GM splitting energies are calculated for each of the two-particle configurations of interest. Our results are compared with the available experimental information to check the location and the configuration assignments in the case of 21 bands postulated from beta-decay, neutron capture, (d,p), and (t, α) studies. Based on this comparison, the following conclusions have been reached.

(a) Our results are in agreement with the assignments of the following nine bands involving the ground state neutron or proton deduced in the particle transfer reaction studies.^{8,14} The ground state $(7^-, 0^-)$ bands with the $K^\pi=7^-$ appearing as an excited band in apparent violation of the GM rule; the first excited neutron bands $(3^+, 4^+)$ starting at 190 and 372 keV, respectively; the 294 keV $K^\pi=6^+$ spin-parallel member of the second excited neutron band pair; the $(0^+, 7^+)$ bands starting at 801 and 915 keV, respectively involving the $\frac{7}{2}^+[404]$ excited proton orbital; and the $(1^+, 6^+)$ bands with bandheads at 1150 and 1272 keV, respectively, involving the $\frac{5}{2}^+[413]$ excited proton.

(b) We agree with the $K^\pi=1^+$ assignments for both the bands starting at 426 and 567 keV, respectively. However, we interchange the configuration assignments suggested earlier. Based on the revision of the β -decay $\log ft$ value by the Nuclear Data Sheets evaluator¹⁰ and our calculations for the bandhead energies along with the GM splitting energy estimates²¹ (in agreement with our calculations), we suggest the 426 keV 1^+ level as the $\{\frac{5}{2}[512]_p - \frac{7}{2}[523]_n\}$ spin-antiparallel $K^\pi=1^+$ bandhead

forming a GM pair with the 294 keV $K^\pi=6^+$ band. The 567 keV 1^+ level is identified as the $\{\frac{3}{2}[523]_p - \frac{7}{2}[523]_n\}$ $K^\pi=1^+$ bandhead with its $K^\pi=6^+$ GM counterpart bandhead predicted to lie around 825 keV.

(c) Although our calculations support the identification by Bollinger and Thomas⁹ of the 592 keV $K^\pi=3^+$ and 719 keV $K^\pi=4^+$ bandheads for the second excited proton bands, the assignment of Dewberry *et al.*¹⁴ placing these $(3^+, 4^+)$ bandheads at 719 and 891 keV, respectively, cannot be completely ruled out. Careful reinvestigation of the proton-pickup reaction and the 592 keV based band are suggested.

(d) Whereas Bollinger and Thomas⁹ and Schilling *et al.*¹¹ had simply labeled the 264 keV $K^\pi=5^+$ band and 430 keV $K^\pi=2^+$ band as the $\{\frac{7}{2}[523]_p \pm \frac{3}{2}[521]_n\}$ bands, we agree with the conclusions of Motz *et al.*⁸ and Dewberry *et al.*¹⁴ that these $(5^+, 2^+)$ bands have admixed structures with $\{\frac{3}{2}[411]_p \pm \frac{7}{2}[633]_n\}$ as the dominant configuration. We further suggest the 655 keV level as the $K^\pi=5^+$ bandhead and the 769 keV level as the $K^\pi=2^+$ bandhead with the dominant $\{[523] \pm [521]\}$ configuration; both these states have been seen in (d,p) spectra and evidence the given J^π character from (\bar{n}, γ) data.

(e) In partial revision of the assignments for the $(4^+, 3^+)$ bands involving the $\frac{1}{2}[510]$ excited neutron, we suggest the $K^\pi=4^+$ bandhead at 736 keV and the $K^\pi=3^+$ bandhead at 814 keV. While we accept the latter Bollinger-Thomas⁹ assignment, their 558 keV 4^+ level as this bandhead is found to be unacceptable. The suggested 736 keV (4^+) state has been seen in the (d,p) spectra and is compatible with the assigned spin from the (\bar{n}, γ) analysis.

(f) No definitive assignments have been experimentally made for any negative parity bands except the $(7^-, 0^-)$ ground state pair. Two out of the three bands proposed by Bollinger and Thomas⁹ had the wrongly assigned K values due to an error in the configuration couplings. The $K^\pi=1^-$ band built on the 373 keV 1^- state proposed by Schilling *et al.*¹¹ is acceptable, but the configuration assigned to it remains ambiguous. Two $K^\pi=1^-$ bands from the coupling of the $\frac{1}{2}^-[521]_n$ orbital to the $\frac{3}{2}^+[411]_p$ and $\frac{1}{2}^+[411]_p$ orbitals, respectively, are predicted to lie very close to each other. The identification of the other 1^- bandhead, the rotational states associated with each of the two bands (to look for the odd-even staggering expected in the latter coupling due to the Coriolis admixture with its $K^\pi=0^-$ counterpart), and the $K^\pi=0^-$ band (predicted to lie about 150 keV above its $K^\pi=1^-$ GM counterpart) represent important experimental challenges for further clarification of the situation. We have tentatively suggested dominant configurations for the two proposed $K^\pi=2^-$ bands with bandheads at 563 and 638 keV. We further predict a rather low lying (~ 500 keV) $K^\pi=4^-$ band and two $K^\pi=3^-$ bands and another $K^\pi=4^-$ band in the 800 keV to 1 MeV energy range. Out of these, a $(3^-, 4^-)$ GM pair shown in Fig. 2 should be accessible through proton transfer reactions. The situation with respect to the negative parity bands, on the whole, remains far from satisfactory and needs specific experimental attention.

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