Simple parametrization for octupole states in spherical and weakly deformed heavy nuclei

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A simple parametrization of the frequencies of low-energy octupole phonons in spherical and weakly deformed heavy nuclei is given. It is shown that this parametrization substantially simplifies the interpretation of octupole states in the $A=70$, $A=100$, and $A=120$ regions and provides information on the ordering and spacing of single particle orbitals in valence shells in the latter region. Finally, this parametrization is used for an investigation of the nature of the low-lying negative parity states in Ra and Th isotopes.

An octupole vibrational state can be simply understood as the coherent sum of a number of one-particle—one-hole (1p-1h) excitations between single particle orbitals differing in orbital angular momentum l, by $3\hbar$. For heavy nuclei (Z and $N>28$), the proton and neutron valence shells each contain such a pair of single particle states—a unique parity orbital with $l=(N+1)\hbar$ (where N is the oscillator quantum number of the common parity orbitals of the shell) and total angular momentum $j=(2N+3)\hbar/2$, and a common parity orbital with $l=(N-2)\hbar$ and $j=(2N-3)\hbar/2$. We shall call such a pair of orbitals a $\Delta l=3$ pair. As pointed out by Bohr and Mottelson,[1] 1p-1h excitations between members of the $\Delta l=3$ pairs (both proton and neutron) generally dominate low-energy octupole excitations in nuclei which are not doubly magic. This behavior leads to a simple qualitative description of the variation of octupole phonon frequency with changing N and Z for weakly deformed nuclei [$\beta_2<0.10$, or roughly, $E(4_1^+)/E(2_1^+)<2.7$]. As particles are added to the lower energy orbital of a $\Delta l=3$ pair and additional 1p-1h excitations become available, the energy eigenvalue of their coherent sum, which is the low-energy collective octupole state, is driven downward. The $N=82$ isotones can be used to demonstrate the relevance of such an interpretation in a striking way (see Fig. 1). In a sequence from $Z=56$ (barium) to $Z=64$ (gadolinium), the excitation energies of the 2_1^+ states are nearly constant, as are the values of $B(E2{:}2_1^+\rightarrow 0_{\rm g.s.}^+)$ (with the exception of ^{146}Gd, for which this matrix element has not been measured); clearly, the ground state quadrupole deformation is nearly constant in this chain. The same sequence, however, displays a drop in the energy eigenvalue of the low-energy octupole state from 2.8 to 1.6 MeV, which can be attributed to the filling of the $\pi d_{5/2}$ orbital, the lower energy orbital in the $\Delta l=3$ pair for the $Z=50$—82 shell. A qualitative analysis based on this description was applied to the lanthanide region and used as a test of the nature of low-lying negative parity states in the radium and thorium isotopes in Ref. 2.

The interpretation suggested in the preceding paragraph lends itself to a simple quantitative treatment. In this paper, we formulate a simple parametrization of low-energy octupole phonon frequencies for spherical and weakly deformed nuclei, and give examples of its utility for the understanding of the systematic behavior of low-energy octupole states in several regions of the periodic table. We conclude by examining the $Z=82$—90 neighborhood with this parametrization to further investigate the radium and thorium isotopes.

The scaling parameter we discuss here takes into account the numbers of valence neutrons ($N_{\rm n}$) and protons ($N_{\rm p}$), the excitation energies of the single particle orbitals in a spherical shell model, ϵ_ν, and the pairing interaction (the importance of which will be illustrated in our first example). First, the occupation numbers of the single particle orbitals are calculated using the Bardeen-Cooper-Schrieffer (BCS) equations, which can be written

$$2\sum_\nu V_\nu^2=N\ ,$$

$$V_\nu^2=(1/2)[1-(\epsilon_\nu-\lambda)/E_\nu]\ ,$$

$$E_\nu=[(\epsilon_\nu-\lambda)^2+\Delta^2]^{1/2}\ ,$$

where Δ is the pairing gap parameter and is set (as sug-

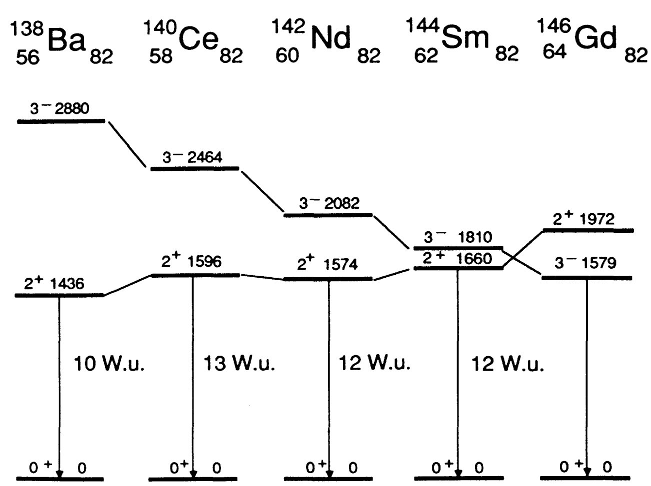

FIG. 1. Excitation energies of the 2_1^+ and 3_1^- states and $B(E2;2_1^+\rightarrow 0_{\rm g.s.}^+)$ values for a series of $N=82$ isotones. These nuclei demonstrate the independence of the octupole phonon frequency from the ground state quadrupole deformation. Data are taken from Refs. 11—16.

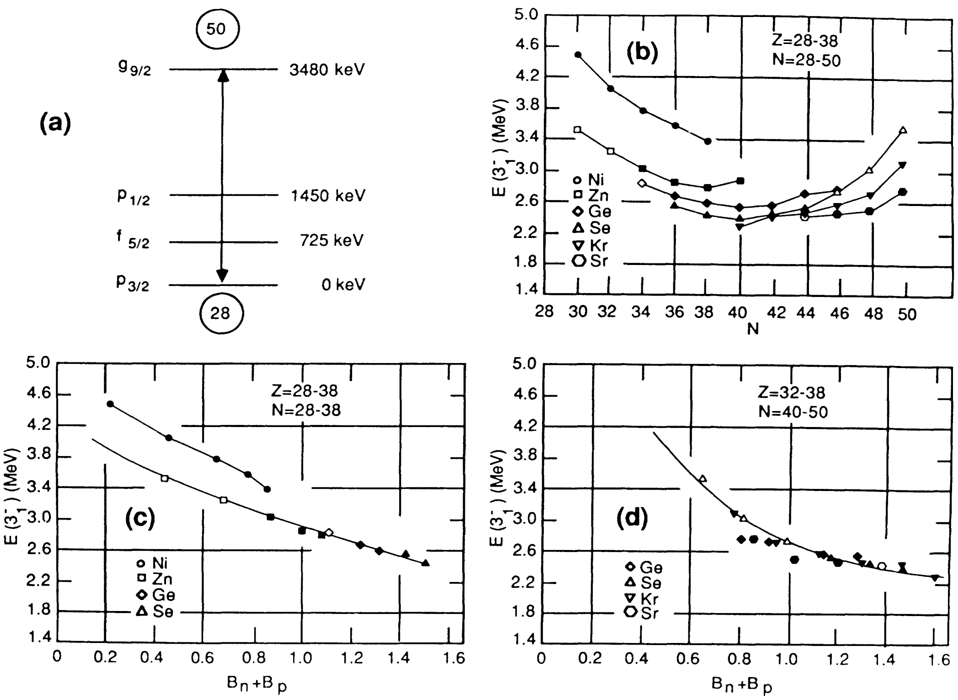

FIG. 2. (a) Single particle level energies used for both protons and neutrons in the calculation for the Ni-Sr region. The $\Delta l = 3$ pair for this region is $p_{3/2}$-$g_{9/2}$: (b) $E(3_1^-)$ vs neutron number for the Ni-Sr region (Ref. 4). Firm and tentative assignments are denoted by filled and open symbols, respectively. (c) $E(3_1^-)$ vs $B_n + B_p$ for $N = 28-38$. (d) $E(3_1^-)$ vs $B_n + B_p$ for $N = 40-50$.

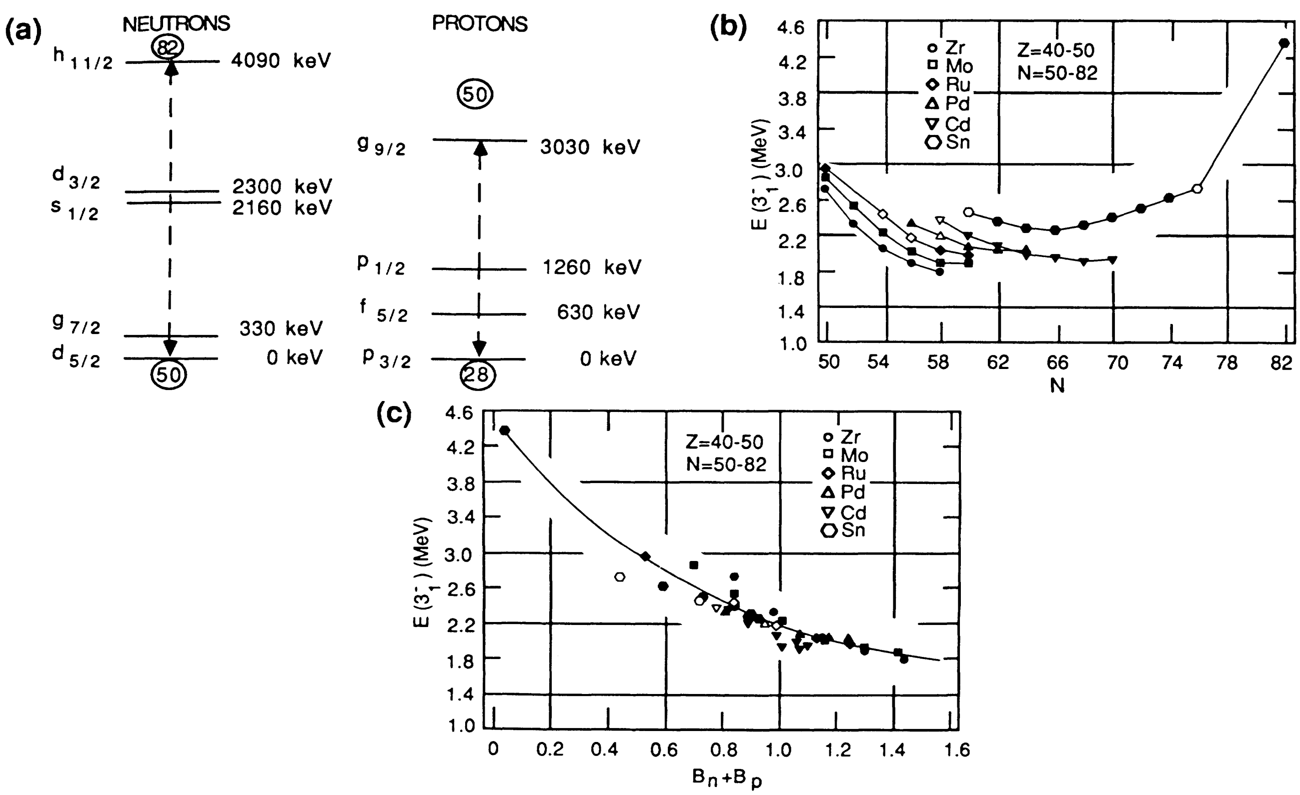

FIG. 3. (a) Single particle level energies used for the Ru region. The $\Delta l = 3$ pairs for this region are $\nu d_{5/2} - \nu h_{11/2}$ and $\pi p_{3/2} - \pi g_{9/2}$. (b) $E(3_1^-)$ vs neutron number for the Ru region (Refs. 4 and 17–21): (c) $E(3_1^-)$ vs $B_n + B_p$ for the Ru region.

gested in Ref. 3) equal to $12/A^{1/2}$ (A is set to an average mass for the region of interest) and N is either $N_{\rm n}$ or $N_{\rm p}$; the occupation numbers, V_ν^2, quasiparticle energies, E_ν, and the chemical potential, λ are determined iteratively by a computer code of approximately ten instructions. If the lower and higher energy orbitals in a $\Delta l=3$ pair are j^1 and j^2, respectively, then we define $B_{\rm n}$ and $B_{\rm p}$ for the neutron and proton shells by

$$B=\min(V_{j^1}^2, 1-V_{j^2}^2) .$$

This function has properties that we require: As the lower orbital fills, "more" 1p-1h states become available, and B increases; however, once the upper orbital begins to fill, 1p-1h states are blocked and B decreases. The resulting scaling parameter is $B_{\rm n}+B_{\rm p}$. We would expect that the octupole phonon frequency would reach a maximum where $B_{\rm n}+B_{\rm p}=0$, and a minimum where this sum is a maximum.

Our technique can be motivated using the $N=28-50$, $Z=28-38$ region as an example of the systematic behavior of octupole states. In this region, the single particle structure is nearly identical for neutrons and protons, the $\Delta l=3$ pair being $p_{3/2}$-$g_{9/2}$ for both; a diagram showing spherical shell model single particle energies is found in Fig. 2(a). The necessity of including pairing in our parametrization is clearly illustrated by Fig. 2(b); even though the $\nu p_{3/2}$ orbital is the lowest in the shell, $E(3_1^-)$ does not reach a minimum until $N=40$, where there are 12 valence neutrons. The dependence on Z is similar.

Figures 2(c) and 2(d) display the simplification resulting from the use of the $B_{\rm n}+B_{\rm p}$ parametrization in the $N=28-50$ region. Most nuclei in this region have $E(3_1^-)$ values that fall close to a single curve, although those of the nickel isotopes are offset from it by roughly 500 keV.

Figure 3 illustrates the use of the $B_{\rm n}+B_{\rm p}$ parametrization in the $Z=40-50$, $N=50-82$ region. Once again, the trends observed in the excitation of the 3_1^- states are considerably simplified, demonstrating the value of the description of low-energy octupole excitations under consideration. It bears emphasis, however, that there are nuclei in this region for which the ground state quadrupole deformation is large enough to invalidate our model (for example, ^{100}Zr); no 3_1^- states of such nuclei appear in recent data compilations.[4]

The procedure described herein is clearly dependent on the ordering and spacing of single particle levels in the valence shells; we can exploit this property to test gross features of the structure of the shells. In Fig. 4, we demonstrate this by using the $B_{\rm n}+B_{\rm p}$ parametrization in the $Z=50-64$, $N=64-82$ region with two different sets of ϵ_ν for the valence proton shell,[5,6] which differ primarily in the strength of the spin-orbit interaction. As a result, the $\pi d_{5/2}$ orbital, which is the lowest in (I), is 770 keV above the $\pi g_{7/2}$ orbital in (II). Furthermore, the $Z=64$ gap in (I) is eradicated by the lowering of the $\pi h_{11/2}$ orbital in (II). Figure 4(c) displays the result of using configuration (II) for the calculation of $B_{\rm n}+B_{\rm p}$: The parametrization fails completely. When configuration (I) is used, however [Fig. 4(d)], the calculation is substantially more successful. The essential characteristic of (I) which yields these improved results is the ordering of the $d_{5/2}$ and $g_{7/2}$ orbitals; the $\pi h_{11/2}$ orbital is only sparsely occupied when $Z\leq 64$ [even for configurations (I)] and, consequently, its precise energy placement has little effect upon $B_{\rm p}$. Five nuclei, however, still fall significantly off the curve in 4(d); of these, four suggest interesting physics. As illustrated in Fig. 4(a), for Nd, Sm, and Gd, $E(3_1^-)$ at $N=82$ is either equal to or less than the value for $N=80$, in contrast to the trends we observe for the other elements shown. This behavior, in addition, is contrary to our expectation that the frequency of the octupole phonon would increase as the $\nu h_{11/2}$ orbital is filled at the end of the shell. However, we can propose an explanation for this using the results of an investigation by Sorenson on the shifting of single particle energy eigenvalues in a spherical shell model.[7] The $\nu d_{3/2}$ orbital in the $N=50-82$ shell and the $\nu h_{9/2}$ orbital in the $N=82-126$ shell also form a $\Delta l=3$ pair. Even though these two orbitals are in different shells, Sorenson predicts a drastic narrowing of the gap between them (to ≈ 2 MeV) as Z approaches 64. For Sm and Gd, the $\nu d_{3/2}$ orbital is below the Fermi level for $N=80$ and can interact strongly with the $\nu h_{9/2}$ orbital. As the $\nu h_{11/2}$ orbital fills and the $\nu d_{5/2}-\nu h_{11/2}$ interaction is progressively blocked, the $\nu d_{3/2}$-$\nu h_{9/2}$ pair becomes an important component of the octupole phonon, and the variation of phonon frequency with neutron number would be expected to be affected significantly.

Finally, we attempt to quantify the discussion of the radium and thorium isotopes presented in Ref. 2 with the parametrization discussed in this paper. The single particle energy levels used for this region are taken from the work of Kuo and Herling,[8] and are illustrated in Fig. 5(a). In the $E(3_1^-)$ vs $B_{\rm n}+B_{\rm p}$ plot for the $Z=82-90$, $N\geq 126$ region, the $E(3_1^-)$ values for the Ra and Th isotopes roughly fall onto a curve which also passes through the octupole vibrational states observed in the Pb and Po isotopes and is not far removed from the octupole states in the Rn isotopes. In this sense, it would appear that the behavior we observe in the Ra and Th isotopes is consistent with an octupole vibrational interpretation. If we regard octupole deformation as the end point of a systematic octupole vibration-to-static octupole deformation transition (as proposed by Nazarewicz *et al.*[9]), then the possibility of a static octupole deformed interpretation of several Ra and Th nuclei cannot be disregarded. The systematic bifurcation evident between the Rn and Ra isotope points in Fig. 5(c) must be considered significant. If the negative parity states in the Ra and Th isotopes are interpreted in terms of octupole vibrations, such branching may indicate that the $\pi f_{7/2}$ orbital is somewhat lower in energy with respect to the $\pi h_{9/2}$ orbital in Ra and Th than we would calculate using the same shell model configuration that we use in the $Z=82-86$ elements. The branching may also, however, be the signature for the onset of static octupole deformation. Finally, we must consider the possibility that yet another mechanism, such as the alpha particle clustering proposed by Iachello and Jackson,[10] is becoming evident in the Ra and Th isotopes, either through mixing with the octupole configuration or

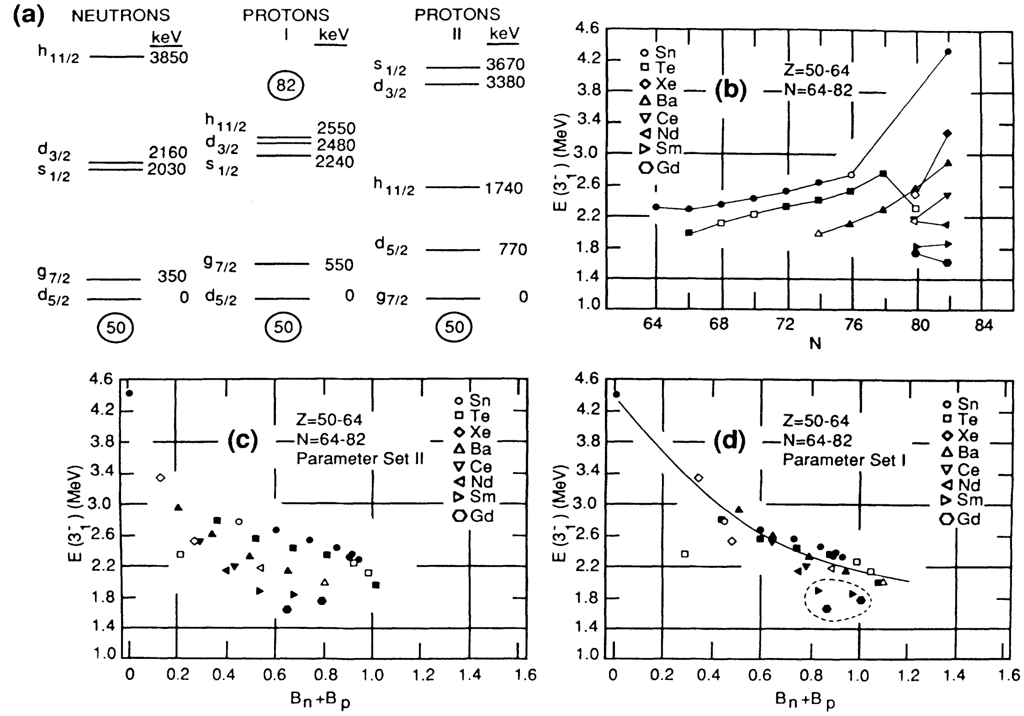

FIG. 4. (a) Single particle level energies used for the Te region. The $\Delta l = 3$ pairs for this region are $d_{5/2}$-$h_{11/2}$ for both neutrons and protons. (b) $E(3_1^-)$ vs neutron number for the Te region (Refs. 4, 17, and 20—24). (c) $E(3_1^-)$ vs B_n+B_p for the Te region using proton parameter set II: (d) $E(3_1^-)$ vs B_n+B_p using proton parameter set I.

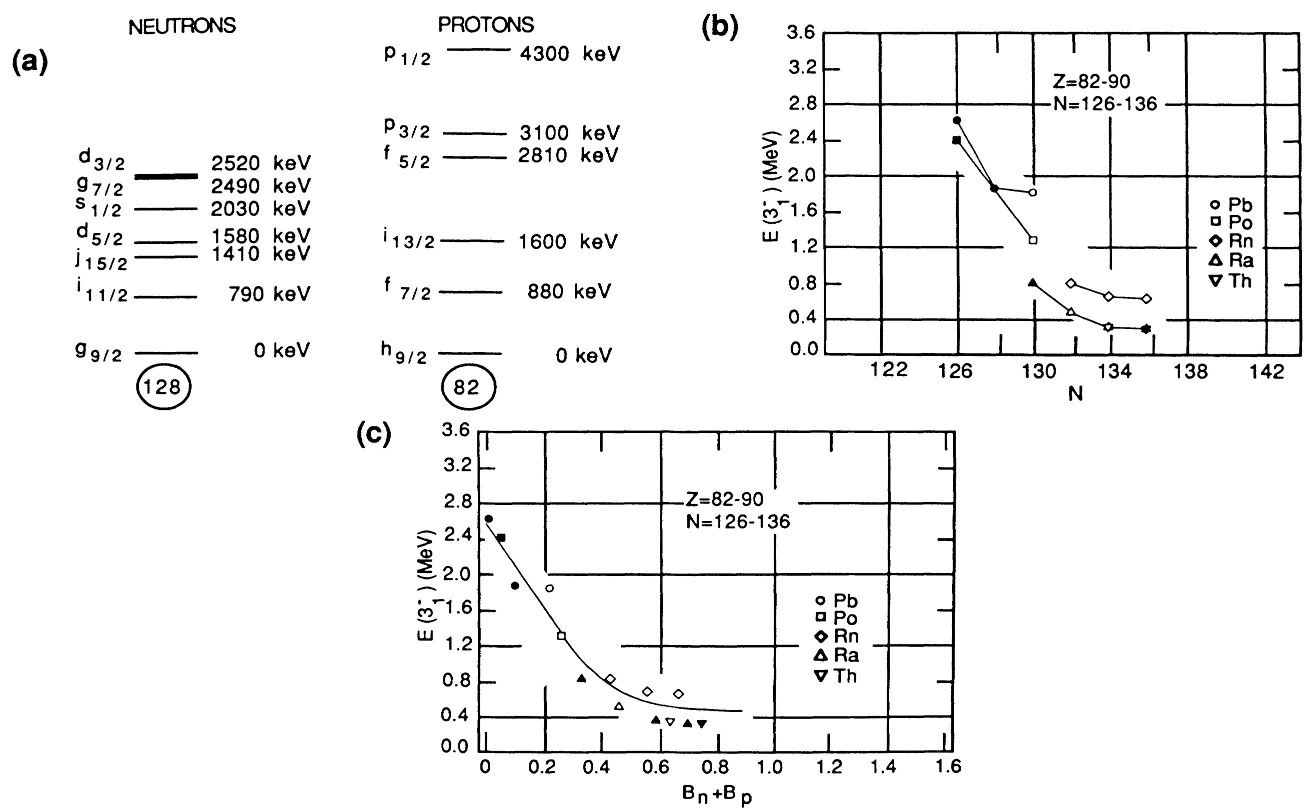

FIG. 5. (a) Single particle level energies used for the light actinide region. The $\Delta l = 3$ pairs for this region are $\nu g_{9/2}$-$\nu j_{15/2}$ and $\pi f_{7/2}$-$\pi i_{13/2}$. (b) $E(3_1^-)$ vs neutron number for the light actinide region (Refs. 4, 17, and 25—28). (c) $E(3_1^-)$ vs B_n+B_p for the light actinide region.

by dominating the nuclear structure in this region. Attempts at resolving these questions would be aided significantly by the observation and study of presently unknown low-spin negative-parity states in the light Rn, Ra, and Th isotopes.

In summary, we have presented a simple parametrization of octupole phonon frequencies in spherical and weakly quadrupole deformed nuclei that illustrates the essential mechanisms driving these excitation modes, and have used this method to analyze the spacing and ordering of single particle orbitals in the $Z=50-82$ shell. Through an examination of the $Z=82-90$ region, we have found that the gross features of the observed systematic behavior of 3_1^- states in Ra and Th isotopes can be explained using this parametrization, but that a more detailed and definitive explanation of the observed behavior requires further experimental investigation.

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