Face-centered-cubic solid-phase theory of the nucleus

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The symmetries of an antiferromagnetic face-centered-cubic lattice with alternating isospin layers reproduce simultaneously several one- and two-dimensional spin- and isospin-ordered states which have previously been shown to be low energy configurations for a condensed nuclear phase. The face-centered-cubic lattice also shows a precise correspondence with the *j-j* model and, consequently, the entire nucleon buildup procedure of the independent-particle model. It is concluded that the study of nuclear condensates at normal nuclear densities should begin with the face-centered-cubic configuration.

I. INTRODUCTION

Over the past 15 years, much work has been done on the possible existence of nucleon or pion condensates at densities near or above that of normal nuclei. The density at which various kinds of condensation might take place remains controversial and depends upon the forces and actual configuration of nucleons which are assumed. Indeed, many different condensates have been studied, including one-, two-, and three-dimensional configurations of pure neutron matter or nuclear matter ($N=Z$ or $N \gg Z$) in spin- and/or isospin-ordered states.

In the present paper, no attempt is made to add to those arguments concerned primarily with the density value at which condensation occurs. Rather, we show that several low-energy states which have been studied by others are simultaneously found in a particular three-dimensional lattice. That lattice is the antiferromagnetic face-centered-cubic configuration with alternating isospin layers (the FCC model). It is also shown that the FCC lattice implies a description of nucleon states which is identical to that of the independent-particle model. As a consequence, the major features of nuclear structure theory which are accounted for in the independent-particle model can also be accounted for in the FCC model.

Whether or not the combined binding contributions of several one- and two-dimensional ordered states can generate sufficient nuclear binding to allow for stability at nuclear densities remains uncertain. Nevertheless, the striking isomorphism between the FCC lattice and the known eigenvalue symmetries of normal nuclei indicates that, *if a condensed nuclear phase is energetically stable,* then investigations of the solid phase should begin with the FCC model. Other solid-phase nuclear models, briefly discussed below, show few advantages and many disadvantages—primarily in their inability to account for the independent-particle nature of nuclei.

II. THE CONDENSATION OF NUCLEAR MATTER

Solid-phase theories of nuclear structure have appeared sporadically since the discovery of the neutron in 1932.[1–9] Their theoretical strengths and weaknesses have been varied, but they all imply a dense nuclear interior from which certain realistic nuclear properties can be deduced. These include essentially all of the properties of the "liquid drop" model [constant nuclear density and the implied saturation of the nuclear force, short mean free path (MFP) of intranuclear nucleons, dependence of the nuclear radius on the number of nucleons present, the incompressibility of nuclear matter, and other collective properties], as well as alpha clustering, which is inherent to the tetrahedral arrangement of nucleons in any close-packed configuration.

Although some of the solid-phase models can also account for the emergence of "magic" numbers in the buildup of nuclei, the validity of the independent-particle description of the nucleus has presented difficulties for most of the solid-phase theories because nucleon "orbiting" is explicitly prohibited within a nucleon solid. The prospects for a solid-phase nuclear theory have been kept alive, however, by recent developments concerning possible nucleon and pion condensates. Theoretical estimates of the density at which pure neutron matter or mixtures of protons and neutrons would solidify vary by more than an order of magnitude,[10] but it has been shown that increased nuclear binding can be achieved through the tensor part of the nuclear force in various ordered states.[11–19] While literal nucleon orbiting remains problematical within any ordered state, "weak" or "strong" condensation at nuclear densities could arise if mechanisms for producing additional internucleon binding can be found.

III. THE NUCLEAR DENSITY, NUCLEON DIMENSIONS, AND THE NUCLEON MEAN FREE PATH

The underlying motivation for pursuing solid-phase nuclear theories stems from the known dimensions of nucleons and nuclei and the extremely short MFP's of nucleons within the nucleus. In contrast to the pointlike nucleons assumed in the independent-particle model, electron and muon scattering studies have shown both protons and neutrons to have charge and magnetic radii of

0.8 fm.[20] Such dimensions are already a large percentage of the *nuclear* rms radius (1.7—5.8 fm) and suggest that models based upon freely-orbiting, pointlike nucleons may not be realistic.

Moreover, the density at the core of the larger nuclei is known to be a value of approximately 0.17 nucleons/fm^3.[21] Even if nucleons were put into a close-packed configuration which produces such a density, a nearest-neighbor internucleon distance of only 2.0 fm is implied (four nucleons within the FCC close-packed unit cell with a cube edge of $\sqrt{8}$ fm; in a less densely packed array, such as cubic, a nearest neighbor distance of only 1.8 fm would be required to obtain a density of 0.17 nucleons/fm^3). In such arrays, space-occupying nucleons would be almost contiguous and very little intranuclear nucleon movement, much less nucleon orbiting, could be expected. As we have reported elsewhere,[7] using an internucleon distance of 2.0 fm and the FCC model, nuclear rms radial values for nuclei over the entire periodic chart ($A > 20$) can be produced within 1.3% of empirical values. Such calculations argue for the viability of the FCC nuclear model, but, more importantly, they demonstrate the validity of the 2.0 internucleon distance.

It is worth recalling that the extremely short MFP's of intranuclear nucleons was one of the early paradoxes posed by the shell model. Although the shell model soon proved itself invaluable in nuclear spectroscopy, it was based fundamentally upon the unimpeded orbiting of nucleons over distances which are "several *nuclear* diameters" (20—30 fm).[22] Empirically, however, it was known that the MFP of low energy (< 30 MeV) nucleon projectiles was 0.4—1.0 fm—or less than one *nucleon* diameter.[22,23] The paradox of having a long MFP in a medium as dense as the nucleus was explained away simply as due to the exclusion principle preventing otherwise inevitable particle interactions,[22] but the MFP problem has not disappeared. Some maintain that a large MFP of 20—30 fm can be theoretically sustained for intranuclear nucleons,[24] while others argue that the empirical evidence indicates a much shorter MFP on the order of one *nucleon* diameter[25]—and implicitly the unlikelihood of nucleon orbiting.

With the spatial extent of nucleons now having become a central issue in quark theory,[26] the MFP question has resurfaced in terms of the magnitude of the interaction between quarks in adjacent nucleons. That is, a quark potential well which is many hundred MeV deep at distances of less than 1 fm could imply a quark contribution of several tens of MeV to *internucleon* binding (1—2 fm). Given a nucleon diameter of 1.6 fm, a center-to-center internucleon distance of 2.0 fm, and an estimated quark radius of 0.5 fm,[27] a small change in the shape and depth of the quark potential well could mean that quarks are responsible not only for the cohesiveness of nucleons themselves, but also for *part* of the attractive force among nucleons.

As will be discussed in Sec. V, the FCC model provides an alternative independent-particle description of nuclei which is consistent with strong nearest-neighbor interactions and which does not require nucleon orbiting. As a consequence, it may obviate the necessity of invoking the "Pauli force" to allow for a long MFP in a substance as dense as the nucleus.

IV. THE STABILITY OF NUCLEON CONDENSATES

The first objection which any solid-phase nuclear theory must address concerns the energetic stability of a nucleon lattice. Calculations based on the uncertainty relations indicate that, when there is an uncertainty of only 1.0 fm in the particle position (half the internucleon distance in a close-packed array), there is an uncertainty in the nucleon momentum which corresponds to some 40 MeV per nucleon—well over the 30 MeV nuclear potential well which is conventionally assumed. Unless a considerable amount of negative potential energy can be generated, this level of nucleon momentum would lead to nuclear instability. Recent speculation has suggested that quark effects may also contribute to nucleon binding,[26] but more concrete results have already been obtained in research on nucleon and pion condensates.

Starting in the early 1970's, Migdal *et al.*[11] and other groups[12–19] showed that previously unanticipated binding energy (and/or diminished repulsion) can be produced in various ordered states, thus making certain condensed configurations more energetically favored than comparable liquid or gas phases. Most approaches, in fact, indicate that solidification can occur only at densities greater than normal nuclear density, but Calogero and Palumbo[12] found a unidimensional ordered state that is favored already at nuclear densities. Of particular interest in the present context is that several of the nucleon configurations which have been shown to be low-energy states are contained simultaneously within the antiferromagnetic FCC lattice (see Fig. 1 and Table I)—suggesting that a combination of binding mechanism could lead to stability at nuclear densities.

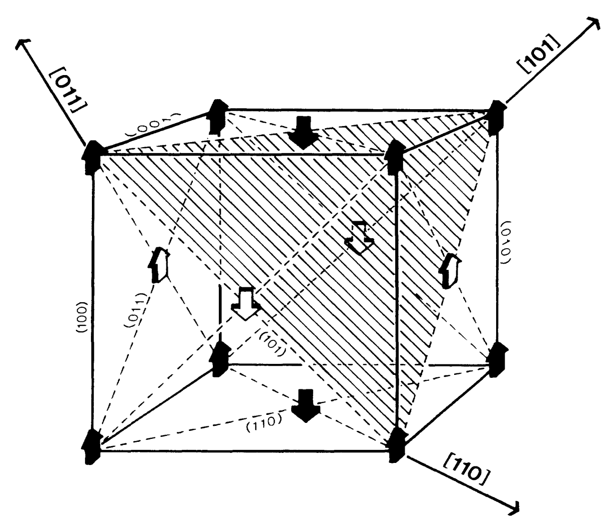

FIG. 1. Planes (in parentheses) and directions (in brackets) of symmetry in the FCC lattice. The shaded plane is (111). See Table I for details of binding effects.

The two principal sources of additional attraction in a nucleon condensate come from (i) the tensor force and (ii) the decreased contribution of the (mainly repulsive) interaction at short range. The separation of nucleons within a three-dimensional lattice can of course operate in all three dimensions. While paying a penalty in terms of kinetic energy, the repulsive contribution among nucleons, and particularly protons, is thereby minimized. Similarly, ordered spin states lead to tensor effects in several directions. Considering only the nearest neighbor interactions in the FCC lattice, attractive tensor effects among nearest neighbors are found in two of the three undimensional *directions* of symmetry, [110] and [011]. Among the *planes* of symmetry, five of the seven planes which include nearest neighbors show attractive tensor interactions. Because of redundancies in the effects listed in Table I and some cancellation of attractive and repulsive effects, the net contribution of the nearest neighbor spin- and isospin-ordered states within the FCC configuration is only twice that which is found in previously studied

TABLE I. The presence of previously studied, one- and two-dimensional condensed states within the FCC lattice.

Antiferromagnetic FCC symmetry	Description	Source of additional binding or decreased repulsion
(111)	Two-dimensional mixed spin, mixed isospin	Alternating isospin layers, attractive tensor effects between nearest neighbor *like* nucleons
(110)	Two-dimensional mixed spin, pure isospin	Alternating isospin layers, attractive tensor effects between nearest and second nearest neighbor *like* nucleons
(101)	Two-dimensional mixed spin, mixed isospin	Alternating isospin layers
(011)	Two-dimensional pure spin, mixed isospin (Ref. 18)	Alternating isospin layers, attractive tensor effects between nearest neighbor *unlike* nucleons
(100)	Two-dimensional mixed spin, mixed isospin (Refs. 13,14,16)	Alternating isospin layers, attractive tensor effects between second nearest neighbor *like* nucleons in different isospin layers
(010)	Two-dimensional pure spin, mixed isospin	Alternating isospin layers, attractive tensor effects between nearest neighbor *unlike* nucleons and between second nearest *like* nucleons in different isospin layers
(001)	Two-dimensional mixed spin, pure isospin	Attractive tensor effects between nearest neighbor *like* nucleons
[110]	One-dimensional mixed spin, pure isospin (Refs. 11,12)	Attractive tensor effects between nearest neighbor *like* nucleons
[101]	One-dimensional mixed spin, mixed isospin (Refs. 11,12)	Alternating isospin layers
[011]	One-dimensional pure spin, mixed isospin (Refs. 11,12)	Alternating isospin layers, attractive tensor effects between nearest neighbor *unlike* nucleons

two-dimensional ordered states, such as the alternating layer spin (ALS) model of Tamagaki *et al.*[13] There are, however, also second neighbor effects working between like-isospin layers, which remain to be included in such calculations.

V. EQUIVALENCE BETWEEN EIGENSTATES OF THE SCHRÖDINGER EQUATION AND THE FCC LATTICE

The solidification density and the source of additional binding energy needed for a solid-phase theory remain uncertain, but pursuit of a solid-phase nuclear theory requires, first of all, some indication that the major, known features of nuclear structure can be maintained within such a theory. In other words, if a solid-phase theory of nuclear structure is to be taken seriously, it must provide a coherent alternative to the powerful independent-particle description of the nucleus, within which most nuclear excited and ground states are currently understood. As we have discussed elsewhere,[7] the systematics of the eigenvalues in the independent-particle model can be reproduced exactly if it is assumed that the nucleus can be represented as a lattice of positions at which nucleons have a high probability of residing: $P(x,y,z)$ along three orthogonal axes. If it is also assumed that the lattice is antiferromagnetic FCC with alternating isospin layers (Fig. 2), then it is found that *all of the restrictions concerning allowed eigenstates of the Schrödinger equation are reproduced within the FCC lattice* (i.e., all of the states and substates with the appropriate eigenvalues and the known number of nucleons with any given eigenvalue or combination of eigenvalues). See Table II.

Specifically, the eigenvalues of nucleons in the FCC model can be redefined as follows. Each nucleon can be given an eigenvalue, n, determined by its position relative to three axes passing through the fixed center of the lattice system [Fig. 3(a)]:

$$n = (|x| + |y| + |z| - 3)/2$$
$$= (r\sin\theta\cos\phi + r\sin\theta\sin\phi + r\cos\theta - 3)/2 . \quad (1)$$

The axial position of the nucleon (from a nuclear "spin

TABLE II. The complete buildup sequence of nucleons within the FCC lattice. The rows correspond to the number of nucleons with each eigenvalue. The sequence and the entire pattern of nucleon eigenvalues are identical to those of the independent-particle model, except for the spacing of levels. In its simplest form, the FCC model predicts magic stability with the closure of all such shells and subshells. This is borne out by the fact that 15 of the first 17 subshells are magic by at least one criterion of magicness. Conventional criteria are as follows: *a*, number of stable isotopes; *b*, number of stable isotones; *c*, number of metastable (half-life > 1 yr) isotopes; *d*, number of metastable isotones; *e*, number of known isotopes; *f*, number of known isotones; *g*, quadrupole moment; *h*, neutron separation energy; *i*, excitation energy of first 2^+ state.

N							J							M							S		
0	1	2	3	4	5	6	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{5}{2}$	$\frac{7}{2}$	$\frac{9}{2}$	$\frac{11}{2}$	$\frac{13}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{5}{2}$	$\frac{7}{2}$	$\frac{9}{2}$	$\frac{11}{2}$	$\frac{13}{2}$	↑	↓	Total
2							2							2							1	1	2 *a,d,i*
								4						2	2						2	2	6 *d,e,f*
	6						2							2							1	1	8 *i*
									6					2	2	2					3	3	14 *h*
		12						4						2	2						2	2	18
							2							2							1	1	20 *a,b,c,d,e,h,i*
										8				2	2	2	2				4	4	28 *b,c,d,h,i*
									6					2	2	2					3	3	34 *d*
			20					4						2	2						2	2	38 *i*
							2							2							1	1	40 *b,e,g,i*
											10			2	2	2	2	2			5	5	50 *a,b,c,d,h,i*
										8				2	2	2	2				4	4	58 *b,f,i*
				30					6					2	2	2					3	3	64 *c,i*
								4						2	2						2	2	68 *e*
							2							2							1	1	70
												12		2	2	2	2	2	2		6	6	82 *b,d,h,i*
											10			2	2	2	2	2			5	5	92 *h*
										8				2	2	2	2				4	4	100
					42				6					2	2	2					3	3	106
								4						2	2						2	2	110
							2							2							1	1	112
													14	2	2	2	2	2	2	2	7	7	126 *h,i*
												12		2	2	2	2	2	2		6	6	138
											10			2	2	2	2	2			5	5	148
						56				8				2	2	2	2				4	4	156
									6					2	2	2					3	3	162
								4						2	2						2	2	166
							2							2							1	1	168

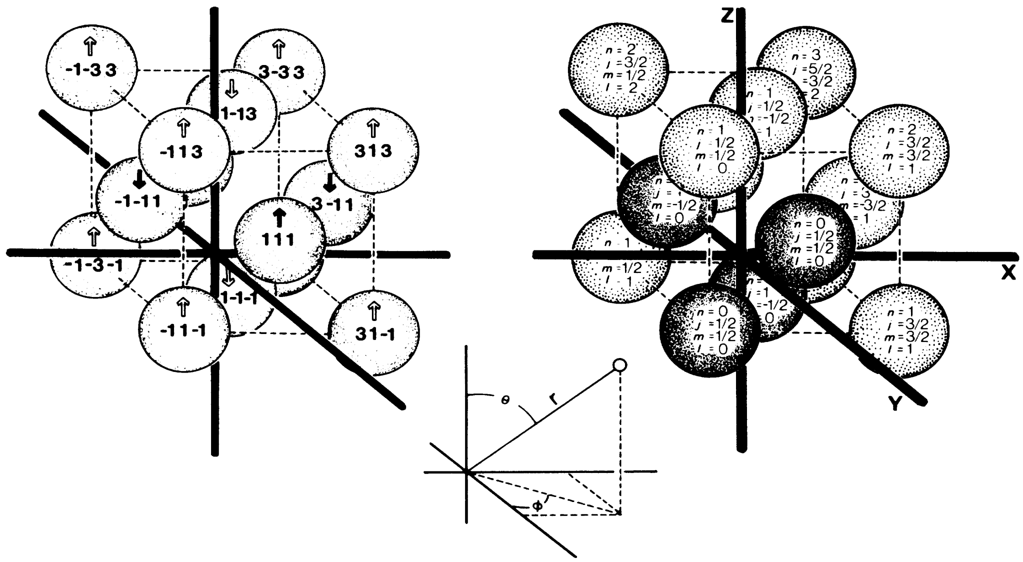

FIG. 2. Two representations of the antiferromagnetic FCC lattice with alternating proton and neutron layers. On the left are shown the nucleon coordinate values (x,y,z) of the 14 nucleons which comprise the unit cube. All coordinate values are odd integers. Spin and isospin are denoted by, respectively, the orientation and shading of the arrows (the assignment being arbitrary since the lattice shows triaxial symmetry around the origin). On the right are shown the n, j, m, and l eigenvalues, which can be deduced directly from the coordinate values of the nucleons and Eqs. (1)—(3). Note that the origin of the coordinate system is not at the center of the cube, but is at the center of a tetrahedron of nucleons (darkened spheres), which comprise the ^{4}He nucleus. The 14-nucleon cube corresponds to a highly unstable ^{14}Be or ^{14}Ne isotope (depending upon the choice of isospin shading) and is presented here only to illustrate the symmetries of the FCC lattice, rather than an actual isotope.

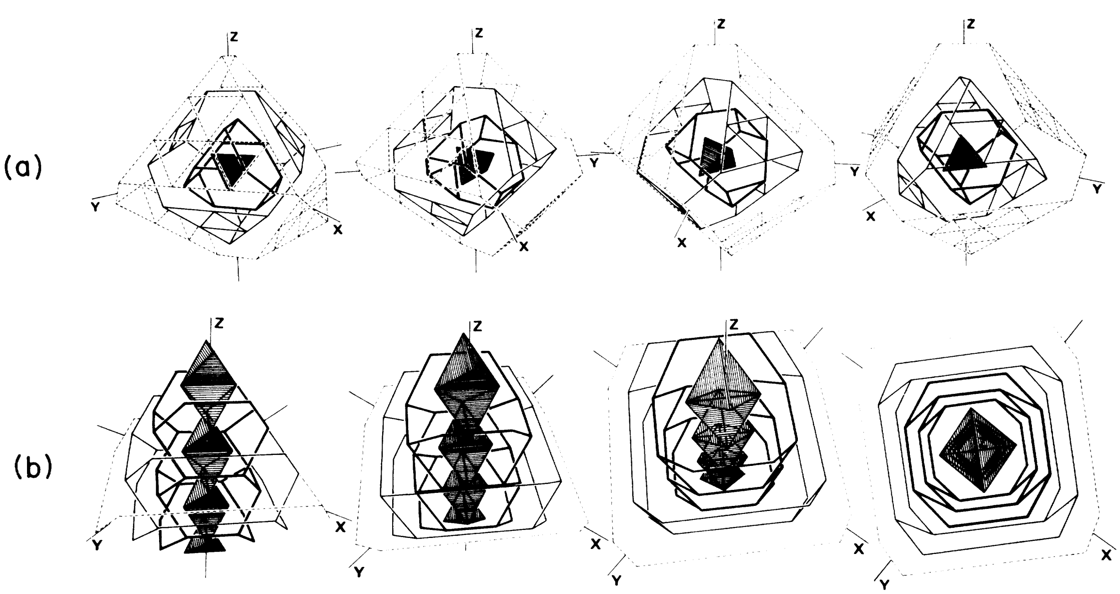

FIG. 3. (a) Illustration of the triaxial symmetry of the principal quantum number for the first four n shells. Bonds are drawn between all nucleons with the same n value, thus producing concentric n shells. The central tetrahedron is the ^{4}He nucleus ($n=0$), and is surrounded by triaxially symmetrical, truncated tetrahedrons which correspond to $n=1$, 2, and 3. (b) Illustration of the symmetry of the angular momentum subshells around the nuclear spin axis. Bonds are drawn between all neighboring nucleons with the same j value, thus producing cylindrical j shells. The shaded structure lying along the z axis corresponds to $j=\frac{1}{2}$ nucleons, and the three surrounding "cylinders" correspond to $j=\frac{3}{2}$, $\frac{5}{2}$, and $\frac{7}{2}$. Similar structures for larger values of n and j values (and other eigenvalues) can be built, and are found to maintain complete isomorphism with the j-j model (see Table II).

axis") can be given an eigenvalue, l, determined by the nucleon's location relative to the two axes orthogonal to the spin axis. Coupled with the intrinsic spin eigenvalue, s (as in the spin-orbit coupling model), a fourth eigenvalue, j, is also defined [Fig. 3(b)]:

$$j = |l+s| = (|x|+|y|-1)/2 = (r\sin\theta\cos\phi + r\sin\theta\sin\phi - 1)/2 \ . \quad (2)$$

A third eigenvalue, m, is determined by the nucleon's position relative to one axis:

$$m = (|x|)/2 = (r\sin\theta\cos\phi)/2 \ . \quad (3)$$

Whereas alternating spin layers are found orthogonal to the x axis (the antiferromagnetic arrangement), alternating isospin layers are found orthogonal to the z axis. This isospin configuration produces a layer of neutrons in between the strongly repelling proton layers, thereby minimizing the total Coulomb repulsion for any nucleus. The antiferromagnetic spin configuration maximizes the magnetic attraction between nearest neighbors, i.e., producing an attractive tensor interaction, and, indeed, the other source of attraction in condensates has been the tensor force between neighboring nucleons.[11–19] Within the context of neutron star research (where gravitation supplies the needed additional binding force), Canuto and Chitre[28] have shown that the *lowest energy state for nucleon condensation* ($N = Z$) *is the antiferromagnetic FCC lattice with alternating isospin layers,* i.e., identical to the configuration which is required in the FCC model to reproduce the known eigenvalue symmetries. Matsui *et al.*[15] have also found the FCC configuration to be the lowest energy condensed state, although their calculations were done using a mixed isospin state for each lattice site.

The principal theoretical attraction of the FCC model [and the reason why all other solid-phase nuclear models (Sec. VI), must be regarded as less likely] is that *only* the FCC model reproduces the entire sequence of allowed nucleon states as found in the independent-particle model. As shown in Table II, the exact isomorphism between the FCC geometry and the nucleon states of the *j-j* model leads to the "shells" and "subshells" of the harmonic oscillator plus spin-orbit coupling model (although the precise spacing of the levels and the production of the magic numbers, which are functions of the potential well, are not directly implied by the FCC description of nucleon states). The significance of the isomorphism is that virtually all of the nuclear features deduced from the independent-particle model can therefore be deduced from the FCC model. More speculatively, it also suggests that the "quantal" nature of fermion energy states, in general, is fundamentally linked with the "quantal" nature of the FCC crystalline structure—i.e., its fundamental unit distances and associated unit energies.

It should be noted that, unlike several previous solid-phase nuclear models, the FCC model is based upon the shell-subshell texture of the *j-j* model, rather than on the so-called magic numbers. Although failing to predict uniquely the 6 (7, 8, or 9) of the textbook magic numbers [i.e., 2, 8, 14(?), 20, 28(?), 40(?), 50, 82, 126], the FCC model implies structural symmetries and more two-body bonds per nucleon with the closure of any shell *or* subshell within the *j-j* nucleon buildup procedure. In other words, it predicts magic stability with the closure of *every j* subshell. As shown in Table II, it is empirically found that 15 of the first 17 subshells are magic by at least one criterion of magicness. In contrast to the inert gases of atomic physics, it is evident that the relative stability of the nuclear closed shells is slight and no single criterion or combination of criteria predicts a unique set of 6 (or so) magic numbers. Rather than being a weakness of the FCC model, the implied "multishell" texture of the FCC lattice is thought to reproduce the known texture more closely than theories built around a small number of magic structures.

VI. OTHER SOLID-PHASE MODELS

The modern (post-shell model) solid-phase theories of nuclear structure include those of Pauling (P),[4] Anagnostatos (A),[5] Lezuo (L),[6] Cook and Dallacasa (FCC),[7] MacGregor (M),[8] and Robson (R).[9] Some of these models can account for cluster effects in the small (R) or small and large (P,M,FCC) nuclei, and some models deal centrally with the magic numbers (P,A,L), but problems concerning the nuclear surface are common (P,A,M,R) and some of the solid-phase theories require *post hoc* adjustment in internucleon distances (A,R) or cluster dimensions (P) and some require the selection of "acceptable" symmetrical structures from a larger number of possibilities (P,A,L). Nevertheless, the varied successes with regard to fission (P,M), electron form factors (L,FCC), total binding energies and magnetic moments (M,FCC), and nuclear rms radial measures (A,FCC) indicate some of the fundamental attractions of a solid-phase conception of the nucleus. No solid-phase theory other than the FCC model, however, can account for the independent-particle nature of nuclei.

VII. DISCUSSION

Provided that the initial question concerning the stability of a nuclear lattice can be answered affirmatively, the solid-phase models can be shown to exhibit a wide range of properties known to exist in real nuclei. The solid-phase models of Pauling and MacGregor show impressive correlations with symmetric and asymmetric fission, but, in not accounting for the independent-particle features of nuclei, such cluster models are inevitably relegated to being "alternative" nuclear models with a limited range of applications, as distinct from unifying models which would eliminate the need to use diverse nuclear models to account for different nuclear properties. In contrast, the truly solid-phase models (L,A,R,FCC) imply the existence of ordered states within the context of a "collective" model. As such, these latter models may be considered candidates as potentially unifying models which bring together cluster, shell, and liquid-drop characteristics within a single theoretical framework.

Unique among the solid-phase models, the FCC model reproduces the entire independent-particle description of the nucleus—based solely upon the geometrical position of the nucleons within an FCC configuration. Conceptually, the FCC and the other solid-phase depictions of the nucleus appear at first consideration to be counter-intuitive since a gaseous phase theory of nuclear structure (the conventional *j-j* model) has been the central paradigm of nuclear theory for three decades. Nevertheless, the precise isomorphism between the *j-j* model and the FCC model clearly indicates that the "orbiting nucleon" conception of the nucleus is not an inevitable implication of the independent-particle description.

The predictive successes of the shell model have been remarkable, yet a theoretical understanding of the spin-orbit interaction has not yet been achieved. As recently as 1980, Bertsch *et al.*[29] noted that "Despite its success in phenomenological description of nuclei, the origin of this spin-orbit interaction ... still remains a puzzle." The FCC model may resolve this problem by retaining a description of individual nucleons identical to that of the spin-orbit coupling model, but without demanding the orbiting of nucleons within a nuclear medium which is so dense that a liquid-drop model has been the basis for quantitative work on nuclear binding energies. It may also answer DoDang's question:[14] "How can one reconcile such a (condensed-phase) structure, which is perfectly valid for an infinite system, with the known shell-model structure of finite nuclei?"

It is worth noting that the principal strength of the FCC model—i.e., its precise mapping onto the independent-particle description of nuclei—is simultaneously its most difficult to appreciate in two dimensions. This is not an inherent weakness in the theory itself, however, and three-dimensional models[30] clearly demonstrate the one-to-one isomorphism with the *j-j* model, as well as the impossibility of alternative crystal structures showing similar correlations with the *n*, *j*, *m*, etc., eigenvalues of the *j-j* model.

The regular polyhedra of Anagnostatos's model and the approximate spherical symmetry of Pauling's models have a "magic" appeal which is not found in the FCC model. Those correlations with magic numbers are attained, however, at the expense of sacrificing all internal symmetries correlating nucleon positions with nucleon eigenvalues. In this respect, such models are fundamentally misleading—suggesting an explanation of (selected) magic numbers, but not relating to the fundamental quantum mechanics which underlie those numbers.

Only the antiferromagnetic FCC lattice with alternating isospin layers shows such internal symmetries (for all eigenvalues) and is therefore the only solid-phase model which is isomorphic with the independent-particle description of nuclei. It can therefore be said that, regardless of whether or not nucleons have actually "condensed" within nuclei into a solid phase, the antiferromagnetic FCC lattice is an accurate and unique representation of "nuclear quantum space"—arguably as valid as the electron orbital depiction of atomic "quantum space." Resolution of questions concerning the validity of the FCC lattice *as a nuclear model* will depend upon future developments regarding the energetic stability of a solid-phase at nuclear densities.

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[30]Low-cost three-dimensional models designed to explain the FCC model are available commercially from Cochrane's Ltd., Leafield, Oxford, England; a computer graphics package is available from U-Micro, Long Lane, Warrington WA2 8PR, England.