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Widths of Σ and Λ hypernuclear states

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We investigate some of the features of Λ and Σ hypernuclei. The widths of Λ- and Σ-hypernuclear resonances are discussed and estimates made. The coupling between a certain class of Λ- and Σ-hypernuclear states is considered and its relevance to the problem of Σ-hypernuclear widths is indicated.

I. INTRODUCTION

The strangeness exchange reactions (K^-,π) on nuclei, performed in the last decade, have led to the observation[1] of excited Λ-hypernuclear states. In more recent experiments bumps in the $(K^-\pi^-)$ and (K^-,π^+) cross sections were observed[2] at energies about 80 MeV above the observed Λ-hypernuclear states. The difference between the mass of a free Λ and a free Σ is about 80 MeV. Therefore these structures in the cross sections were interpreted as evidence for the formation of Σ-hypernuclear states in these reactions. In the lighter nuclei studied, ^{6}Li, ^{9}Be, ^{12}C, and ^{16}O, the various structures that appeared usually had widths of several MeV. The above experiments have prompted a great deal of theoretical work concerning Σ hypernuclei[3–12] and, in particular, the question of Σ-nuclear widths was examined.[3–9]

The Σ-hypernuclear states are coupled through the conversion interaction $\Sigma+N\to\Lambda+N$ to the Λ-hypernuclear states. This coupling, it was stressed,[3–6] should contribute very significantly to the widths of the Σ-hypernuclear excitations.

Also, recently there have been attempts to describe the low-lying (relative to the Σ-hypernuclear ground state) Σ-hypernuclear excitations using a small space shell-model basis.[12] This basis includes only a few (1–3) Σ-particle nucleon-hole (ΣN^{-1}) configurations. However, the ΣN^{-1} configurations lie at 80 MeV excitation energy in the Λ hypernucleus, and if the conversion process is indeed important, then the coupling between the $|\Sigma N^{-1}\rangle$ configurations and the more complicated surrounding Λ-hypernuclear states will affect not only the widths but also the energy positions of the Σ-hypernuclear excitations. In other words, the conversion process should contribute to both the widths and shifts of the Σ-hypernuclear levels.

In this paper we do not attempt to improve upon these calculations and extend them to larger spaces. Rather, we apply some methods taken from the calculation of widths of "usual" nuclear resonances to estimate widths of the Λ and Σ hypernuclei. In the last section we consider the coupling of certain Λ and Σ hypernuclear states and the relevance of this coupling to the problem of the widths of hypernuclear states.

II. THE HYPERNUCLEAR HAMILTONIAN

The Hamiltonian that describes the single-Λ or single-Σ nucleus[13] contains the following parts:

$$H=H_N+H_\Lambda+H_\Sigma+H_{\Sigma\Lambda}\ , \tag{1}$$

where H_N denotes the purely nucleonic Hamiltonian and is given by

$$H_N=\sum_{i=1}^{A}(T_i+m_{Ni})+\frac{1}{2}\sum_{i\neq j}V_{ij}(\mathbf{r}_i,\mathbf{r}_j)\ . \tag{2}$$

The Λ part of H is

$$H_\Lambda=m_\Lambda+T_\Lambda+\sum_{i=1}^{A}V_{\Lambda N}(\mathbf{r}_\Lambda,\mathbf{r}_i)\ , \tag{3}$$

and the Σ part is analogously

$$H_\Sigma=m_\Sigma+T_\Sigma+\sum_{i=1}^{A}V_{\Sigma N}(\mathbf{r}_\Sigma,\mathbf{r}_i)\ . \tag{4}$$

The last term in Eq. (1) is the ΣΛ transition interaction representing the strong interaction conversion process $\Sigma+N\to\Lambda+N$,

$$H_{\Sigma\Lambda}=\sum_{i=1}^{A}V_{\Sigma\Lambda}(\mathbf{r}_h,\mathbf{r}_i)\ . \tag{5}$$

In all the above equations T denotes the kinetic energy, $\mathbf{r}_i,\mathbf{r}_j$ nuclear coordinates, and $\mathbf{r}_h$ the coordinate of the Λ or Σ.

In our discussion we assume that each part of the Hamiltonian (except, of course, for $H_{\Sigma\Lambda}$) can be expressed in terms of a one-body part that contains the kinetic energy T_B, mass m_B, one-body nuclear potential U_B $(B\equiv N,\Lambda,\Sigma)$, and a residual two-body interaction.

III. THE WIDTHS OF Σ-HYPERNUCLEAR STATES

There are some tentative experimental indications[2] from (K^-,π^-) and (K^-,π^+) reactions that relatively narrow Σ-hypernuclear states exist. The experiments point to the possibility that in these reactions "substitutional" Σ resonances are observed in light nuclei and that the widths of these resonances are of the order of several MeV.

These resonances are assumed to have a simple p-h structure, i.e., of the form $|\psi_{jl}^{\Sigma}(\phi_{jl}^{N})^{-1}JT\rangle$, where a nucleon in a state ϕ_{jl}^{N} is replaced by a Σ in a state ψ_{jl}^{Σ} to form a Σ-particle nucleon-hole excitation. These states, it is believed, should decay through the conversion process $\Sigma+N\to\Lambda+N$. The most naive estimates[3] of the conversion widths give numbers of the order of 15–20 MeV, considerably larger than the widths of the bumps found in the $(K^-,\pi^\pm)$ experiments. Of course, these simple theoretical estimates do not take into account various selection rules, correlations in the wave function, and all kinds of cancellations which could result.

There have been several attempts to produce smaller conversion widths. While there are some indications that the Σ hypernuclei have a long lifetime, the explanations of this fact[3–9] have met serious criticism[14] and in most cases were short lived.

It is well known that the calculation of the width of a nuclear resonance is very sensitive to the detailed structure of the wave function of the initial state and of the states into which it decays. For example, when one considers the isobaric analog resonance (IAR) in a naive estimate that does not take into account the precise particle-hole composition of the analog state, one finds widths of the order of several MeV, while the actual spreading widths ($\Gamma^{\downarrow}$) of IAR's are 100 times smaller. It is only when the cancellations[15–17] in the different ph contributions to $\Gamma^{\downarrow}$ are taken into account that a substantial reduction occurs and the $\Gamma^{\downarrow}$ width turns out to be several hundreds of keV. These cancellations are a result of the underlying isospin symmetry. But even in this case the widths are a factor 5–8 too large compared to the experimental ones.[15,17] Only after additional correlations are included in the calculation[17] are the $\Gamma^{\downarrow}$ for the IAR's reduced further.

Another example is collective multipole giant resonances in nuclei.[18] The basic components that make up such states are 1p-1h excitations. The spreading widths of such excitations are calculated by coupling the particle and hole each to vibrational states. These couplings give rise to self-energies for the particle and hole. The imaginary parts of these self-energies are the individual widths of the particle and hole. However, in addition to these one should also take into account a correlation correction, namely the exchange of a vibration between the particle and hole.[18] When the amplitude for this contribution is added to the self-energy amplitudes, one often obtains a reduction in the widths, sometimes by a factor of 2.[19]

We now try to apply some of the knowledge we gained in the study of widths of "usual" nuclear states to the discussion of hypernuclear excitations. We will also present some preliminary but simple estimates for these widths.

The width of nuclear (or hypernuclear) resonances can be decomposed into two parts:[20]

$$\Gamma = \Gamma^{\uparrow} + \Gamma^{\downarrow} , \qquad (6)$$

where $\Gamma^{\uparrow}$, the escape width, represents the decay of a state due to the direct emission of the particle into the continuum, and $\Gamma^{\downarrow}$, the spreading width, represents the spread of the simple (doorway) nuclear state into the vast spectrum of surrounding nuclear excitations. The escape widths for the Λ strangeness exchange (or substitutional) states were calculated in the past[21] using a Λ-nucleon continuum random-phase approximation (RPA). For these low-lying states the Λ emission width $\Gamma_{\Lambda}^{\uparrow}$ turned out to be of the order of several hundred keV for ${}^{12}_{\Lambda}$C or ${}^{16}_{\Lambda}$O. Only in certain cases[22] in lighter nuclei was the $\Gamma_{\Lambda}^{\uparrow}$ of the order of 2 MeV.

Because of the similarity of the $\Sigma + N \rightarrow \Sigma + N$ and $\Lambda + N \rightarrow \Lambda + N$ interactions, the following statements can be made. The average Σ potential is similar to the Λ potential and therefore the onset of the continuum will be close in the two cases. Also, the matrix elements connecting bound states and continuum states should not differ very much for Λ and Σ states. Therefore one should expect escape widths for Σ hypernuclei ($\Gamma_{\Sigma}^{\uparrow}$) to have similar values as $\Gamma_{\Lambda}^{\uparrow}$.

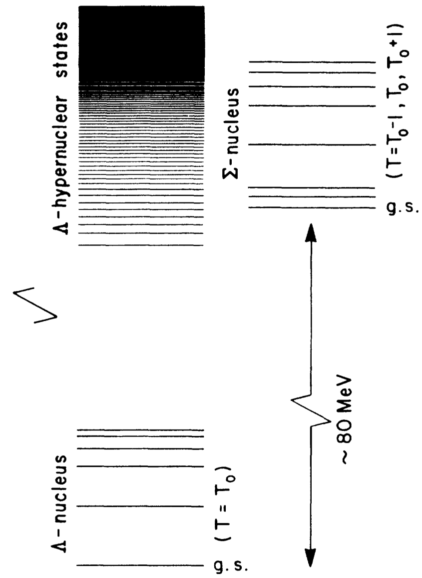

FIG. 1. Schematic representation of the Λ- and Σ-hypernuclear spectrum, for a nucleus with $N-Z=2T_0>0$.

The problem of spreading width is different in the two cases. In the Λ hypernuclei the low-lying states (including the substitutional states) are surrounded by a dense spectrum of nuclear states but cannot connect to the above states because of strangeness conservation in the strong interaction. The number of Λ-hypernuclear states surrounding the simple $|\Lambda N^{-1}\rangle$ configurations is small and the ΛN interaction is weaker than the NN force. One should expect therefore the $\Gamma_{\Lambda}^{\downarrow}$ for the low-lying states to be small, of the order of a few MeV or less. In the case of $|\Lambda N^{-1}\rangle$ states involving deep nucleon holes (such as the

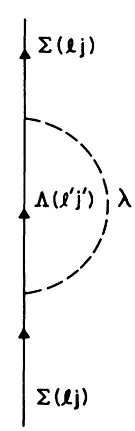

FIG. 2. Diagram representing the coupling of a Σ-hypernuclear single-particle state to a Λ single particle plus a λ vibration of the nuclear one.

$1s_{1/2}$), the spreading width could be larger because of the substantial fragmentation of the nucleon-hole strength. Also, here, as in the nuclear case, cancellations between the spreading amplitudes of the hole and particle could occur.

In the case of Σ-hypernuclear states the spreading width $\Gamma_\Sigma^\downarrow$ could be subdivided into two parts:

$$\Gamma_\Sigma^\downarrow = \Gamma_{\Sigma\Sigma}^\downarrow + \Gamma_{\Sigma\Lambda}^\downarrow . \tag{7}$$

The part denoted by $\Gamma_{\Sigma\Sigma}^\downarrow$ is the result of the interaction $\Sigma + N \rightarrow \Sigma + N$ and, because of the similarity of this interaction to the $\Lambda + N \rightarrow \Lambda + N$ one, we expect $\Gamma_{\Sigma\Sigma}^\downarrow$ to be of the same order as $\Gamma_\Lambda^\downarrow$, i.e., several MeV or less.

The new ingredient in the Σ hypernucleus case is the fact that low-lying Σ-hypernuclear states (relative to the Σ-nucleus ground state) find themselves at an 80-MeV excitation energy in the Λ-nucleus system and are embedded in a vast spectrum of Λ-hypernuclear states (see Fig. 1). The conversion interaction $\Sigma + N \rightarrow \Lambda + N$ will couple the Λ-hypernuclear states to the Σ-hypernuclear states and give rise to $\Gamma_{\Sigma\Lambda}^\downarrow$. It is $\Gamma_{\Sigma\Lambda}^\downarrow$ that attracted so much attention[3–10] in recent years.

It is clear that many of the Λ-nuclear states surrounding the Σ states are of np-nh nature with $n > 3$ and will not couple to the latter through a two-body force. Also, spin and isospin selection rules will greatly reduce the number of states that can couple to given Σ states.

Guided by the theory of spreading width in usual nuclei, we will now apply the same methods to $\Gamma_{\Sigma\Lambda}^\downarrow$. For simplicity, we will first assume a single-particle state for the Σ, $|\psi_{lj}^\Sigma\rangle$. In nuclear physics the spreading width of one-particle (or one-hole) states is calculated by first coupling these to vibrational states.[18] The particle plus vibrations are not eigenstates, but serve as doorways,[15,18,20] and one has to assign widths to these doorways.

Let the $|\psi_{lj}^\Sigma 0JT\rangle$ states be coupled to Λ states of the form $|\psi_{l'j'}^\Lambda \lambda JT\rangle$, where $\psi_{l'j'}^\Lambda$ is a single-particle Λ state and λ is a vibrational state (a giant resonance) of the nuclear core (see Fig. 2). The spreading width $\Gamma_{\Sigma\Lambda}^\downarrow$ in this approximation is given by[15–18]

$$\Gamma_{\Sigma\Lambda}^\downarrow = 2\,\mathrm{Im} \sum_{\lambda, l'j'} \frac{\langle \psi_{lj}^\Sigma 0JT \mid V_{\Sigma\Lambda} \mid \psi_{l'j'}^\Lambda \lambda JT\rangle \langle \psi_{l'j'}^\Lambda \lambda JT \mid V_{\Sigma\Lambda} \mid \psi_{lj}^\Sigma 0JT\rangle}{(E_0 + m_\Sigma + \epsilon_{lj} - E_\lambda - m_\Lambda - \epsilon_{l'j'}) - i\Gamma_{\Lambda\lambda}^\downarrow(E_\Sigma)/2} . \tag{8}$$

E_0 and E_λ denote the nuclear core ground- and excited-state energies, m_Σ, m_Λ are the masses of the Σ and Λ, and ϵ_{lj} and $\epsilon_{l'j'}$ are the single-particle energies of the Σ and Λ in their respective potentials. The parameter $\Gamma_{\Lambda\lambda}(E_\Sigma)$ is the width of the $\Lambda + \lambda$ state evaluated at the excitation energy (E_Σ) of the Σ-hypernuclear state. The interpretation of this parameter is that if we place a ($\Lambda + \lambda$) state at the excitation energy of the Σ (i.e., ~ 80 MeV) in the Λ hypernuclear system, then it would acquire the width $\Gamma_{\Lambda\lambda}^\downarrow(E_\Sigma)$. It is quite reasonable to use the weak coupling approximation and write that

$$\Gamma_{\Lambda\lambda}^\downarrow(E_\Sigma) \simeq \Gamma_\lambda^\downarrow(E_\Sigma) , \tag{9}$$

where $\Gamma_\lambda^\downarrow(E_\Sigma)$ is the width of a nuclear vibration at the energy of $E_\Sigma \simeq 80$ MeV excitation.

In the case in which there is a single dominant doorway $|\psi_{l'j'}^\Lambda \lambda JT\rangle$ [i.e., we drop the sum in Eq. (8)] and for

$$\Gamma_{\Lambda\lambda}^\downarrow \ll (m_\Sigma - m_\Lambda + E_0 - E_\lambda + \epsilon_{lj} - \epsilon_{l'j'}) ,$$

Eq. (8) can be simplified to

$$\Gamma_{\Sigma\Lambda}^\downarrow \simeq \frac{|\langle \psi_{lj}^\Sigma 0JT \mid V_{\Sigma\Lambda} \mid \psi_{l'j'}^\Lambda \lambda JT\rangle|^2 \Gamma_{\Lambda\lambda}^\downarrow(E_\Sigma)}{(m_\Sigma - m_\Lambda + E_0 - E_\lambda + \epsilon_{lj} - \epsilon_{l'j'})^2} . \tag{10}$$

The meaning of this expression is the following: the Σ-hypernuclear state mixes with surrounding, complicated, multiparticle-multihole Λ-nuclear states only via the rather small component of the 2p-1h doorway state $|\psi_{l'j'}^\Lambda \Lambda JT\rangle$ admixed into the $|\psi_{lj}^\Sigma 0JT\rangle$. The spread of the doorway into the more complicated states is represented by $\Gamma_{\Lambda\lambda}^\downarrow(E_\Sigma)$. For "substitutional" states of the form $|\psi_{lj}^\Sigma(\phi_{lj}^N)^{-1}JT\rangle$, the same considerations hold; however, one has to add the width of the single nucleon-hole and the contribution which stems from the exchange of a vibration between the Σ particle and the nucleon hole.[18]

We now make a crude estimate of $\Gamma_{\Sigma\Lambda}^\downarrow$ using Eq. (10). First, the denominator: $m_\Sigma - m_\Lambda \simeq 80$ MeV. We note that the isovector λ vibrations are the main contributions to matrix elements in Eq. (8) or (10). Therefore $E_0 - E_\lambda \simeq -30$ MeV, for medium heavy nuclei, and the denominator in Eq. (10) is about 50 MeV. (We have neglected the small differences stemming from $\epsilon_{lj} - \epsilon_{l'j'}$.) The matrix element in the numerator for a zero range force $V_{\Sigma\Lambda} = V_0^{(\Sigma\Lambda)}\delta(\mathbf{r} - \mathbf{r}')$ can be written in the form

$$\langle \psi_{lj}^\Sigma 0JT \mid V_{\Sigma\Lambda} \mid \psi_{l'j'}^\Lambda \lambda JT\rangle = V_0^{(\Sigma\Lambda)} \int \psi_{lj}^\Sigma(\mathbf{r}) \rho_{\mathrm{tr}}^{(\lambda)}(\mathbf{r}) \psi_{l'j'}^\Lambda(\mathbf{r}) d\mathbf{r} , \tag{11}$$

where $\rho_{\mathrm{tr}}^{(\lambda)}(\mathbf{r})$ is the transition density[18] between the nuclear ground state and the vibrational state λ. In the nuclear case (when only nucleons are involved) the above matrix element is of the order of a few MeV. In the case considered here we expect this matrix element not to exceed the nuclear one because $|V_{\Sigma\Lambda}| < |V_{NN}|$.

Thus the ratio of the matrix element in Eq. (11) to the denominator of 50 MeV when squared will be of the order of $\frac{1}{100}$ or less. The upper limit for the parameter $\Gamma_{\Lambda\lambda}^\downarrow(E_\Sigma)$ can be estimated crudely by first using the approximation in Eq. (9) and using hydrodynamical models which yield a power law dependence of the width on the excitation energy:[23,24]

$$\Gamma^\downarrow \simeq \Gamma_0 E^\delta , \tag{12}$$

with Γ_0 constant. Certain models[24] predict $\delta \simeq 1-1.5$. From the above considerations we obtain

$$\Gamma_\lambda^\downarrow(E_\Sigma) \simeq \left[\frac{E_\Sigma}{E_\lambda}\right]^\delta \Gamma_\lambda^\downarrow(E_\lambda) . \tag{13}$$

For $E_\lambda \simeq 30$ MeV and $E_\Sigma \simeq 80$ MeV and $\delta \simeq 1-1.5$,

$$\Gamma^{\downarrow}_{\Lambda\lambda} \simeq \Gamma^{\downarrow}_{\lambda}(E_\Sigma) \simeq (2.7-4.3)\Gamma^{\downarrow}_{\lambda}(E_\lambda) \; . \tag{14}$$

From experimental studies of nuclear giant resonances, one finds that $\Gamma^{\downarrow}_{\lambda}(E_\lambda)$ ranges between 2 and 7 MeV and so $\Gamma^{\downarrow}_{\Lambda\lambda}(E_\Sigma) \simeq 5-30$ MeV. (We consider these numbers to be definitely upper limits.) We reach the conclusion that for a single vibration λ and single $\psi^{\Lambda}_{l'j'}$ orbit,

$$\Gamma^{\downarrow}_{\Sigma\Lambda} \le \text{a few hundreds of keV} \; .$$

Several different vibrational states λ and also several different $\psi^{\Lambda}_{l'j'}$ orbits may contribute in Eq. (8). In this case a reasonable upper limit is

$$\Gamma^{\downarrow}_{\Sigma\Lambda} < \text{several MeV} \; ,$$

and more probably of the order of 1 MeV. The primary reason that the spreading widths of the Σ hypernuclei turn out to be so small in this estimate is the fact that the main doorways (the nuclear giant resonances) in the calculation of $\Gamma^{\downarrow}_{\Sigma\Lambda}$ are far removed in energy from the Σ-hypernuclear resonance considered. This should be contrasted with the usual nuclear single-particle spreading widths where the resonances are close in energy to the vibrational states λ that contribute to processes analogous to the ones in Fig. 2.

In summary, the width of a Λ-hypernuclear state has an usually small escape width $\Gamma^{\uparrow}_{\Lambda}$ which is of the order of several hundred keV. The spreading width $\Gamma^{\downarrow}_{\Lambda}$ of the single Λ is also probably of the same order and thus the main contribution to the observed widths in (K^-,π^-) reactions of Λ-substitutional states is the spreading width of the neutron hole. In the Σ-hypernuclear states we expect $\Gamma^{\uparrow}_{\Sigma}$ again to be of the order of hundreds of keV and also $\Gamma^{\downarrow}_{\Sigma\Sigma}$ is expected to be of the same size as $\Gamma^{\downarrow}_{\Lambda}$. The additional $\Gamma^{\downarrow}_{\Sigma\Lambda}$ conversion width is, as estimated here, of the order of several MeV or less. As in the case of substitutional Λ-hypernuclear states, one should add also in the case of Σ-substitutional states the same spreading widths resulting from the nucleon hole. Altogether, the Σ-hypernuclear states in light nuclei should according to these estimates be only slightly broader than the corresponding Λ states.

IV. THE COUPLING BETWEEN THE Λ AND Σ HYPERNUCLEI IN $N > Z$ NUCLEI

We will now consider a very special type of coupling between Λ-hypernuclear and Σ-hypernuclear states. As we will see, it is a coherent effect and involves a large portion of the nucleons in the nucleus. The Λ is an isoscalar ($\tau=0$) particle and therefore when coupled to an $N=Z$ nucleus in a $T_0=0$ state will produce isoscalar hypernuclear states (i.e., $T \equiv T_0+\tau=0$). On the other hand, the Σ is an isovector triplet ($\tau=1$) and when coupled to a $T_0=0$ nuclear state it produces isovector ($T \equiv T_0+\tau=1$) excitations in the hypernucleus. The above-mentioned Λ-hypernuclear states and the Σ-hypernuclear ones, although having the same strangeness $S=-1$, cannot mix via the strong force. At high excitation energies in the Λ hypernucleus (certainly around 70–80 MeV excitation, which corresponds to the low energies in the Σ hypernucleus), many Λ-nucleus states exist in which the Λ is coupled to $T_0=1$ nuclear excitations forming $T=1$ states, which then can couple to the low-lying Σ-nucleus states (see Fig. 1).

The situation is changed when we consider $N > Z$ nuclei to which either a Λ or Σ is added. (To this category belong also the so-called "substitutional" states in which a nucleon in an $N=Z$ nucleus is substituted by a Λ or Σ; in this case, $T_0=\frac{1}{2}$.) In $N > Z$ nuclear cores, $T_0 \neq 0$, and the Λ will form low-lying excitations with total isospin $T=T_0$, while in the Σ hypernucleus low-lying (relative to the Σ-hypernucleus ground state) excitations with $T=T_0+1$, T_0, T_0-1 will be formed (see Fig. 1). (For $T_0=\frac{1}{2}$ the T_0-1 state is, of course, not present.) Now the $T=T_0$ Λ-hypernuclear low-lying states (for example, the ground state) will mix through the strong force with the $T=T_0$, Σ-hypernuclear states.

The coupling between Λ and Σ states was considered in the context of charge symmetry breaking in ${}^4_\Lambda$H and ${}^4_\Lambda$He in Ref. 25 and in the context of nuclear matter in Refs. 26 and 27.

Let us consider (as shown in Fig. 3) a Λ in a single particle state ψ^{Λ}_{lj} and the nuclear core in its ground state, i.e., $|\psi^{\Lambda}_{lj} 0 T_0\rangle$ and the corresponding $T=T_0$, Σ-hypernuclear state $|\psi^{\Sigma}_{lj} 0 T_0\rangle$. The wave function $|\psi^{\Sigma}_{lj} 0 T_0\rangle$ will have the form

$$|\psi^{\Sigma}_{lj} 0 T_0\rangle = \left[\frac{T_0}{T_0+1}\right]^{1/2} |\psi^{\Sigma^0}_{lj} 0 T_0\rangle - \left[\frac{1}{T_0+1}\right]^{1/2} |\psi^{\Sigma^-}_{lj} \mathcal{A} T_0\rangle \; , \tag{15}$$

where $\mathcal{A}$ denotes the isobaric analog state of the g.s. $|0\rangle$. The matrix element

$$\left\langle \psi^{\Sigma}_{lj} 0 T_0 \left| \sum V_{\Sigma\Lambda} \right| \psi^{\Lambda}_{lj} 0 T_0 \right\rangle \tag{16}$$

is of one-body nature involving a $\Sigma \rightarrow \Lambda$ nuclear transition potential as indicated in Fig. 4. In calculating this matrix element we take into account the isovector nature of the $\Sigma N \rightarrow \Lambda N$ interaction. Let us denote by $v(\mathbf{r}_i, \mathbf{r}_h)$ this interaction in the $T=\frac{1}{2}$ state. One can then express the

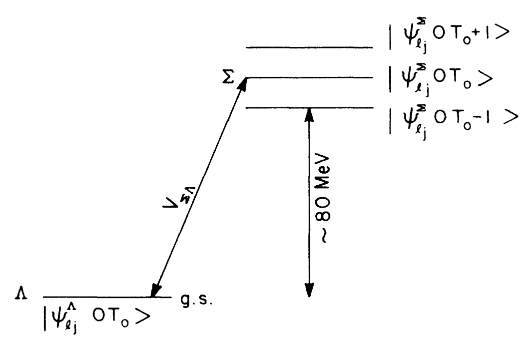

FIG. 3. Schematic representation of the coupling between a Λ and Σ single-particle state in a hypernucleus with $N-Z>0$.

matrix element in Eq. (16) in terms of $V_{\Sigma\Lambda}$ written now in the form

$$V_{\Sigma\Lambda}=\frac{1}{\sqrt{3}}\sum_i v(\mathbf{r}_i,\mathbf{r}_h)\boldsymbol{\tau}_i\cdot\boldsymbol{\phi}\ , \tag{17}$$

where $\boldsymbol{\tau}_i$ denotes the nucleon isospin operator and $\boldsymbol{\phi}$ is defined as $|\boldsymbol{\Sigma}\rangle=\boldsymbol{\phi}\,|\Lambda\rangle$ in isospin space. The factor $1/\sqrt{3}$ in Eq. (17) comes from the fact that

$$\langle \Sigma\mathrm{N}, T=\tfrac{1}{2}\,|\,\boldsymbol{\tau}\cdot\boldsymbol{\phi}\,|\,\Lambda\mathrm{N}, T=\tfrac{1}{2}\rangle=-\sqrt{3}\ .$$

Using these definitions and evaluating Clebsch-Gordan coefficients, one obtains

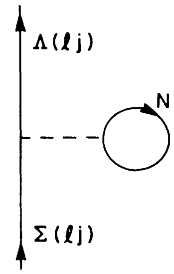

FIG. 4. Diagram for the one-body Λ↔Σ transition potential.

$$\left\langle \psi_{lj}^{\Lambda}0T_0\left|\sum V_{\Sigma\Lambda}\right|\psi_{lj}^{\Sigma}0T_0\right\rangle=\frac{1}{\sqrt{3}}\left[\frac{T_0+1}{T_0}\right]^{1/2}\int \psi_{lj}^{\Lambda}(\mathbf{r})\psi_{lj}^{\Sigma}(\mathbf{r})\rho_{\mathrm{ex}}(\mathbf{r}')v(\mathbf{r},\mathbf{r}')d\mathbf{r}\,d\mathbf{r}'\equiv\int \psi_{lj}^{\Lambda}(\mathbf{r})U_{\Sigma\Lambda}(\mathbf{r})\psi_{lj}^{\Sigma}(\mathbf{r})d\mathbf{r}\ , \tag{18}$$

where $\rho_{\mathrm{ex}}(\mathbf{r}')$ is the excess neutron density (which is approximately equal to the isovector part of the nuclear ground state density), normalized to $N-Z$ and

$$U_{\Sigma\Lambda}(\mathbf{r})=\frac{1}{\sqrt{3}}\left[\frac{T_0+1}{T_0}\right]^{1/2}\int \rho_{\mathrm{exc}}(\mathbf{r}')v(\mathbf{r},\mathbf{r}')d\mathbf{r}' \tag{19}$$

is the *one-body* $\Sigma\rightarrow\Lambda$ nuclear transition operator involving all the excess neutrons.

The admixed Λ-nucleus and $T=T_0$, Σ-nucleus wave functions are now

$$\Phi_{lj}^{\Lambda}(T_0)=(1-\alpha^2)^{1/2}\,|\,\psi_{lj}^{\Lambda}0T_0\rangle+\alpha\,|\,\psi_{lj}^{\Sigma}0T_0\rangle\ , \tag{20a}$$

$$\Phi_{lj}^{\Sigma}(T_0)=(1-\alpha^2)^{1/2}\,|\,\psi_{lj}^{\Sigma}0T_0\rangle-\alpha\,|\,\psi_{lj}^{\Lambda}0T_0\rangle\ , \tag{20b}$$

where, in perturbation theory,

$$\alpha\simeq\frac{\left\langle \psi_{lj}^{\Lambda}0T_0\left|\sum V_{\Sigma\Lambda}\right|\psi_{lj}^{\Sigma}0T_0\right\rangle}{m_\Lambda-m_\Sigma}\equiv\frac{\langle\psi_{lj}^{\Lambda}\,|\,U_{\Sigma\Lambda}\,|\,\psi_{lj}^{\Sigma}\rangle}{m_\Lambda-m_\Sigma}\ . \tag{21}$$

All the low-lying Λ-hypernuclear states will experience a downward shift (compared to the unperturbed positions) of the order of

$$\Delta E_\Lambda\simeq-\alpha^2(m_\Sigma-m_\Lambda)\ , \tag{22}$$

while the $T=T_0$ Σ-hypernuclear low-lying levels will be shifted upwards by

$$\Delta E_\Sigma\simeq\alpha^2(m_\Sigma-m_\Lambda)\ . \tag{23}$$

In the case of the Λ-hypernuclear states, all of the states are shifted by approximately the same amount and therefore such shifts due to Λ-Σ mixing can probably be incorporated in the Λ-nucleon (ΛN) effective interaction.[13] For the Σ hypernuclei, however, it is only the $T=T_0$ levels that are shifted (in the "substitutional" states in ^{12}C, for example, these are only the $T=\frac{1}{2}$ and not $T=\frac{3}{2}$ that experience the shift). Therefore, the shift is not an overall one for the entire Σ-hypernucleus spectrum.

Another parameter that would be affected by this Λ-Σ mixing is the width of a Σ-hypernuclear state. In the discussion of Sec. III we have not specified the isospin of the Σ-nucleus state. If we consider the Σ states with isospin $T=T_0$, then an additional modification will occur because of the mixing of Σ-Λ configurations as in Eq. (20b). In Eq. (8) or (10) one should replace $|\,\psi_{lj}^{\Sigma}0JT\rangle$ with the admixed state $\Phi_{lj}^{\Sigma}(T_0)$ as given in Eq. (20b). Therefore

$$\Gamma_{\Sigma\Lambda}^{\downarrow}\simeq\frac{[(1-\alpha^2)^{1/2}\langle\psi_{lj}^{\Sigma}0JT_0\,|\,V_{\Sigma\Lambda}\,|\,\psi_{l'j'}^{\Lambda}\lambda JT_0\rangle-\alpha\langle\psi_{lj}^{\Lambda}0JT_0\,|\,V_{\Lambda\mathrm{N}}\,|\,\psi_{l'j'}^{\Lambda}\lambda JT_0\rangle]^2}{(m_\Sigma-m_\Lambda+E_0-E_\lambda+\epsilon_{lj}-\epsilon_{l'j'})^2}\Gamma_{\Lambda\lambda}^{\downarrow}(E_\Sigma)\ . \tag{24}$$

Let us now assume, as before, that $\psi_{lj}^{\Sigma}\simeq\psi_{lj}^{\Lambda}$ and that $V_{\Sigma\Lambda}\simeq V_{\Lambda\mathrm{N}}$. Then for $1-\alpha^2\simeq1$,

$$\Gamma_{\Sigma\Lambda}^{\downarrow}\simeq\frac{|\langle\psi_{lj}^{\Sigma}0JT_0\,|\,V_{\Sigma\Lambda}\,|\,\psi_{l'j'}^{\Lambda}\lambda JT_0\rangle|^2}{(m_\Sigma-m_\Lambda+E_0-E_\lambda+\epsilon_{lj}-\epsilon_{l'j'})^2}(1-\alpha)^2\Gamma_{\Lambda\lambda}^{\downarrow}(E_\Sigma)\ . \tag{25}$$

We note that now (25) has an additional factor of $(1-\alpha)^2$ as compared to Eq. (10). Thus one gets a reduction of the conversion width for $T=T_0$ as compared to, say, $T=T_0+1$ states.

We now make a very simplistic estimate of the matrix element in Eq. (16). We assume that the $\Sigma+\mathrm{N}\rightarrow\Lambda+\mathrm{N}$ interaction is similar to the isospin averaged $\Sigma+\mathrm{N}\rightarrow\Sigma+\mathrm{N}$ interaction, i.e., $|V_{\Sigma\Lambda}|\simeq|\bar{V}_{\Sigma\mathrm{N}}|$. Approximating $\rho_{\mathrm{exc}}\simeq[(N-Z)/A]\rho$, where ρ is the nuclear density, and, for $T_0\gg1$,

$$|U_{\Sigma\Lambda}(\mathbf{r})|\simeq\frac{1}{\sqrt{3}}\frac{N-Z}{A}|U_\Sigma(\mathbf{r})|\ , \tag{26}$$

where U_Σ is the Σ-nucleus potential,

$$U_\Sigma(\mathbf{r}) = \int \rho(\mathbf{r}')\bar{V}_{\Sigma N}(\mathbf{r},\mathbf{r}')d\mathbf{r}' . \quad (27)$$

Using the approximation in Eq. (26) and a homogeneous density distribution of radius R and also assuming that the depth of Σ-nucleus potential U_Σ is equal to the depth of the Λ-nucleus potential (approximately 30 MeV), we can write

$$\left| \left\langle \psi_{lj}^{\Lambda} 0 T_0 \left| \sum V_{\Sigma\Lambda} \right| \psi_{lj}^{\Sigma} 0 T_0 \right\rangle \right| \simeq \frac{30 \text{ MeV}}{\sqrt{3}} \left[\frac{N-Z}{A} \right] \int \psi_{lj}^{\Lambda}(r)\psi_{lj}^{\Sigma} d\mathbf{r} , \quad (28)$$

where the integration is over the sphere of radius R. The Σ-nucleus and Λ-nucleus potentials are probably quite similar and therefore we make an additional approximation that $\psi_{lj}^{\Sigma} \simeq \psi_{lj}^{\Lambda}$ and that the integral in Eq. (28) is close to 1. Thus,

$$\left| \left\langle \psi_{lj}^{\Lambda} 0 T_0 \left| \sum V_{\Sigma\Lambda} \right| \psi_{lj}^{\Sigma} 0 T_0 \right\rangle \right| \simeq \frac{30}{\sqrt{3}} \left[\frac{N-Z}{A} \right] \text{MeV} . \quad (29)$$

This matrix element will be larger in nuclei in which the ratio $(N-Z)/A$ is large. Typically, for medium and heavy mass nuclei this ratio is $\frac{1}{6}-\frac{1}{5}$. The resulting matrix elements are therefore of the order 3—4 MeV. Such matrix elements would produce Σ-Λ mixing amplitudes α [see Eqs. (20) and (21)] of the order of 0.05 (i.e., 0.25% admixtures). The shifts in Eqs. (22) and (23) would be of the order of 200 keV in heavy nuclei. The effect of this type of Σ-Λ mixing will reduce the widths for the T_0 Σ-hypernuclear states as given by Eq. (25) by 10%. This crude estimate indicates that this special type of Σ-Λ mixing is small; however, it is based on a numerical choice for the Σ-nucleus potential, a parameter that has not yet been determined. The estimate also neglects the spin dependence of the $\Sigma + N$ interaction. It is of interest to consider this kind of coupling when performing spectroscopic calculations of Σ-hypernuclear states.

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