RAPID COMMUNICATIONS

PHYSICAL REVIEW C VOLUME 35, NUMBER 4 APRIL 1987

^{28}Si + ^{12}C fusion reactions at high energies

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(Received 22 January 1987)

We present a model based on the direct reaction concept which is able to describe the fusion cross section consistently throughout the entire range of the measured energies from the subbarrier region to the very high energies where the cross section decreases linearly as the inverse of the center-of-mass energy. The model is applied to the fusion reaction of ^{28}Si with ^{12}C where Harmon *et al.* measured the cross section at energies up to three times the Coulomb barrier.

The most general feature of the fusion cross sections σ_F in light heavy-ion systems near the Coulomb barrier is the monotonic increase in σ_F (Region I) as the center-of-mass energy $E_{\text{c.m.}}$ increases. Recently a simple direct reaction approach was proposed by Udagawa, Kim, and Tamura[1] (UKT) to reproduce this behavior. σ_F in this model can be written as

$$\sigma_F = \frac{\pi}{k^2}\sum_l (2l+1)T_l \ , \tag{1}$$

with the partial transmission coefficient T_l given by

$$T_l = \frac{8}{\hbar v}\int_0^\infty |\chi_l(r)|^2 W_F(r)dr \ , \tag{2}$$

where v and k are, respectively, the relative velocity and the wave number in the incident channel, and $\chi_l(r)$ denotes the partial distorted wave function. The fusion potential $W_F(r)$ in Eq. (2) is defined as a part of the imaginary potential $W(r)$ and is supposed to be responsible for fusion. UKT chose $W_F(r)$ the same as $W(r)$ for the radial distance r smaller than the fusion radius R_F, which is given by

$$R_F = r_F(A_1^{1/3} + A_2^{1/3}) \tag{3}$$

and vanishes elsewhere, where A_1 and A_2 are the mass numbers of colliding heavy-ion partners and r_F is in practice taken as an adjustable parameter to fit data. This sharp cutoff choice works surprisingly well for both sub- and above-barrier fusions with essentially the same value of $r_F = \sim 1.45$ fm for a variety of heavy-ion partners A_1 and A_2.

For most systems, the fusion cross section begins to saturate rather suddenly (Region II) when $E_{\text{c.m.}}$ reaches around twice the s-wave barrier height. This observed saturation was well handled in the direct reaction description by imposing an additional constraint. For example, Udagawa, Hong, and Tamura[2] recently calculated the fusion cross sections for the ^{40}Ca + ^{16}O system for $E_{\text{lab}} = 40-215$ MeV. It was shown that the data in the whole range of E_{lab}, which includes the saturation region, were fitted rather nicely by taking the energy dependent fusion radius, i.e., by assuming that the fusion radius R_F decreases as $E_{\text{c.m.}}$ increases. Another analysis of the same system has been made by Park and Kim,[3] where they replaced the condition of the energy dependent fusion radius with the assumption that fusion does not take place unless the orbital angular momentum of the colliding ions is less than a critical value. In other words, the fusion cross section was calculated by giving the upper limit in the summation of Eq. (1), which is the appropriate angular momentum for forming a compound nucleus. As a result, an excellent fit with the data was obtained by limiting the partial waves by the known "shifted" yrast line[4] in the compound nuclear formation.

Recently Harmon *et al.*[5] measured the fusion cross sections for the ^{28}Si + ^{12}C system at energies up to three times higher than the s-wave Coulomb barrier. The linearly decreasing fusion cross sections are observed in the high energy region (Region III) as the inverse of the incident center-of-mass energy decreases. This trend suggests that there exists[6] a maximum angular momentum for fusion which is a constant independent of $E_{\text{c.m.}}$. Indeed, they found the critical angular momentum of $22\hbar$ as a limit for fusion of the system, which is related to the orbital angular momentum for an orbiting dinuclear system[7] formed in the collision between ^{28}Si and ^{12}C. In this paper, we will investigate Region III by extending the direct reaction description and show that the theory is able to cope with the fusion mechanism consistently throughout Regions I–III.

Let us consider the fusion reactions in the systems which lead to the same fused nucleus of ^{40}Ca. Examples include reactions such as ^{28}Si + ^{12}C, ^{24}Mg + ^{16}O, and ^{20}Ne + ^{20}Ne. In fact, they have been rather extensively studied[5,8–10] experimentally as well as theoretically in the last several years. The measured fusion cross sections for ^{28}Si + ^{12}C and ^{24}Mg + ^{16}O reactions are displayed in Figs. 1 and 2, respectively. We can immediately observe that the two cross sections behave quite similarly in Region I with a little difference in the absolute magnitude at the tail region, which simply reflects barrier and Q-value differences as pointed out by Lesko *et al.*[8] For Region II and above, we would expect an even more similar behavior of the cross sections for the two systems, since the compound nucleus limitation model would be more favorable

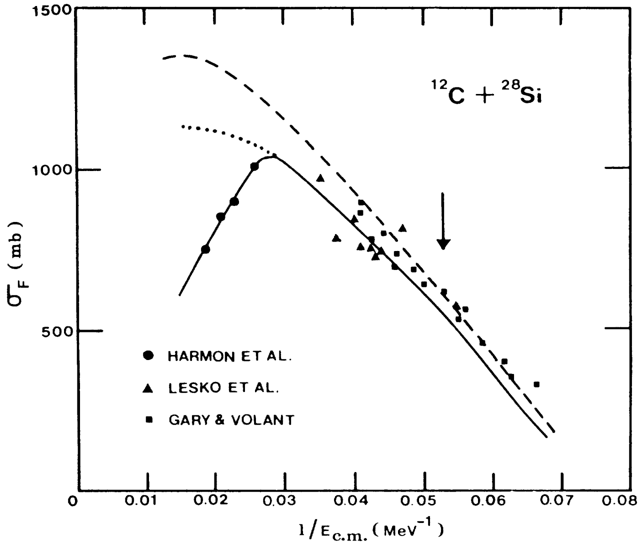

FIG. 1. Fusion cross sections for the ^{28}Si + ^{12}C system as a function of the reciprocal of $E_{\mathrm{c.m.}}$. The dotted curve represents the calculated fusion cross sections with an angular momentum limit by the shifted yrast line. The solid curve is the result of calculations with an energy independent upper limit of $22\hbar$ at energies higher than $E_{\mathrm{c.m.}}=35$ MeV. The calculated total reaction cross sections are displayed by the dashed curve. The arrow indicates the height of the s-wave Coulomb barrier.

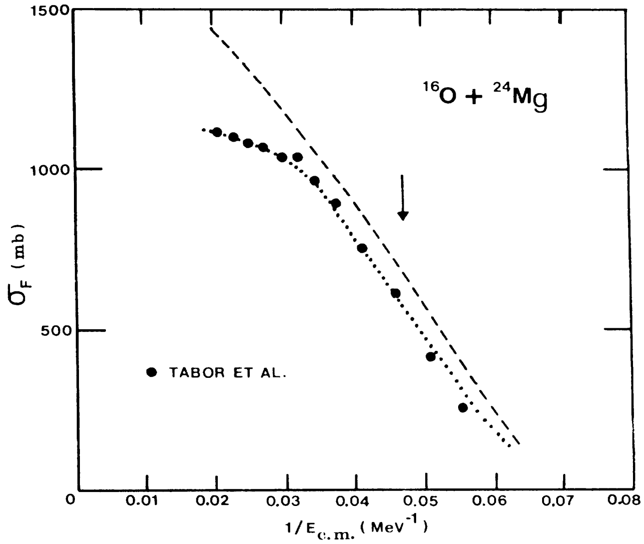

FIG. 2. Same as Fig. 1 but for the ^{24}Mg + ^{16}O system.

in such energies and they populate the same compound nucleus. However, according to the data, σ_F for the ^{24}Mg + ^{16}O system show saturation in the highest part of the measured $E_{\mathrm{c.m.}}$, while those for the ^{28}Si + ^{12}C system decrease sharply without showing any plateau.

We shall first present predictions of the direct reaction description by limiting the partial waves by a "shifted" yrast line,[4] which was successful up to Region II as mentioned earlier. In the calculations, the distorted waves are generated with an energy independent optical potential given by Gary and Volant[9] for the ^{28}Si + ^{12}C system and by Tabor *et al.*[10] for the ^{24}Mg + ^{16}O system, which reproduce the elastic scattering data reasonably well. The fusion radii are chosen by $r_F=1.46$ and 1.50 fm, respectively, for the ^{28}Si + ^{12}C and ^{24}Mg + ^{16}O systems. We used the "shifted" yrast line given by

$$E^*=\frac{\hbar^2}{2\mathcal{J}}J_{\mathrm{cr}}(J_{\mathrm{cr}}+1)+E_0 \ , \tag{4}$$

where $\mathcal{J}=4.3\times10^5$ MeV fm$^2/c^2$ and $E_0=27$ MeV. The excitation energy E^* versus the critical angular momentum J_{cr} according to Eq. (4), which is for the ^{40}Ca nucleus, is displayed in Fig. 3 as the solid curve.

The results of our calculations are plotted as the dotted curves in Figs. 1 and 2 for the ^{28}Si + ^{12}C and ^{24}Mg + ^{16}O systems, respectively. As expected, the two theoretical curves look very much alike and work pretty well at the low energies for both systems. A little deviation from the data at very low energies in the ^{28}Si + ^{12}C system can be improved by considering the coupled-channel effect as studied by UKT. It is worth noting that while the data of the ^{24}Mg + ^{16}O system in Fig. 2 follow exactly the theoretical dotted curve throughout the measured energies, those of the ^{28}Si + ^{12}C system resist following the curve at high energies above $E_{\mathrm{c.m.}}=35$ MeV but fall down rapidly as the inverse of $E_{\mathrm{c.m.}}$ decreases, as can be seen in Fig. 1. This sudden decrease in the cross section clearly indicates a departure from the compound nucleus limitation model, and Harmon *et al.*[5] argued that this is the effect of the entrance channel limit for the system.

It is common to express the fusion cross section σ_F in

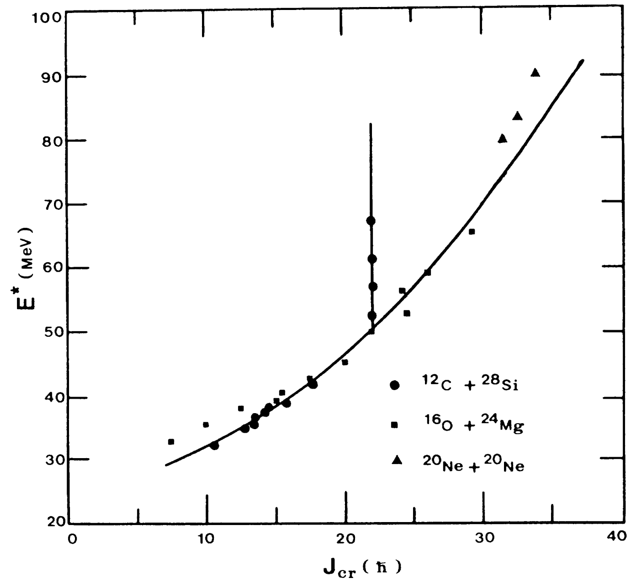

FIG. 3. E^*-J_{cr} diagram for the ^{40}Ca nucleus formed by the ^{28}Si + ^{12}C, ^{24}Mg + ^{16}O, and ^{20}Ne + ^{20}Ne systems. The solid curves are the angular momentum limit used in the calculations. The vertical line at $22\hbar$ indicates the energy independent limit at energies higher than $E_{\mathrm{c.m.}}=35$ MeV for the ^{28}Si + ^{12}C system.

terms of the critical angular momentum $J_{\rm cr}$ as

$$\sigma_F = \frac{\hbar^2}{2\mu}\frac{J_{\rm cr}^2}{E_{\rm c.m.}} , \qquad (5)$$

where μ is the reduced mass of the system. By making use of the above equation, we can assign a critical angular momentum $J_{\rm cr}$ to each experimental cross section. Such measured $J_{\rm cr}$ are shown in Fig. 3 for the ^{28}Si + ^{12}C, ^{24}Mg + ^{16}O, and ^{20}Ne + ^{20}Ne reactions. In the figure, it is distinctively shown that the ^{28}Si + ^{12}C system possesses a maximum critical angular momentum $J_{\max}$ above $E^* = 50$ MeV, which is a constant independent of $E_{\rm c.m.}$, while the data from other systems follow more or less the "shifted" yrast line for the whole measured energy range. From such indications, Harmon *et al.*[5] extracted the critical angular momentum of $22\hbar$ as the maximum angular momentum for the fusion of the ^{28}Si + ^{12}C system at high energies.

With the above evidence, we perform our calculation again by limiting the partial waves further above $E_{\rm c.m.} = 35$ MeV by the constant upper limit of $22\hbar$ for the ^{28}Si + ^{12}C system. The result is displayed in Fig. 1 as the solid curve. The agreement with the data, especially for Region III, is very impressive. Thus our model, based on the direct reaction concept, can provide a consistent explanation for the fusion excitation function starting from the sub-barrier region up to Region III.

One shortcoming of the model is that the limit of the partial waves in the summation of Eq. (1) has to be made by hand afterwards. However, for low energies (Regions I and II), the reaction time between the colliding nuclei would be long and it is plausible to employ the limit of the partial waves from the compound nucleus limitation model.[4] On the other hand, for high energies it is reasonable to adopt the simple minded barrier penetration model (BPM), which demands an almost constant $J_{\max}$ for fusions.

The maximum angular momentum $J_{\max}$ in BPM is known to be approximately l_p/f, where l_p is the largest orbital angular momentum for which there is a pocket in the effective potential that is the sum of nuclear, Coulomb, and centrifugal terms in the entrance channel, and f is the dissipation parameter with the usual value of $0.7 \sim 0.85$, depending on the specific model for the nuclear friction.[6] The reasonable nuclear potential, like the Bass potential,[11] gives $l_p = 24 \sim 28\hbar$ for the systems considered here. Harmon *et al.*[5] also deduced $l_p = 19\hbar$ for the ^{28}Si + ^{12}C system by a similar method. This apparently small l_p in fact resulted from modifying one of the original parameters in the Bass model in order to reproduce the measured fusion cross section and the behavior of the damped scattering in the ^{28}Si + ^{12}C reaction. However, for other systems considered in this paper, we could imagine a much larger maximum angular momentum. For example, Fig. 3 suggests that $J_{\max}$ for fusion is not yet reached until $J_{\rm cr} = \sim 30\hbar$ and $\sim 35\hbar$ for the ^{24}Mg + ^{16}O and ^{20}Ne + ^{20}Ne systems, respectively. Harmon *et al.* also observed that more symmetric systems have considerably larger $J_{\max}$. Also, in the similar systems such as ^{27}Al + ^{12}C and ^{27}Al + ^{16}O, where the fusion cross sections in Region III have been measured, we can extract the experimental $J_{\max}$ of about $35 \sim 36\hbar$. Therefore it is not surprising to observe that the fusion data of ^{24}Mg + ^{16}O do not show a decrease in the cross section until the highest energy measured at present. But what is surprising is to find the very small $J_{\max}$ in the ^{28}Si + ^{12}C reaction compared to those in other systems which lead to the common compound nucleus. This particular entrance channel effect is taken as evidence for an orbiting phenomenon in the dinuclear complex formed during the collision. In fact, Shapira *et al.*[7] observed angular momentum saturation due to orbiting in the study of the deep inelastic process of ^{28}Si + ^{12}C. We suppose that this early saturation of the angular momentum is caused by the relatively large reduced moment of inertia in the entrance channel, and a detailed investigation in this direction is in progress.

In summary, we have demonstrated that our model, based on the direct reaction theory first proposed by UKT for the low energy fusion cross section, can be extended successfully to give a consistent explanation for the fusion reaction in the whole range of the measured center-of-mass energy. The model consists of two ingredients. One is the fusion potential $W_F(r)$ in Eq. (1), which plays the role of a main apparatus for the fusion mechanism. The original choice of $W_F(r)$ by UKT for Region I, namely, a part of the imaginary potential up to the fusion radius is shown to work equally well for energies up to Region III. The other is the upper limit in the summation of partial waves in Eq. (1), which is found to be very important in reproducing the specific shape of the measured fusion excitation function. As for the upper limit, we have taken the "shifted" yrast line for Regions I and II and the measured $J_{\max}$ for Region III, and obtained a remarkably good description for the ^{28}Si + ^{12}C fusion reaction.

This work was supported in part by the Ministry of Education, Korea, through the Basic Science Research Institute Program, 1986.

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