Liquid-gas and superconducting phase transitions of nuclear matter calculated with real time Green's function methods and Skyrme interactions

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(Received 4 February 1986; revised manuscript received 20 November 1986)

Real-time finite temperature Green's function methods with pair cutoff approximations are applied to the calculation of the equation of state of symmetric nuclear matter. The liquid-gas and the superconducting second-order phase transitions of nuclear matter are studied using, respectively, the normal and abnormal pair cutoff approximations. Several versions of the Skyrme effective interactions are employed. Significant differences are found between the pressure-density isotherms at finite temperature given by different Skyrme interactions, although they give quite similar ground state nuclear matter properties. The critical temperatures $k_B T_c^{(1)}$ for the liquid-gas phase transition given by various Skyrme interactions range from ~ 15 to ~ 20 MeV. A strong dependence of $k_B T_c^{(1)}$ on the combination $3t_1+5t_2$, t_1 and t_2 being two parameters of the Skyrme interaction, is observed. For nuclear matter at normal density, nonvanishing energy gap is obtained only for the Skyrme interactions SkI and SkVI. The critical temperatures for the superconducting second-order phase transitions for these two cases are, respectively, 0.5 and 0.345 MeV. Dependence of the energy gap on the nuclear matter density is discussed.

I. INTRODUCTION

Finite temperature Green's function methods are very useful tools for studying thermodynamical properties of many-body systems. These methods may be classified into two general types: one using imaginary time variables and the other using real time variables. They are usually referred to, respectively, as the imaginary-time (IMT) and the real-time (RT) Green's function methods. In statistical and solid-state physics, these methods have been used rather frequently.[1-5] However, their applications to nuclear physics problems have been relatively scarce. There have been nuclear structure calculations based on the finite-temperature IMT Green's function methods.[6] However, as far as we know, the rather powerful and interesting RT finite-temperature Green's function method has not yet been used to treat nuclear physics problems. In this paper, we wish to use this method to study the first-order (liquid-gas) and second-order (superconducting) phase transitions of symmetric nuclear matter, within the nonrelativistic framework of treating nuclear matter as a system of nucleons interacting with some effective nucleon-nucleon potential.

Phase transitions in nuclear matter are a topic of much current interest. Recent medium energy heavy-ion experiments have indicated the occurrence of the liquid-gas phase transition of nuclear matter, its critical temperature being estimated to be $k_B T_c \approx 15$ MeV.[7,8] The Green's function methods mentioned above provide a convenient theoretical framework for carrying out such phase transition calculations. Here we have a choice of two methods—the IMT and the RT Green's function method. An advantage of the IMT Green's function method is its systematic perturbation (diagrammatic) expansions for various thermodynamic quantities such as the grand potential $\hat{\Omega}$. On the other hand, the RT Green's function method is particularly suited for nonperturbative treatments of phase transitions. An example is its treatment of the superconducting phase transition, where the order parameter is the expectation values of the abnormal Cooper pairs.[9]

The choice of the nucleon-nucleon effective interactions to be used in treating the phase transitions also plays an important role in deciding which of the above two Green's function methods is more convenient for calculation. If one intends to perform a microscopic phase transition calculation of nuclear matter starting from a realistic nucleon-nucleon potential such as the Paris interaction,[10] then the IMT Green's function method is apparently a better choice. This is because in such calculations one usually needs to make partial summations of certain classes of diagrams to all orders, and the diagrammatic formulation of the IMT Green's function method is particularly suited for doing so. Such a calculation is, in fact, being carried out by Kuo, Yang, and Song.[11]

It is well known that gross properties of nuclear systems can often be described rather successfully by model-space nuclear calculations using empirical effective interactions such as the Skyrme effective interactions.[12,13] An undisputed advantage of such calculations is that many results can be obtained analytically and are therefore physically more transparent than those obtained from microscopic calculations using realistic nucleon-nucleon interactions. Thus there are definitely advantages in first carrying out nuclear phase transition calculations using empirical effective interactions. For them, the RT Green's function method appears clearly to be a convenient and natural choice. Together with the normal and abnormal pair cutoff (to be denoted by NPC and ANPC, respectively) approximations to be discussed later, this

method provides an expedient scheme for calculating the Green's functions directly and from them various thermodynamical properties can be conveniently calculated. In fact, this method, together with NPC and ANPC approximations, has been used rather successfully in treating various problems in statistical and elementary particle physics.[14–16]

In this paper, our primary aim is to study and calculate the first- and second-order phase transitions of symmetric nuclear matter using RT Green's function methods with, respectively, the NPC and ANPC approximations. To simplify our calculations and, moreover, to make them physically more transparent, we will carry out our calculations using the rather successful Skyrme effective interactions. In fact, we will use several (seven) different versions of such interactions and study and compare the results obtained.

To a large extent, our study is motivated by the recent work of Jaqaman, Mekjian, and Zamick (JMZ).[17] They have carried out a very interesting calculation of the first-order (liquid-gas) phase transition of nuclear matter based on finite temperature Hartree-Fock theory. As explained later, the theoretical framework provided by the RT Green's function method with the NPC approximation and that by the finite temperature Hartree-Fock theory are equivalent to each other. But there is an essential difference between their calculation and ours. In their work, they started out with an interaction of a general Skyrme effective interaction type. Based on this, they have obtained an interesting result that the critical temperature, pressure, and density of nuclear matter can be expressed directly in terms of the ground-state nuclear gross properties—the binding energy per nucleon, $\hat{E}^0_B$, the kinetic energy per particle, $\hat{E}^0_K$, and the effective mass $(m^*/m)_0$. Thus their calculations were formulated and carried out mainly in terms of $\hat{E}^0_B$, $\hat{E}^0_K$, and $(m^*/m)_0$. Hence it is a main interest of their work to study the dependence of the liquid-gas phase transition of nuclear matter on the above mentioned ground state nuclear gross properties, but not directly on the characteristics and strengths of the various components of the Skyrme effective interactions.

Our interest and purpose are somewhat different and more microscopic. We are interested in studying the roles of the various components of the Skyrme effective interactions in the liquid-gas phase transition of nuclear matter. Thus we will carry out our calculations using the Skyrme interactions throughout, and give our results directly in terms of the Skyrme interaction parameters. In this way, we will be able to study, for example, which components of the Skyrme effective interactions are most influential to the liquid-gas phase transition of nuclear matter.

Jaqaman and co-workers have not studied the second-order superconducting phase transitions of nuclear matter. To our knowledge, this type of calculation seems to have not yet been done for nuclear matter. As stated earlier, we will carry out such calculations using the RT Green's function method and ANPC approximation. Again, the Skyrme interactions will be employed. It is our hope that our calculations may lead to some experimental investigation of this type of nuclear matter phase transitions.

In the following, we first briefly describe some essential features of the RT Green's function method and the NPC and ANPC approximations. This is carried out in Secs. II–IV. The theoretical formulation obtained in these sections is then applied, in Sec. V, to calculate the equation of state, critical temperatures for liquid-gas and superconducting phase transitions, and other thermodynamic properties of nuclear matter. Certain thermodynamic properties of nuclear matter have been calculated by Brack *et al.*[18] using the Skyrme interaction SkM*, and by Sauer *et al.*[19] using the Skyrme interaction SkyIII′. Our results will be compared with theirs. Some concluding remarks are presented in Sec. VI.

II. FINITE TEMPERATURE REAL TIME GREEN'S FUNCTION METHOD

There are basically two types of finite temperature Green's function methods, one with imaginary time variables and the other with real time variables. The former is more familiar. Because the finite temperature real time Green's function method has been relatively unfamiliar to nuclear physicists, we describe briefly some of its basic features in this section. This is also necessary for discussing the normal and abnormal pair cutoff approximations on which our calculations of nuclear matter phase transitions are based.

We consider the retarded thermal Green's function

$$G_{AB}(t,t') = -i\Theta(t-t')[\langle A(t)B(t')\rangle - \eta\langle B(t')A(t)\rangle] \equiv \langle\langle A(t),B(t')\rangle\rangle \ . \tag{2.1}$$

Here, Θ is the usual step function, and t and t' are both real time variables. The grand canonical ensemble average $\langle\cdots\rangle$ is defined as $\mathrm{tr}\{\cdots\exp[\beta(\Omega-\tilde{H})]\}$, where $\tilde{H}=H-\mu N$, μ and N being, respectively, the chemical potential and the number operator, and Ω is the thermodynamic potential. $A(t)$ and $B(t')$ are both Heisenberg operators, i.e.,

$$A(t)=e^{i\tilde{H}t}Ae^{-i\tilde{H}t} \ , \tag{2.2}$$

and similarly for $B(t')$. We consider A and B each to be a product of Fermion operators; then, η in Eq. (2.1) is given by $(-1)^p$, where p is the number of permutations needed to reorder AB into BA.

We intend to calculate $G_{AB}(t-t')$ by way of its equation of motion, which is obtained from Eq. (2.1) as

$$i\frac{d}{dt}G_{AB}(t-t') = \delta(t-t')[\langle A(t)B(t')\rangle - \eta\langle B(t')A(t)\rangle] + \langle\langle [A,\tilde{H}]_t,B(t')\rangle\rangle \ , \tag{2.3}$$

where $[\cdots]_t$ stands for $(\exp(i\tilde{H}t))(A\tilde{H}-\tilde{H}A)\times(\exp(-i\tilde{H}t))$. It is easily seen that Eq. (2.3) does *not* form a closed set of equations. Suppose A is a single Fermion operator a_l. The commutator $[a_l,\tilde{H}]_t$ contains, in general, terms of the form $a_j^\dagger a_k a_l$. Then, clearly, Eq. (2.3) tells us that G_{AB} is coupled to higher order Green's functions of the form $G^3=\langle\langle a_j^\dagger a_k a_l,B\rangle\rangle$. Similarly, the equation of motion for G^3 will be coupled to G^5, and so on.

In this way, they form an hierarchy of infinitely many coupled equations, and from them we cannot get the solution of G_{AB} unless some truncation approximations are made. The pair cutoff approximations to be discussed in later sections are for this truncation purpose.

The above thermal Green's functions are useful in calculating various thermodynamic properties. This is usually carried out by way of the time correlation function

$$F_{BA}(t-t')\equiv\langle B(t')A(t)\rangle \ . \tag{2.4}$$

If we have obtained the Green's function G_{AB}, then F_{BA} is given by[4]

$$F_{BA}(t-t')=i\int_{-\infty}^{\infty}\frac{G_{AB}(E+i0^+)-G_{AB}(E-i0^+)}{e^{\beta E}-\eta}\times e^{-iE(t-t')}dE \ , \tag{2.5}$$

where the Fourier transform of G_{AB} is defined as

$$G_{AB}(E)=\frac{1}{2\pi}\int_{-\infty}^{\infty}G_{AB}(t)e^{iEt}dt \ . \tag{2.6}$$

As usual, β is $1/k_BT$, where T is the temperature and k_B the Boltzmann constant. 0^+ denotes an infinitesimal positive number. As demonstrated later, various thermodynamic quantities such as the Fermi-Dirac distribution function and internal energy can be calculated directly from the above time correlation function. In the following, we will describe two types of pair cutoff approximations for calculating the Green's functions.

III NORMAL PAIR CUTOFF (NPC) APPROXIMATION

The normal pair cutoff (NPC) approximation for the single particle Green's function is used in our first order phase transition calculations and is briefly described here. In fact, the single particle Green's function is the case where the NPC approximation can be most transparently explained. We start from a Hamiltonian $\tilde{H}=H-\mu N$ given in second quantized form as

$$\tilde{H}=\sum_i(\epsilon_i-\mu)a_i^\dagger a_i+\tfrac{1}{2}\sum_{ijkl}\langle ij\mid V\mid kl\rangle a_i^\dagger a_j^\dagger a_l a_k \ , \tag{3.1}$$

where the single particle states (i,j,k,l) and energy ϵ are defined by the unperturbed Hamiltonian H_0.

Following Eq. (2.1) we define the retarded single particle Green's function as

$$G_{12}(t-t')\equiv -i\Theta(t-t')[\langle a_1(t)a_2^\dagger(t')\rangle-\eta\langle a_2^\dagger(t')a_1(t)\rangle]\equiv\langle\langle a_1(t),a_2^\dagger(t')\rangle\rangle \ . \tag{3.2}$$

The equation of motion for $G_{12}(t-t')$ is

$$i\frac{d}{dt}G_{12}(t-t')=\delta_{12}\delta(t-t')+\langle\langle[a_1,\tilde{H}]_t,a_2^\dagger(t')\rangle\rangle \ . \tag{3.3}$$

Using Eq. (3.1), the above becomes

$$i\frac{d}{dt}G_{12}(t-t')=\delta_{12}\delta(t-t')+(\epsilon_1-\mu)G_{12}(t-t')+\tfrac{1}{2}\sum_{ijkl}\{\delta_{i1}\langle ij\mid V\mid kl\rangle\langle\langle(a_j^\dagger a_l a_k)_t,a_2^\dagger(t')\rangle\rangle-\delta_{j1}\langle ij\mid V\mid kl\rangle\langle\langle(a_i^\dagger a_l a_k)_t,a_2^\dagger(t')\rangle\rangle\} \ , \tag{3.4}$$

where $(\cdots)_t$ stands for $(\exp(i\tilde{H}))(\cdots)(\exp(-iHt))$.

Clearly, Eq. (3.4) does not form a closed set of equations for $G_{12}(t-t')$. Now we introduce the normal pair cutoff (NPC) approximations under which the $\langle\langle\cdots\rangle\rangle$ quantities of Eq. (3.4) are approximated according to

$$\langle\langle(a_j^\dagger a_l a_k)_t,a_2^\dagger(t')\rangle\rangle\cong\delta_{jl}\langle a_j^\dagger a_l\rangle\langle\langle a_k(t),a_2^\dagger(t')\rangle\rangle-\delta_{jk}\langle a_j^\dagger a_k\rangle\langle\langle a_l(t),a_2^\dagger(t')\rangle\rangle \ , \tag{3.5}$$

and similarly for $\langle\langle(a_i^\dagger a_l a_k)_t,a_2^\dagger(t')\rangle\rangle$. Under these approximations and using the symmetry property $\langle ij\mid V\mid kl\rangle=\langle ji\mid V\mid lk\rangle$, Eq. (3.4) becomes

$$i\frac{d}{dt}G_{12}(t-t')=\delta_{12}\delta(t-t')+(\epsilon_1-\mu)G_{12}(t-t')+\sum_{jk}\{\langle 1j\mid V\mid kj\rangle-\langle 1j\mid V\mid jk\rangle\}\langle a_j^\dagger a_j\rangle G_{k2}(t-t') \ . \tag{3.6}$$

We see clearly that the above forms a closed set of equations of motion for the finite temperature single particle Green's function.

For nuclear matter, the single particle wave functions are of definite linear momentum p, and G_{12} is diagonal in p. We consider the interaction to be spin and isospin independent. Thus we rewrite Eq. (3.6) as

$$i\frac{d}{dt}G_{p\alpha}(t-t')=\delta(t-t')+(\epsilon_p-\mu)G_{p\alpha}(t-t')+\sum_{k\alpha'}\langle p,k\mid V\mid p,k\rangle\langle a_{k,\alpha'}^\dagger a_{k,\alpha'}\rangle G_{p\alpha}(t-t')-\sum_{q\alpha'}\langle p,p-q\mid V\mid p-q,p\rangle\langle a_{p-q,\alpha'}^\dagger a_{p-q,\alpha'}\rangle G_{p\alpha}(t-t') \ , \tag{3.7}$$

where α and α' represent the spin isospin variables (s_z,τ_z) and (s_z',τ_z'), respectively. We now make a Fourier transform as indicated by Eq. (2.6). Then Eq. (3.7) becomes

$$G_{p\alpha}(E)=\frac{1}{2\pi}\frac{1}{E-E_p}\ , \tag{3.8}$$

where E_p is given by

$$E_p=\epsilon_p-\mu+g\sum_k\langle p,k\mid V\mid p,k\rangle n_k - \sum_q\langle p,p-q\mid V\mid p-q,p\rangle n_{p-q}\ . \tag{3.9}$$

Here, g is the single particle spin-isospin degeneracy, i.e., $g=(2s+1)(2\tau+1)=4$, and n_k is the occupation number

$$n_k\equiv\langle a_{k,\alpha}^\dagger a_{k,\alpha}\rangle\ , \tag{3.10}$$

and similarly for n_{p-q}. Note that E_p and n_k are both independent of α.

The above is just the finite temperature Hartree-Fock theory.[2] Thus we see that the NPC approximation described above is equivalent to the finite temperature Hartree-Fock theory. The Hartree-Fock single particle energy E_p is to be calculated self-consistently. This is because n_k of Eq. (3.10) is itself dependent on E_k. Consider the time correlation function $\langle a_{k,\alpha}^\dagger(t')a_{k,\alpha}(t)\rangle$. Using Eqs. (2.4), (2.5), and (3.8), we readily obtain

$$\langle a_{k,\alpha}^\dagger(t')a_{k,\alpha}(t)\rangle=\frac{e^{-iE_k(t-t')}}{e^{\beta E_k}+1}\ . \tag{3.11}$$

Taking the limit of $t'\to t$, we obtain the Fermi-Dirac distribution function under the NPC approximation as

$$n_k=\langle a_{k,\alpha}^\dagger a_{k,\alpha}\rangle=\frac{1}{e^{\beta E_k}+1}\ . \tag{3.12}$$

The internal energy $U=\langle H\rangle$ is calculated using the same NPC approximation. The result is

$$U=g\sum_k\epsilon_k n_k+\frac{g}{2}\sum_{kk'}\langle k,k'\mid V\mid k,k'\rangle n_k n_{k'} + \tfrac{1}{2}\sum_{kq}\langle k,k-q\mid V\mid k-q,k\rangle n_k n_{k-q}\ . \tag{3.13}$$

IV. ABNORMAL PAIR CUTOFF (ANPC) APPROXIMATION

We consider again the retarded single particle Green's function $\langle\langle a_1(t),a_2^\dagger(t')\rangle\rangle$ of Eq. (3.2), and similarly we study its equation of motion (3.4). Unlike the NPC approximation indicated by Eq. (3.5), we now approximate the Green's function within the curly braces of Eq. (3.4) according to

$$\langle\langle(a_j^\dagger a_l a_k)_{t'},a_2^\dagger(t')\rangle\rangle\cong\delta_{jl}\langle a_j^\dagger a_l\rangle\langle\langle a_k(t),a_2^\dagger(t')\rangle\rangle-\delta_{jk}\langle a_j^\dagger a_k\rangle\langle\langle a_l(t),a_2^\dagger(t')\rangle\rangle+\delta_{l\underline{k}}\langle a_l a_k\rangle\langle\langle a_j^\dagger(t),a_2^\dagger(t')\rangle\rangle\ , \tag{4.1}$$

where $\underline{k}$ denotes the time-reversed state of k. We note that Eqs. (4.1) and (3.5) are identical, except that Eq. (4.1) has an extra third term which contains the factor $\langle a_l a_k\rangle$. This is a "number nonconserving" factor, a feature well known in the BCS theory of superconductivity. To be more explicit, the basis wave functions used here are not the eigenfunctions of the number operator. Consequently, the average value $\langle a_{\underline{k}}a_k\rangle$ is generally nonvanishing. $\langle a_{\underline{k}}a_k\rangle$ is the order parameter for the second order phase transition.

Substituting Eq. (4.1) into Eq. (3.4) gives the equation of motion for $G_{12}(t-t')$ as

$$i\frac{d}{dt}G_{12}(t-t')=\delta_{12}\delta(t-t')+(\epsilon_1-\mu)G_{12}(t-t') + \sum_{j,k}\{\langle 1j\mid V\mid kj\rangle-\langle 1j\mid V\mid jk\rangle\}\langle a_j^\dagger a_j\rangle G_{k2}(t-t')+\sum_{j,k}\langle 1j\mid V\mid k\underline{k}\rangle\langle a_{\underline{k}}a_k\rangle F_{j2}(t-t')\ , \tag{4.2}$$

where the new retarded Green's function F is defined as

$$F_{12}(t-t')\equiv\langle\langle a_1^\dagger(t),a_2^\dagger(t')\rangle\rangle\ . \tag{4.3}$$

Similarly, we can derive an equation of motion for $F_{12}(t-t')$. Thus, under the ANPC approximation, we have a closed set of coupled equations for the Green's functions G and F.

Similar to the derivation of Eq. (3.8), we obtain the Gorkov-type[2] coupled equations for G and F as

$$\begin{aligned}&(E-E_p)G_{p\alpha}(E)+\Delta_p F_{p\alpha}(E)=\frac{1}{2\pi}\ ,\\ &\Delta_p G_{p\alpha}(E)+(E+E_p)F_{p\alpha}(E)=0\ ,\end{aligned} \tag{4.4}$$

where E_p and Δ_p, the energy gap, are given by

$$E_p=(\epsilon_p-\mu)+g\sum_k\langle p,k\mid V\mid p,k\rangle n_k - \sum_q\langle p,p-q\mid V\mid p-q,p\rangle n_{p-q} \tag{4.5}$$

and

$$\Delta_p=\sum_k\langle p,\underline{p}\mid V\mid k,\underline{k}\rangle\nu_k\ , \tag{4.6}$$

with

$$n_k=\langle a_{k,\alpha}^\dagger a_{k,\alpha}\rangle\ \text{ and }\ \nu_k=\langle a_{\underline{k},\underline{\alpha}}a_{k,\alpha}\rangle\ . \tag{4.7}$$

Note that $(\underline{k},\underline{\alpha})$ is the time reversed state of (k,α). From Eq. (4.4) we have

$$G_{p\alpha}(E)=\frac{1}{2\pi}\frac{E+E_p}{E^2-(E_p^2+\Delta_p^2)} \tag{4.8}$$

and

$$F_{p\alpha}(E)=-\frac{1}{2\pi}\frac{\Delta_p}{E^2-(E_p^2+\Delta_p^2)} . \tag{4.9}$$

n_k and v_k are calculated from the respective time correlation functions, as indicated by Eqs. (2.4) and (2.5). The results are

$$n_p=\frac{1}{2}\left\{1-\frac{E_p}{(E_p^2+\Delta_p^2)^{1/2}}\tanh[\tfrac{1}{2}\beta(E_p^2+\Delta_p^2)^{1/2}]\right\} , \tag{4.10}$$

and

$$v_p=\frac{1}{2}\frac{\Delta_p}{(E_p^2+\Delta_p^2)^{1/2}}\tanh[\tfrac{1}{2}\beta(E_p^2+\Delta_p^2)^{1/2}] . \tag{4.11}$$

Equations (4.5), (4.6), (4.10), and (4.11) form a set of self-consistent equations. As temperature tends to zero, they reduce to the familiar BCS gap equations (at zero temperature).

The energy gap Δ_p is now temperature dependent. Suppose there is enough pairing interaction in nuclear matter; then we expect to have a nonvanishing gap near zero temperature. A question of much interest is whether this gap may vanish as temperature increases? The temperature T_c at which this gap vanishes is the critical temperature for the second-order superconducting to normal phase transition[20] of nuclear matter.

V. RESULTS

A. Liquid-gas phase transition

In this section we apply the NPC method described in Sec. III to calculate the liquid-gas phase transition and some other thermodynamical properties of nuclear matter. We have chosen to use the Skyrme effective interactions[12,13] in our calculations for two considerations. First, they are relatively simple to calculate with and their usage will enable us to obtain analytic results for many physical quantities of our interest. Second, as pointed out by Beiner *et al.*,[13] the Skyrme effective interactions have been quite successful in describing the ground-state (zero temperature) properties, such as binding energies and charge radii and densities, of a large number of nuclei. A drawback of these interactions is that they contain a number of adjustable parameters and these parameters are not uniquely determined. In fact, there exist many different sets of such parameters which are practically equivalent in reproducing certain ground-state nuclear properties. Thus it will be of interest to calculate the thermodynamical (finite temperature) properties of nuclear matter using these different versions of the Skyrme effective interaction and compare their results. For example, it should be of interest to compare the critical temperatures k_BT_c for the liquid-gas phase transition of nuclear matter given by these different versions of the Skyrme effective interaction.

The Skyrme effective interactions used here are of the general form

$$V_{12}=t_0\delta(\mathbf{r}_1-\mathbf{r}_2)+\tfrac{1}{2}t_1[\delta(\mathbf{r}_1-\mathbf{r}_2)k^2+k'^2\delta(\mathbf{r}_1-\mathbf{r}_2)]+t_2\overleftarrow{\mathbf{k}}'\cdot\delta(\mathbf{r}_1-\mathbf{r}_2)\overrightarrow{\mathbf{k}}+\tfrac{1}{6}t_3\rho^\sigma\delta(\mathbf{r}_1-\mathbf{r}_2) , \tag{5.1}$$

where t_0, t_1, t_2, and t_3 are parameters determined phenomenologically. ρ is the nuclear matter density, and the exponent σ is also taken as a parameter.[17,18] $\overrightarrow{\mathbf{k}}$ and $\overleftarrow{\mathbf{k}}'$ are, respectively, $(\nabla_1-\nabla_2)/2i$ and $-(\overleftarrow{\nabla}_1-\overleftarrow{\nabla}_2)/2i$. As shown in Table I, we employ seven versions of Skyrme interactions,[13,18] denoted SkI–SkVI and SkM*.

To carry out our calculation, we need to solve Eqs. (3.9) and (3.12). Thus, a first step is to evaluate the plane-wave matrix elements of V_{12}. Using Eq. (5.1), we have readily

$$\langle \mathbf{q}_1\mathbf{q}_2 \mid V_{12} \mid \mathbf{q}_3\mathbf{q}_4\rangle=\frac{1}{V_0}\delta_{\mathbf{K}_{12},\mathbf{K}_{34}}\left[t_0+\frac{t_3}{6}\rho^\sigma+\frac{t_1}{2}(\mathbf{k}_{12}-\mathbf{k}_{34})^2+(t_1+t_2)\mathbf{k}_{12}\cdot\mathbf{k}_{34}\right] , \tag{5.2}$$

where V_0 denotes the volume of the nuclear matter system. The relative and center-of-mass moment are defined, respectively, as $\mathbf{k}_{12}=(\mathbf{q}_1-\mathbf{q}_2)/2$, $\mathbf{K}_{12}=\mathbf{q}_1+\mathbf{q}_2$, and similarly for $\mathbf{k}_{34}$ and $\mathbf{K}_{34}$. It is convenient to change the momentum variables in Eq. (5.2) as $\mathbf{q}_1\to\mathbf{k}'+\mathbf{q}$, $\mathbf{q}_2\to\mathbf{k}-\mathbf{q}$, $\mathbf{q}_3\to\mathbf{k}'$, and $\mathbf{q}_4\to\mathbf{k}$. Then we are dealing with the matrix element

TABLE I. Skyrme effective interactions. Parameters for SkI–SkVI are taken from Ref. 13, and SkM* from Ref. 18, and similarly for the saturation Fermi level k_F and incompressibility coefficient K.

	t_0 (MeV fm^3)	t_1 (MeV fm^5)	t_2 (MeV fm^5)	t_3 (MeV fm$^{3+3\sigma}$)	k_F (fm^{-1})	K (MeV)	σ
SkI	−1057.3	235.9	−100	14463.5	1.32	370	1
SkII	−1169.9	585.6	−27.1	9331.1	1.30	342	1
SkIII	−1128.75	395	−95	14000	1.29	356	1
SkIV	−1205.6	765	35	5000	1.31	325	1
SkV	−1248.29	970.56	107.22	0	1.32	306	1
SkVI	−1101.81	271.67	−138.33	17000	1.29	364	1
SkM*	−2645.0	410.0	−135.0	15595	1.33	216.7	$\frac{1}{6}$

$\langle \mathbf{k}+\mathbf{q}, \mathbf{k}'-\mathbf{q} \mid V_{12} \mid \mathbf{k},\mathbf{k}' \rangle$, where q is the momentum transfer and $\mathbf{k}+\mathbf{k}'$ is the conserved center-of-mass momentum. Substituting Eq. (5.2) into Eq. (3.9) gives

$$E_p=(\epsilon_p-\mu)+\frac{g-1}{(2\pi)^3}\left[t_0+\frac{t_3}{6}\rho^\sigma\right]\int d^3k\, n_k+\frac{1}{(2\pi)^3}\left[\frac{(g-1)t_1}{4}+\frac{(g+1)t_2}{4}\right]\int (\mathbf{p}-\mathbf{k})^2 n_k d^3k\ , \tag{5.3}$$

where n_k was given by Eq. (3.12). Remember that Eqs. (5.3) and (3.12) are to be solved self-consistently. This may be facilitated by using an effective mass approach, where E_p is expressed as

$$E_p=\left[\epsilon_0+\frac{\hbar^2p^2}{2m^*}\right]-\mu\ , \tag{5.4}$$

and then n_k is given as

$$n_k=(1+\tilde{Z}^{-1}e^{\beta\hbar^2k^2/2m^*})^{-1}\ , \tag{5.5}$$

where $\tilde{Z}$ is the fugacity

$$\tilde{Z}=e^{\beta(\mu-\epsilon_0)}\ . \tag{5.6}$$

By comparing Eq. (5.3) with Eq. (5.4), it is readily found that

$$\epsilon_0=\tfrac{3}{4}\rho\left[t_0+\frac{t_3}{6}\rho^\sigma\right]-\frac{3}{2}\frac{k_BT}{m^*}\frac{dm^*}{d\rho}\frac{4}{\tilde{\lambda}^3}f_{5/2}(\tilde{Z})\ , \tag{5.7}$$

and

$$m^*=m\left[1+\frac{m\rho}{8\hbar^2}(3t_1+5t_2)\right]^{-1}\ . \tag{5.8}$$

In the above, $f_{5/2}(Z)$ is[21]

$$f_{5/2}(\tilde{Z})=\frac{4}{\sqrt{\pi}}\int_0^\infty dx\, x^2\ln(1+\tilde{Z}e^{-x^2})\ , \tag{5.9}$$

and for small Z it can be expanded as

$$f_{5/2}(\tilde{Z})=\sum_{l=1}^{\infty}\frac{(-1)^{l+1}\tilde{Z}^l}{l^{5/2}}\ . \tag{5.9'}$$

$\tilde{\lambda}$ is the thermal de Broglie wavelength $[2\pi\hbar^2/(m^*k_BT)]^{1/2}$. We note that in the limit of $T\to 0$, ϵ_0 of Eq. (5.7) becomes

$$\epsilon_0=\tfrac{3}{4}\rho\left[t_0+\frac{t_3}{6}\rho^\sigma\right]+\frac{3t_1+5t_2}{8\pi^2}\frac{k_F^5}{5}\ . \tag{5.10}$$

Now we are in a position to obtain the equation of state $p(\rho,T)$ for nuclear matter, p, ρ, and T being, respectively, the pressure, density, and temperature. For a system of *noninteracting* nucleons with a HF (Hartree-Fock) external potential, the pressure is given as[21]

$$P_{\rm id}(m^*)=k_BT\frac{4}{\tilde{\lambda}^3}f_{5/2}(\tilde{Z})\ . \tag{5.11}$$

We are, however, considering a system of interesting nucleons. In zero-temperature HF calculations, it is well known that the ground state energy is given by the sum of the HF single particle energies and the lowest order potential energy $\langle\phi \mid V-U^{\rm HF}\mid\phi\rangle$. $U^{\rm HF}$ is the HF single particle potential, and ϕ is the unperturbed ground state. Here the situation is similar and we need to take into account the interaction $V-U^{\rm HF}$ in calculating $p(\rho,T)$, and this is done as outlined below.

We use the relation $p=-f+\rho\mu$, where f is the free energy density. Within the context of the finite temperature HF approximation,[2,5] the thermodynamic potential Ω is given by $\Omega_0+\Omega_1^{\rm HF}$, where Ω_0 is for the noninteracting HF system and $\Omega_1^{\rm HF}$ is due to the interaction $V-U^{\rm HF}$. Usually, one determines the chemical potential from the relation

$$\rho=\frac{2}{\pi^2}\int n_k k^2dk\ . \tag{5.12}$$

By inverting this equation, one obtains

$$\mu_0=\tfrac{3}{4}\rho\left[t_0+\frac{t_3}{6}\rho^\sigma\right]-\frac{3}{2}\frac{k_BT}{m^*}\frac{dm^*}{d\rho}\sum_{n=1}^{\infty}\tilde{B}_n\rho^n +k_BT\left[\ln\left[\frac{\tilde{\lambda}^3\rho}{4}\right]+\sum_{n=2}^{\infty}\frac{n}{n-1}\tilde{B}_n\rho^{n-1}\right]\ . \tag{5.13}$$

Note that we use μ_0 to denote the above chemical potential, because Eq. (5.12) is obtained from $n=-(\partial\Omega_0/\partial\mu)_{TV}$, i.e., it is for the noninteracting system. To obtain an approximate chemical potential μ_1 with internucleon interactions, we first calculate the approximate free energy $f_1=(\Omega_0+\Omega_1^{\rm HF})/V_0+\mu_0\rho$, which is obtained as

$$f_1=\tfrac{3}{8}t_0\rho^2+\frac{t_3}{16}\rho^{\sigma+2}+k_BT\left[\rho\ln\left[\frac{\tilde{\lambda}^3\rho}{4}\right]-\rho\right] +k_BT\sum_{n=2}^{\infty}\frac{\tilde{B}_n}{n-1}\rho^n\ . \tag{5.14}$$

In the above equations, $\tilde{B}_n$ denotes the expansion coefficients in

$$\frac{4}{\tilde{\lambda}^3}f_{5/2}(\tilde{Z})=\sum_{n=1}^{\infty}\tilde{B}_n\rho^n\ . \tag{5.15}$$

From f_1, we calculate $\mu_1=(\partial f_1/\partial\rho)_T$. Then the pressure is given by $p=\rho\mu_1-f_1$, which leads to

$$p=\tfrac{3}{8}t_0\rho^2+\frac{\sigma+1}{16}t_3\rho^{\sigma+2}+\left[1-\frac{3}{2}\frac{\rho}{m^*}\frac{dm^*}{d\rho}\right]P_{\rm id}(m^*)\ , \tag{5.16}$$

where $P_{\rm id}$ can be expressed as a power series in ρ using Eqs. (5.11) and (5.15). The above is an iterative scheme for calculating the pressure for a system of interacting nucleons. It is of interest to note that using a different method of derivation Ref. 17 has obtained the same equation of state as above.

As the temperature approaches zero, Eq. (5.16) becomes

$$p=\tfrac{3}{8}t_0\rho^2+\frac{\sigma+1}{16}t_3\rho^{\sigma+2}+\frac{2}{5}\frac{\hbar^2}{2m}\left[\frac{3\pi^2}{2}\right]^{2/3}\rho^{5/3} +\tfrac{1}{16}(3t_1+5t_2)\left[\frac{3\pi^2}{2}\right]^{2/3}\rho^{8/3}\ . \tag{5.17}$$

The internal energy $U=\langle H\rangle$ can be calculated from Eqs. (3.13) and (5.2), giving

$$U=\frac{V_0}{(2\pi)^3}\int d^3k\, n_k\epsilon_k+\frac{g-1}{2}\frac{V_0}{(2\pi)^6}\int d^3k\, d^3k' n_k n_{k'}\left[t_0+\frac{t_3}{6}\rho^\sigma\right]+\frac{1}{2}\frac{V_0}{(2\pi)^6}\int d^3k\, d^3k'\frac{3t_1+5t_2}{4}(\mathbf{k}'-\mathbf{k})^2 n_k n_{k'} . \tag{5.18}$$

In the limit of $T\to 0$, U becomes the nuclear matter ground-state energy E_0, namely

$$\frac{E_0}{N}=\tfrac{3}{5}\epsilon_F+\frac{3}{8}\left[t_0+\frac{t_3}{6}\rho^\sigma\right]\rho+\tfrac{3}{80}(3t_1+5t_2)\rho k_F^2 . \tag{5.19}$$

This agrees with the result given by Ref. 22. The pressure at zero temperature is $p=\rho^2\partial(E_0/N)/\partial\rho$, which agrees exactly with Eq. (5.17). This provides a consistency check for our method for obtaining the pressure as given by Eq. (5.16).

Using the Skyrme interactions given in Table I, we have calculated the pressure-density isotherms of nuclear matter using Eqs. (5.11), (5.15), and (5.16). (We consider low-density regions, and hence terms of the order ρ^7 and higher are neglected.) Our results for the Skyrme I, III, VI, and M* interactions are shown in Figs. 1–4. (Our results for SkIII and SkM* are in good agreement with those of Refs. 18 and 19, respectively.) As shown, the isotherms are generally of the form given by a typical van der Waals interaction. Generally speaking, the shapes of the isotherms shown by these figures look similar to each other, but a more detailed comparison reveals that the isotherms given by different Skyrme interactions can, in fact, be very different.

As an example, we compare in Fig. 5 the $k_BT=6$ MeV isotherms given by the seven Skyrme interactions of Table I. A shown, the isotherms given by the interactions II, IV, and VI are rather close to each other, but the others are indeed quite different. Thus, if some of the finite-temperature isotherms of nuclear matter can be determined by or deduced from experiments, then they may be used as a sensitive test of the various Skyrme interactions—to see which set of the Skyrme interaction parameters can best describe the finite-temperature properties of nuclear matter. We note that the ground state ($T=0$) properties of nuclear matter predicted by the various Skyrme interaction parameters of Table I are really not significantly different from each other, and therefore they cannot provide a sensitive test of the Skyrme interaction parameters.

Now we turn to the critical temperature $k_BT_c^{(1)}$ and density ρ_c. As is well known, they are determined by the condition

$$\left[\frac{\partial p}{\partial\rho}\right]_T=\left[\frac{\partial^2 p}{\partial\rho^2}\right]_T=0 . \tag{5.20}$$

Using this relation and Eq. (5.16), we have calculated the above quantities using the Skyrme interactions of Table I. Our results are given in Table II, together with the critical pressures. The effective masses at the critical densities and at the ground state saturation densities are also given. As shown, there is a rather large variation of the critical temperatures, ranging from 14.60 to 20.12 MeV. The re-

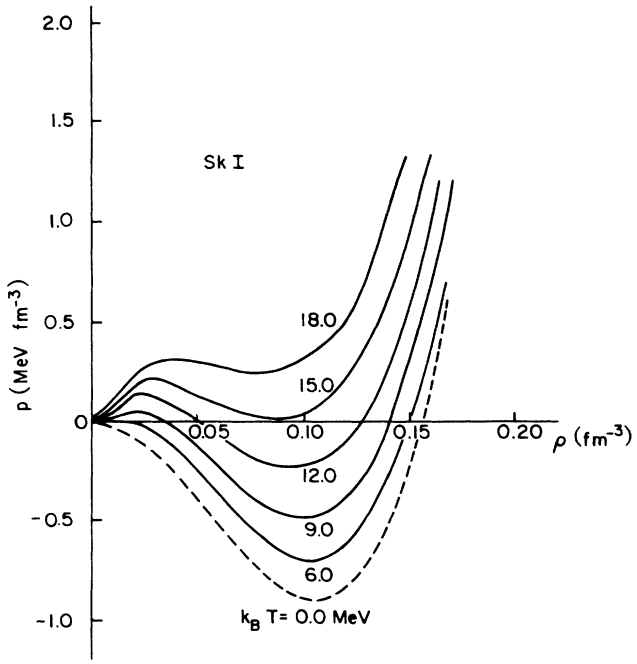

FIG. 1. Nuclear matter isotherms calculated with the Skyrme I interaction. Pressure p, density ρ, and temperature k_BT are in units of MeV fm^3, fm^{-3}, and MeV, respectively.

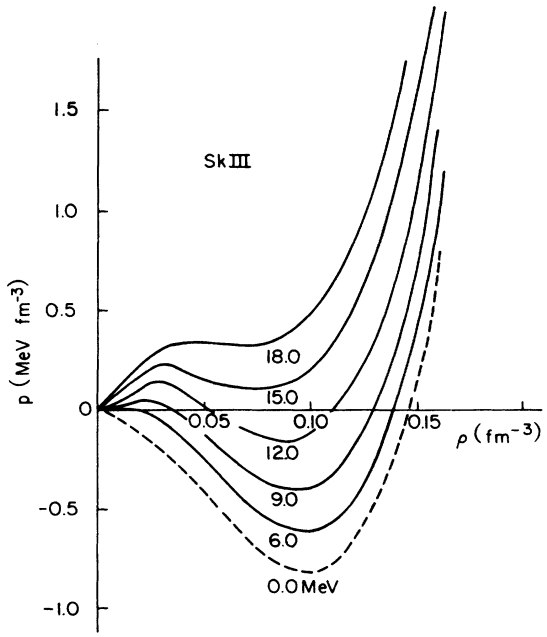

FIG. 2. Same as Fig. 1, except for the Skyrme III interaction.

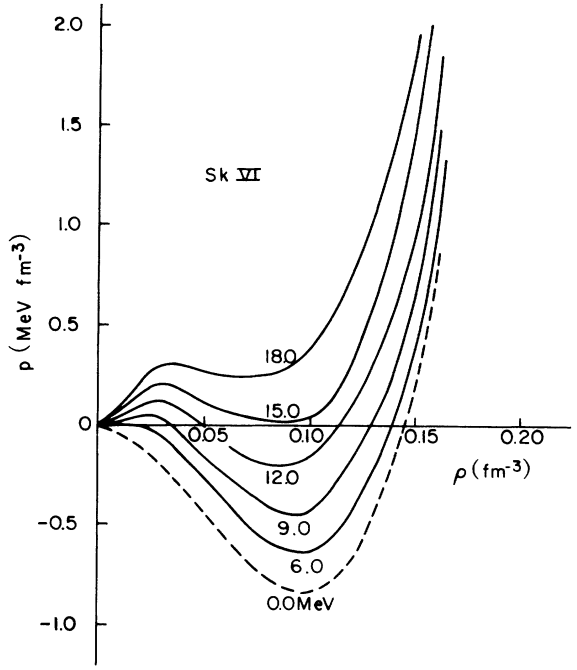

FIG. 3. Same as Fig. 1, except for the Skyrme VI interaction.

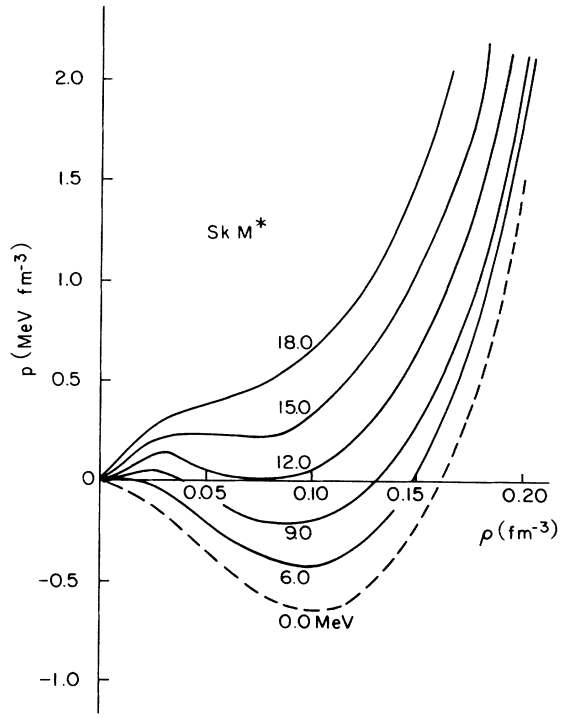

FIG. 4. Same as Fig. 1, except for the Skyrme M* interaction.

sults for the critical densities have much less variation.

In Fig. 6 we plot the critical temperatures versus the ground state effective masses. Results for the Skyrme interactions I–VI almost lie on a straight line, while the result for the Skyrme M* interaction deviates largely from this trend. This is a rather interesting result and we have not found a satisfactory explanation. It may have some connection with the fact that the compression modulus K for SkI–SkVI are all in the vicinity of 300 MeV, while that for SkM* is about 200 MeV. From Eq. (5.8), we see that m^* is dependent on the combination $3t_1+5t_2$. Our results seem to indicate that the critical temperatures $k_B T_c^{(1)}$ have a rather strong dependence on this combination.

B. Second-order phase transition

As discussed in Sec. IV, the energy gap Δ given by the ANPC approach is temperature dependent and the critical temperature $k_B T_c^{(2)}$ is the temperature at which the energy gap becomes zero. We calculate here $k_B T_c^{(2)}$ for nuclear matter using the Skyrme effective interactions given in Table I. To carry out this calculation, we need to first express the gap equations (4.5), (4.6), (4.10), and (4.11) in terms of the Skyrme interaction parameters. This leads to

$$E_p=(\epsilon_p-\mu)+\frac{1}{(2\pi)^3}\int d^3k\,[3t_0+\tfrac{1}{2}t_3\rho^\sigma+\tfrac{1}{4}(3t_1+5t_2)(p^2+k^2)]\frac{1}{2}\left[1-\frac{E_k}{(E_k^2+\Delta_k^2)^{1/2}}\tanh(\beta/2)(E_k^2+\Delta_k^2)^{1/2}\right], \quad (5.21)$$

$$\Delta_p=-\frac{1}{2(2\pi)^3}\int d^3k\,[t_0+\tfrac{1}{6}t_3\rho^\sigma+\tfrac{1}{2}t_1(p^2+k^2)]\frac{\Delta_k}{(E_k^2+\Delta_k^2)^{1/2}}\tanh(\beta/2)(E_k^2+\Delta_k^2)^{1/2}, \quad (5.22)$$

and

$$\rho=\frac{2}{(2\pi)^3}\int d^3k\left[1-\frac{E_k}{(E_k^2+\Delta_k^2)^{1/2}}\times\tanh(\beta/2)(E_k^2+\Delta_k^2)^{1/2}\right]. \quad (5.23)$$

In the above, ϵ_p is $\hbar^2p^2/2m$, and Eq. (5.23) is obtained from Eq. (4.10) and the relation $\rho=4\int 4\pi k^2dk\,n_k/(2\pi)^3$.

The above are a set of rather complicated self-consistent equations for determining E_p, Δ_p, and μ. It should be emphasized that to have a meaningful solution we must impose a strong requirement on the shape of Δ_p. For electrons in a usual superconductor, it is well known that the energy gap is a sharply peaked function centered around the Fermi surface. We assume that the energy gap for nucleons in nuclear matter has the same behavior. As a preliminary study, we take it as a square barrier func-

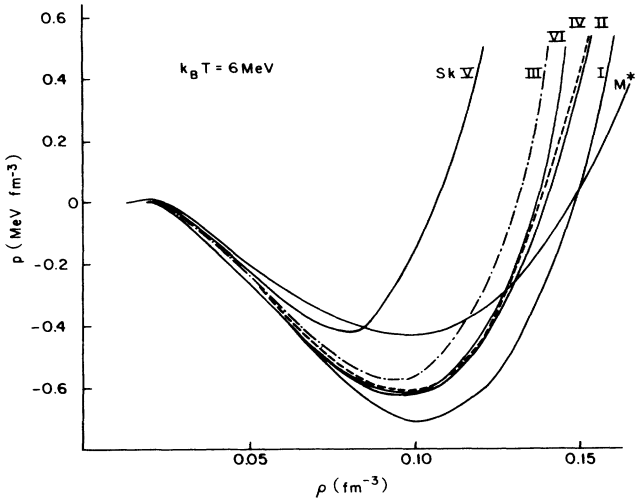

FIG. 5. Comparison of the $k_BT=6$ MeV isotherms given by various Skyrme interactions.

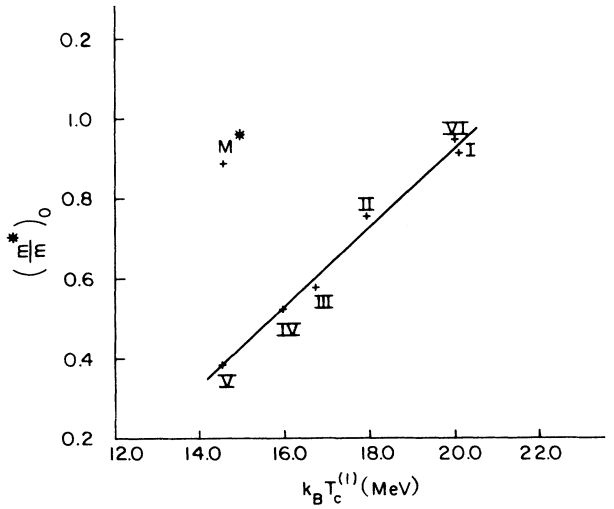

FIG. 6. Correlation between the critical temperature $k_BT_c^{(1)}$ and the effective mass $(m^*/m)_0$, for the various Skyrme interactions.

tion centered at the Fermi surface, namely

$$\Delta_k=\begin{cases}\Delta, & k_0-D<k<k_0+D\\ 0, & \text{otherwise}\end{cases} \tag{5.24}$$

where k_0 is determined by $n_{k_0}=\frac{1}{2}$, n being the Fermi-Dirac distribution function. We use $D=0.1$ fm^{-1}, which is a reasonable choice because the energy gap should be significant only within a narrow shell around the Fermi surface. Equation (5.24) is a strong but reasonable requirement imposed on the energy gap. With it, Eq. (5.22) becomes

$$1=-\frac{1}{2(2\pi)^3}\int_{k_0-D}^{k_0+D}d^3k\,g(k)\frac{1}{(E_k^2+\Delta^2)^{1/2}} \times\tanh(\beta/2)(E_k^2+\Delta^2)^{1/2}\,, \tag{5.25}$$

with

$$g(k)=t_0+\tfrac{1}{6}t_3\rho^\sigma+\tfrac{1}{2}t_1(k_0^2+k^2)\,. \tag{5.26}$$

We can now carry out an iterative solution of the gap equations (5.21), (5.23), and (5.25), using the following procedure. First, we assume a set of initial values for Δ, μ, and E_k. (We use a set of momentum space Gaussian mesh points to discretize E_k.) Using them, we calculate new values of E_k and μ from Eqs. (5.21) and (5.23). The new value for Δ is then determined from Eq. (5.25) based on the new E_k values. The procedure is repeated until satisfactory convergence is reached.

We have calculated the above gap equations using the Skyrme interactions of Table I. At normal nuclear matter density (i.e., $\rho=2k_F^3/3\pi^2$ with k_F given by Table I for the respective interactions), nonzero energy gaps are found for SkI and SkVI only. As displayed in Fig. 7, at zero temperature the gaps are 0.99 and 0.67 MeV for SkI and SkVI, respectively. As temperature increases, the gaps stay constant for a while and then abruptly drop to zero at the critical temperatures $kT_c^{(2)}$ as indicated by the arrows in the figure. (Goodman[23] has studied heated rotating nuclei using a finite temperature Hartree-Fock-Bogoliubov approach and has observed similar behavior for the pairing energy gap.) The values of $kT_c^{(2)}$ for SkI and SkVI are, respectively, 0.5 and 0.345 MeV. Thus for these two interactions our calculations indicate that nuclear matter at normal density and temperature below the respective $kT_c^{(2)}$ values is in a superconducting state.

To get some insight into the above results, it is helpful

TABLE II. Critical temperatures ($k_BT_c^{(1)}$), densities (ρ_c), and pressure (p_c) for the liquid-gas phase transition of nuclear matter. The effective masses at ρ_c and at ρ_0—the ground state saturation density—are also given, denoted by $(m^*/m)_c$ and $(m^*/m)_0$, respectively.

	SkI	SkII	SkIII	SkIV	SkV	SkVI	SkM*
$k_BT_c^{(1)}$ (MeV)	20.12	16.75	17.95	16.00	14.55	20.05	14.60
ρ_c (fm^{-3})	0.061	0.58	0.056	0.057	0.048	0.055	0.052
p_c (MeV fm^3)	0.399	0.331	0.330	0.333	0.276	0.367	0.210
$(m^*/m)_c$	0.963	0.779	0.893	0.703	0.744	0.980	0.920
$(m^*/m)_0$	0.913	0.577	0.760	0.471	0.382	0.949	0.789

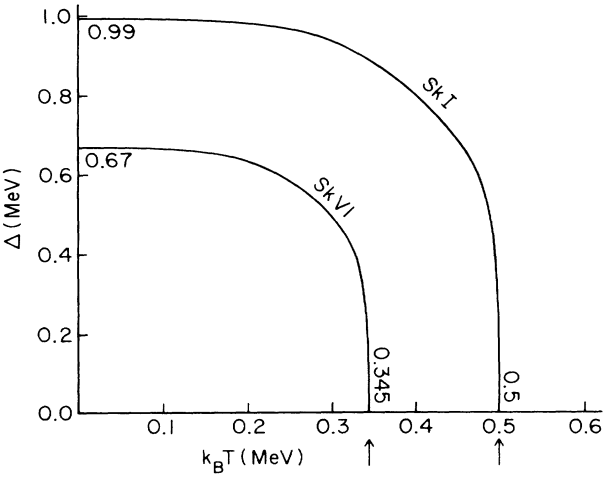

FIG. 7. Energy gaps Δ for nuclear matter at normal density.

to carry out an approximate analytic solution of the gap equations, in close analogy to that described in Fetter and Walecka[2] for the case of electron gas. Let us consider the case of zero temperature. Then the tanh term of Eq. (5.25) becomes one. Since D is small, we may replace $g(k)$ by $g(k_0)$, and approximate E_k by $(k^2-k_0^2)\hbar^2/2m^*$. Note that $k_0=k_F$. The integral of Eq. (5.25) can then be evaluated analytically, leading to

$$\Delta=\frac{2\hbar^2k_FD/m^*}{\left\{\exp\left[\frac{-1}{g(k_F)N_0}\right]-\exp\left[\frac{1}{g(k_F)N_0}\right]\right\}}, \quad (5.27)$$

with $N_0=m^*k_F/(2\pi^2\hbar^2)$. Note that this relation is for $T=0$ and $D\ll k_F$.

Within the above approximation, it is clearly seen that to have a nonvanishing gap at zero temperature we must have $g(k_F)<0$. From Table I, we see that t_0 is negative while t_3 and t_1 are both positive and t_3 is very large. Thus it is easy to have $g(k_F)>0$ and, consequently, we will not have solutions with nonvanishing gap. This is indeed the case. From Table I, it is readily found that $g(k_F)<0$ for SkI, SkIII, and SkVI only. The values of Δ calculated from Eq. (5.27) are 0.99, 0.027, and 0.69 MeV for these interactions, respectively. These values are, in fact, quite close to those of Fig. 7 obtained from exact numerical calculations. In a very similar way, the critical temperature $kT_c^{(2)}$ can also be determined approximately from Eq. (5.25) by setting $\Delta=0$ in this equation. The results so obtained are also in good agreement with those of Fig. 7.

The existence of a pairing energy gap of about 1 MeV in medium- and heavy-mass nuclei is well known, as pointed out a long time ago by Bohr, Mottelson, and Pines.[24] Subsequent microscopic nuclear structure calculations of the BCS type, such as that of Kisslinger and Sorensen,[25] who employed a pairing plus p_2 interaction, and that of Kuo, Baranger, and Baranger,[26] who used a realistic Gaussian interaction, have reached the same general result. Nuclear matter may be looked upon as an extremely large nucleus, and one may naturally expect that nuclear matter should also have an energy gap of about 1 MeV. This argument is clearly not valid because it has not taken the density dependence of the energy gap into account. For finite nuclei, we deal with nucleons near the Fermi surface where the nuclear matter density is approximately one-half as large as the normal nuclear matter density. From the present work, one can clearly see that the energy gap Δ depends rather sensitively on the nuclear matter density.

As discussed earlier, the function $g(k)$ plays an important role in determining Δ, and it contains a term $t_3\rho^\sigma/6$ where t_3 is strongly positive. Thus Δ's calculated with different densities can be quite different, and this is indeed borne out by our numerical solutions of the gap equations (5.21), (5.23), and (5.25). We show a typical result in Fig. 8, for SkI and $k_BT=0.1$ MeV. For large k_F (i.e., large ρ), there is no gap and this is easily understood in terms of the sign of $g(k)$. For low k_F, there is also no gap. This is because at very small density the chemical potential becomes very large and negative. This makes the right-hand side of (5.25) less than one. At $k_F\cong1.1$ fm^{-1} ($\rho\cong0.09$ fm^{-3}), Δ has its maximum value of $\cong2.5$ MeV. Our results indicate that for nuclear matter at normal density ρ_0 the energy gap is small or not existing. But at a density of $\sim\rho_0/2$, there should be an energy gap of ~1–2 MeV, as has been observed for nucleons near the surface of large even-even nuclei. Our results also indicate an interesting prospect, namely that nuclear matter at a density of $\sim\rho_0/2$ may exist in a superconducting phase. Further studies in this direction are being carried out.[27]

The density dependence of the pairing energy gap of nuclear matter at zero temperature has been studied by Emery and Sessler;[28] the trend of our results is in good agreement with theirs.

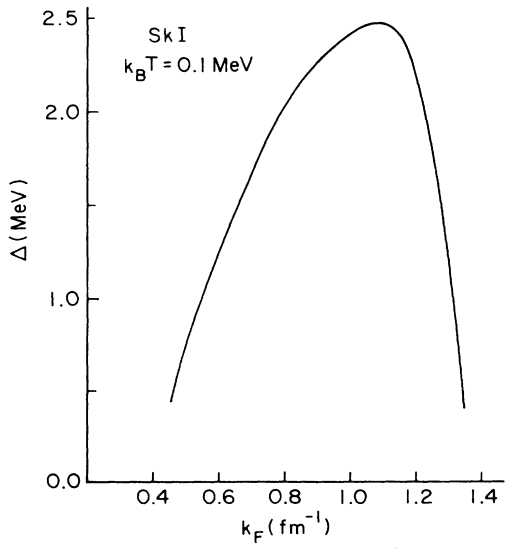

FIG. 8. Density dependence of the energy gap.

VI. DISCUSSION AND CONCLUSION

We have applied the real-time finite temperature Green's function method with pair cutoff approximations to the derivation of the equation of state of symmetric nuclear matter from the Skyrme effective interactions.

First, we have considered the normal pair cutoff approximation. This enables us to obtain in close form the single-particle Green's function from which the nuclear matter chemical potential μ is derived. The free energy of nuclear matter is calculated from μ, and then we obtain an equation of state expressing the nuclear matter pressure as an analytic function of its density and temperature and the parameters of the Skyrme effective interactions. The entire procedure is rather straightforward, indicating that the finite temperature pair cutoff Green's function method can be a very useful tool for treating nuclear matter.

We have found that the equations of state given by the various Skyrme effective interactions are significantly different from each other, although these interactions give quite similar ground state properties of nuclear matter. The critical temperatures $kT_c^{(1)}$ for the liquid-gas phase transition given by them are also significantly different from each other. The lowest value for $kT_c^{(1)}$ is given by SkV as 14.55 MeV, while SkI gives the highest value as 20.12 MeV. If some of these finite temperature nuclear matter properties can be determined experimentally, then they may be very useful in discriminating the parameters of the Skyrme effective interaction.

The abnormal pair cutoff approximation is then employed to treat the superconducting (section order) phase transition of nuclear matter. This leads to a set of self-consistent equations for deriving the pairing energy gap or the order parameter Δ as a function of temperature, density, and the parameters of the Skyrme interactions. At normal density, only SkI and SkVI give nonvanishing energy gap at zero temperature, with superconducting critical temperatures being 0.5 and 0.345 MeV, respectively. Since the other Skyrme interactions of Table I do not give such energy gaps, and among them SkM* is probably a more realistic effective interaction than SkI and SkVI, our results seem to indicate that nuclear matter at zero temperature and normal density is more likely in a normal state than in a superconducting state.

We have found that there is a strong dependence of the energy gap on the nuclear matter density. At normal nuclear matter density ρ_0, the energy gap is either zero or rather small as indicated above. But at a density $\sim\rho_0/2$, there is usually a sizable energy gap of $\sim 1-2$ MeV. This means that nuclear matter with density in the vicinity of $\rho_0/2$ may well be in a superconducting phase. It should be of much interest to further study the thermodynamic properties of nuclear matter in this density region. We are carrying out such studies and will report our results in a future publication.[27]

The present Green's function method can be extended to treat asymmetric nuclear matter. A preliminary study in this direction is in progress.[29]

ACKNOWLEDGMENTS

One of us (R.K.S.) was supported by a Lee Hysan Fellowship through the Committee for Educational Exchange with China at the State University of New York at Stony Brook. The authors wish to thank Prof. G. E. Brown for a number of helpful discussions. Two of us (R.K.S. and S.D.Y.) are particularly grateful to him for his encouragement, hospitality, and support during their visits at Stony Brook. R.K.S. also thanks Prof. C. N. Yang for his encouragement and support. This work was supported in part by the U.S. Department of Energy under Contract DE-AC02-76ER3001.

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