Peculiarity of the charge-exchange quadrupole excitation in nuclei

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The experimental isovector quadrupole strength in the charge exchange channels is smaller than predicted by random phase approximation calculations using conventional nuclear interactions. The introduction of nonlocal components in the interaction combined with the existence of a low-lying $0\hbar\omega$ rotational 2^+ quadrupole state in the daughter nucleus can explain this behavior. These nonlocal interactions do not play a role in the properties of the isovector monopole and dipole resonance.

Recent[1] experimental work has shown that no compact isovector quadrupole strength is needed to fit pion charge exchange data. This result contrasts with the situation for the isospin components of the isovector monopole and dipole resonances whose excitation energies, widths, and strengths are well described by random phase approximation (RPA) calculations using Skyrme forces.[2] Calculations using the same Skyrme forces predict large compact isovector quadrupole resonances in the $\Delta T_3=1$ (π^-,π^0) and $\Delta T_3=-1$ (π^+,π^0) channels. Experimental upper limits for isovector quadrupole strengths are considerably smaller than the corresponding strengths for the monopole and dipole resonances. This result poses a problem. The Skyrme forces used in the RPA calculations are able to reproduce the average excitation energy of the $\Delta T_3=0$ isovector dipole resonance. It was a considerable success when the same force predicted properties of the $\Delta T_3=\pm 1$ isovector monopole and dipole resonances. Why do similar calculations fail to reproduce the strength of the $\Delta T_3=+1$ isovector quadrupole? Note, for example, that in the $\Delta T_3=0$ channel the energy weighted sum rule, apart from a geometrical factor, is the same for the quadrupole and monopole operators. Also, the double commutator $\langle 0|[F^+,[H,F^-]]|0\rangle$, giving the sum of the energy weighted strengths in the $\Delta T_3=-1$ and $\Delta T_3=+1$ channels, yields quite similar results for the two modes if one uses standard forces, for example Skyrme forces, and it seems impossible, as we shall see, to solve the above puzzle simply by adjusting the parameters of the force.

Stimulated by these problems, in this paper we propose a possible mechanism responsible for the observed difference between the quadrupole and the other modes in charge exchange reactions.

In contrast to the isobaric analog isovector monopole and dipole excitation, the existence of a low-lying state in the daughter nucleus combined with the introduction of a nonlocal component in the interaction can both reduce the quadrupole strength in the $T+1$ channel and fragment it in the $T-1$ channel. In fact, the isovector quadrupole operator $\sum_i r_i^2 Y_{2m}\tau_i^-$, applied to the ground state of the parent nucleus can excite both a low-lying $0\hbar\omega$ (rotational) state and a high-lying 2 (vibrational) state. Nothing similar occurs for the isovector dipole operator which excites a giant $1\hbar\omega$ state only whereas the monopole operator $(\sum_i r_i^2\tau_i^-)$ can excite the $0\hbar\omega$ isobaric analog state (IAS) in addition to the giant $2\hbar\omega$ vibrational one. In the monopole case, the excitation of the IAS can be decoupled from the giant state, projecting off it through a local projector. In fact, if one defines $F'=\sum_i (r_i^2-\langle r_v^2\rangle)\tau_i^-$, where $r_v^2=\langle 0|\sum_i r_i^2\tau_i^3|0\rangle/(N-Z)$, one immediately verifies that $\langle \mathrm{IAS}|F'|0\rangle=0$. In the quadrupole case, the situation is different. One cannot remove the rotational excitation simply by introducing a local projector and one must take explicitly into account nonlocal terms. The problem can be investigated simply in the schematic plus harmonic oscillator potentials model:[3,4]

$$H=\sum_i^A [p_i^2/2m+\tfrac{1}{2}m\omega_0^2 r_i^2 + V_1 T\langle r_v^4\rangle \tau_i^3/(A\langle r^4\rangle)]+\xi\langle \mathbf{F}\rangle\cdot\mathbf{F}\ , \tag{1}$$

where $\omega_0=41A^{-1/2}$ MeV, $\mathbf{F}$ is the quadrupole operator $\sum_{i=1}^A y_i z_i \tau_i$, and $\xi=15V_1/(4A\langle r^4\rangle)$ with

$$\langle r_v^4\rangle=\left\langle 0\left|\sum_i r_i^4\tau_i^3\right|0\right\rangle\Big/(N-Z)\ ,$$

$$\langle r^4\rangle=\frac{1}{A}\left\langle 0\left|\sum_i r_i^4\right|0\right\rangle\ ,$$

and $V_1\simeq 120$ MeV.

If one solves the RPA equations of motion

$$[H,0^+]|0\rangle=\omega 0^+|0\rangle\ , \tag{2}$$

with Hamiltonian (1), one can explicitly determine three solutions at the energies ω_0^- (low lying) and ω_2^- and ω_2^+ (giant states), where the superscripts $+$ or $-$ refer to $\Delta T_3=\pm 1$ modes, respectively.

We straightforwardly obtain the following commutation relations:

$$[H,F^\pm]=iG^\pm\ ,$$

$$[H,iG^\pm]=2\omega_0^2 M^\pm + \left[2\omega_0^2+(\xi/m)\left\langle 0\left|\sum_i (y_i^2+z_i^2)\right|0\right\rangle\right]F^\pm\ ,$$

$$[H,M^\pm]=iG^\pm\ ,$$

TABLE I. Energies (in MeV) and strength distributions (fm^4) for the isovector quadrupole mode.

	ω_0^-	σ_0^-	ω_2^-	σ_2^-	ω_2^+	σ_2^+
$V_1=0$ MeV	0	490	18.3	1462	18.3	972
$V_1=120$ MeV	-9	68	23	1319	36.4	407
Ref. 2			24.3	1200	31.9	697

where the operators F,G,M are defined by

$$F^\pm=\sum_i^A y_i z_i \tau_i^\pm ,$$
$$G^\pm=-\frac{1}{m}\sum_{i=1}^A (y_i p_i^z + z_i p_i^y)\tau_i^\pm , \quad (3)$$
$$M^\pm=\frac{1}{m^2\omega_0^2}\sum_{i=1}^A p_i^y p_i^z \tau_i^\pm \quad (\tau^+ \mid p\rangle=\sqrt{2}\mid n\rangle) .$$

The normal modes are determined by solving Eq. (2) with the ansatz

$$0^\pm=\alpha F^\pm+\beta M^\pm+\gamma i G^\pm . \quad (4)$$

One gets the following equation for the eigenvalues ω:

$$\omega^3=2\xi S_0\omega^2-\omega(\omega_q^2-S_0^2\xi^2)-S_0\xi(\omega_q^2-2\omega_0^2)=0 , \quad (5)$$

where $\omega_q^2=4\omega_0^2+\frac{2}{3}A\langle r^2\rangle\xi/m$ is the energy of the giant quadrupole mode in the $\Delta T_3=0$ channel, $\langle r^2\rangle=(1/A)\langle 0\mid\sum_i r_i^2\mid 0\rangle$, and S_0 is the model independent sum rule

$$S_0=\langle 0\mid[F^+,F^-]\mid 0\rangle=\tfrac{2}{15}(N-Z)\langle r_v^4\rangle .$$

To calculate the quadrupole charge exchange strengths $\sigma_0^-,\sigma_2^-,\sigma_2^+$ associated with the solutions ω_0^-, ω_2^-, and ω_2^+, one can invert the equations

$$\sigma_0^-+\sigma_2^--\sigma_2^+=S_0 , \quad (6)$$

$$\omega_0^-\sigma_0^-+\omega_2^-\sigma_2^-+\omega_2^+\sigma_2^+=\langle 0\mid[F^+,[H,F^-]]\mid 0\rangle\equiv S_1 , \quad (7)$$

$$(\omega_0^-)^2\sigma_0^-+(\omega_2^-)^2\sigma_2^--(\omega_2^+)^2\sigma_2^+ =\langle 0\mid[[F^+,H],[H,F^-]]\mid 0\rangle\equiv S_2 . \quad (8)$$

The sum rules S_1 and S_2 are model dependent and for the Hamiltonian (1) take the values $S_1=2A\langle r^2\rangle/(3m)$ and $S_2=2\omega_0^2 S_0$, respectively.

In the case of the pure harmonic oscillator potential [$V_1=0$ in Eq. (1)], Eqs. (5) and (7) yield the solutions

$$\omega_0^-=0,\quad \omega_2^-=\omega_2^+=2\omega_0$$

and

$$\sigma_0^-=\tfrac{1}{2}S_0,\quad \sigma_2^\mp=(S_1\pm\omega_0 S_0)/4\omega_0 .$$

One remarks that the zero energy state takes half of the non-energy-weighted sum rule and that σ_2^+ is quenched with respect to σ_2^-. The effect of the interaction ($V_1\neq 0$) pushes down the energy of the low-lying state to negative values[5] and strongly quenches its strength. Furthermore, the energies of the giant states are split. In Table I we report the predictions of Eqs. (5) and (7) for ^{90}Zr, taking $V_1=0$ and 120 MeV. The predictions of the RPA calculation of Ref. 2 are also given in the last line of the table for comparison. From the table one can appreciate the good agreement between the predictions of the schematic model (1) and the calculations of Ref. 2 using a Skyrme force. In particular, one remarks that both calculations strongly quench the strength of the low-lying state so that it practically disappears. However, this result depends on the form assumed for the schematic quadrupole force or equivalently on the structure of Skyrme-type forces.

Now we will try to generalize the force to find a possible mechanism that depresses and/or spreads the quadrupole strength carried by the giant states $\omega_2^\pm$, as suggested by experiments.

The solutions of the equations of motion (2) with Hamiltonian (1) indicate that the relevant quadrupole operators [Eq. (4)] have an important local (yz) as well as a nonlocal ($p_y p_z$) component. It is then natural to generalize the Hamiltonian (1) to include the local as well as the nonlocal interaction terms. One can write H in the most general form as

$$H=\sum_i\left[\frac{p_i^2}{2m}+\tfrac{1}{2}m\omega_0^2 r_i^2+\tfrac{1}{2}V_1 T\langle r_v^4\rangle\tau_i^3/(A\langle r^4\rangle)\right] +a\langle\mathbf{F}\rangle\cdot\mathbf{F}+b\langle\mathbf{F}\rangle\cdot\mathbf{M}+c\langle\mathbf{M}\rangle\cdot\mathbf{F}+d\langle\mathbf{M}\rangle\cdot\mathbf{M} , \quad (9)$$

where a,b,c,d are strength parameters.

Notice that the most general schematic force that one can construct using the operators of Eq. (3) would also include a term of the form const$\times\langle\mathbf{G}\rangle\cdot\mathbf{G}$. Such a current-dependent interaction term which is the analog of the nonlocality terms in Skyrme forces, leads to an enhancement factor in the energy weighted sum rules and does not play a crucial role in the mechanism we are looking for.

Starting from Eq. (9) one again solves the equation of motion (2). One gets the following equation for the energy of the normal modes:

$$\omega^3+(3\xi^2-a-d)S_0\omega^2-\omega\left\{4\omega_0^2+\frac{2}{3m}A\langle r^2\rangle(a+b+c+d)-S_0^2[3\xi^2-2\xi(a+d)+ad-bc]\right\}$$
$$-S_0\left[\xi(4\omega_0^2+\frac{2}{3m}A\langle r^2\rangle(a+b+c+d)+2\omega_0^2(b+c-a-d)-\frac{2}{3}A\frac{\langle r^2\rangle}{m}(ad-bc)\right]+S_0^3\xi[\xi^2-\xi(a+d)+ad-bc]=0 . \quad (10)$$

S_0 is given by Eq. (6) together with the following expressions for the sum rules S_1 and S_2:

$$S_1=\frac{2}{3m}A\langle r^2\rangle+(a-\xi)S_0^2 ,$$
$$S_2=2\omega_0^2S_0+\frac{4}{3m}A\langle r^2\rangle S_0(a+b-\xi)+S_0^3[(a-\xi)^2+b^2] . \quad (11)$$

One obtains a relation between the parameters a,b,c,d requiring that the energy of the giant isovector quadrupole resonance in the $\Delta T_3=0$ channel be the same as the energy calculated with the traditional schematic model (1). This relation is

$$a+b+c+d=\xi=15V_1/(4A\langle r^4\rangle) . \quad (12)$$

Let us note that when the parent nucleus is itself intrinsically deformed then, beyond the giant (vibrational) quadrupole state, the parent nucleus itself possesses a low-lying rotational state (the so-called $M1$ rotational state).[6] In this last case a low-lying state is excited not only in the $\Delta T_3=-1$ quadrupole excitation but in the $\Delta T_3=0$ too. A further relation between a,b,c,d can be obtained by imposing that in deformed (parent) nuclei the low-lying $\Delta T_3=0$ solution remains unchanged at the energy[7]

$$\omega_{M1}=\omega_0\delta(1+\chi)^{1/2}(1+\chi/2)^{-1/2}$$
$$\text{where } \chi=5V_1\langle r^2\rangle(4m\omega_0^2\langle r^4\rangle)^{-1}$$

(δ is the usual nuclear deformation) predicted by Hamiltonian (1), which reproduces experimental data quite well. Since Hamiltonian (9) yields, for this state, the result

$$\omega_{M1}=\omega_0\delta\{[1+(a+d)\chi/\xi]+(ad-bc)\chi^2/\xi^2\}^{1/2}(1+\chi/2)^{-1/2} , \quad (13)$$

one immediately gets, comparing the two equations,

$$a+d=\xi,\ \ ad-bc=0 . \quad (14)$$

Conditions (12) and (14), when used in Eq. (10), yield the very nice result that Eq. (10) reduces to Eq. (5). In other words, the generalized Hamiltonian (9) fixes the energies ω_0^-, ω_2^-, and ω_2^+ to the values given by traditional local forces. However, it changes the sum rules S_1 and S_2 and hence, the quadrupole strength distribution via Eq. (7). In Table II we report the results for σ_0^-, σ_2^-, and σ_2^+ for ^{90}Zr as a function of the free parameter a. This strength parameter is constrained by either of the conditions $a\leqq0$ and $a\geqq\xi$. The case $a\geqq\xi$ practically reduces to $a=\xi$ and hence $b=c=d=0$; that is, it reduces to the traditional solution which is reported in Table II for comparison. In fact, larger values of a immediately yield negative values for the quadrupole strengths. Hence, we have explored the case $a\leqq0$.

TABLE II. Quadrupole strength distribution (fm^4) as a function of a/ξ.

a/ξ	σ_0^-	σ_2^-	σ_2^+	Total strength
0	1232	598	849	2679
−0.3	762	759	541	2062
−0.5	620	788	427	1835
−0.8	397	835	252	1484
−1	230	875	125	1230
−1.18	64	919	2	985
1	68	1319	407	1794

From Table II we find situations where the total quadrupole strength in the $\Delta T_3=-1$ channel can be spread to the low energy region, the strength in the $\Delta T_3=+1$ channel remaining practically unchanged (for example, in the case $a/\xi=-0.5$). We also see that the total quadrupole strength can be strongly quenched and present only in the $\Delta T_3=-1$ channels in the high energy state (this is the limiting situation reached at $a/\xi\simeq-1.2$).

In conclusion, we have investigated a possible mechanism which can both decrease and spread the quadrupole strength in the charge exchange channels, differently from the prediction of usual forces. This mechanism is connected with the presence of nonlocal components in the nuclear interaction that are absent in forces, for example, of Skyrme type, generally used to perform RPA calculations. More precise data on both the $\Delta T_3=\pm1$ components of the isovector quadrupole in spherical and deformed nuclei could determine the strength of the nonlocal force components and test the consistency of the model.

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[5] Notice that the energies must be corrected for the Coulomb energy shift ΔE_C before comparison with the experiments. For $^{90}Z_r$, $\Delta E_C=12.3$ MeV.

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