Crossing symmetric self-consistent πN t matrix

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Previous calculations of the self-consistent effective πN t matrix are extended to include a complete, consistent treatment of the pion crossing symmetry. The self-consistent πN t matrix equation is rewritten as a set of four coupled integral equations in order to facilitate the inclusion of crossing symmetry. The πN amplitudes are obtained by means of a Low expansion as in previous papers. The insertion of the crossed πN scattering amplitudes into the nuclear medium is studied in the context of time ordered perturbation theory in order to determine the correct energy at which the crossed amplitudes should be evaluated in the medium. The self-consistent equations are evaluated in nuclear matter. The resonant contribution to the optical potential is found to be broadened by an additional 15—25 % when compared to similar calculations which do not include the crossed amplitudes.

I. INTRODUCTION

The strong absorption of the pion by the nucleus complicates attempts to describe π-nucleus scattering theoretically. One approach to an organization of the π-nucleus scattering problem is to introduce medium modifications of the πN interaction in the nucleus. This is the primary characteristic of the isobar-hole model[1] which has achieved considerable success in predicting π-nucleus elastic scattering by introducing a phenomenological Δ isobar optical potential to account for the coupling of the elastic channel to absorption and knock out channels. Attempts at a more microscopic description of the medium modified effective πN interaction have centered on the self-consistent πN t matrix which has been calculated[2-9] or estimated[10] by a number of authors using a variety of models and approximations. The self-consistent πN t matrix is found as a solution to an integral equation which can be represented schematically as

$$T_{\pi N}=T^{0}_{\pi N}+T^{0}_{\pi N}(G_{\pi N}-G^{0}_{\pi N})T_{\pi N} \ , \tag{1}$$

where $T^{0}_{\pi N}$ and $T_{\pi N}$ are the free and self-consistent πN t matrices, and $G^{0}_{\pi N}$ is the free πN propagator. $G_{\pi N}$ is a dressed πN propagator represented by the equation

$$G_{\pi N}=QG^{0}_{\pi N}Q+QG_{\pi N}Q\Sigma G_{\pi N} \ . \tag{2}$$

Here the projector Q modifies the propagator by Pauli blocking the nucleon propagation and the pion self-energy Σ includes pion distortion due to the nuclear medium. The self-consistency arises from the requirement that the pion self-energy be calculated by folding the self-consistent t matrix over the nuclear ground state.

In order to determine the physical content of the self-consistent t matrix, it is useful to first consider some properties of the free πN t matrix. Since the pion is a self-charge-conjugate particle, the πN t matrix must be crossing symmetric. This means that the t matrix can be represented as the sum of two terms which are related by the interchange $k\leftrightarrow -k'$, where k and k' are the initial and final pion four-momenta, and by the interchange of initial and final isospin quantum numbers $\alpha\leftrightarrow\beta$ (see Fig. 1). These two contributions will be referred to as the direct or right hand contribution and the crossed or left hand contribution. Clearly, the values of the right- and left-hand contributions as functions of the external variables will differ considerably.

Each of these terms can be further divided according to the intermediate states which contribute to the amplitude. The most important of these are the pion pole term, where the pion is absorbed and then reemitted by a nucleon [see Fig. 2(b)], and the rescattering or cut term, which has intermediate states which contain at least one pion. By including these contributions in the self-consistent t matrix equation and iterating, it can be seen that the inclusion of the right hand pole contribution to the free πN t matrix leads to intermediate states of the self-consistent t matrix where the pion has been absorbed on one or more nucleons. The need to include the crossed terms is implied by the physical importance of the absorption channels

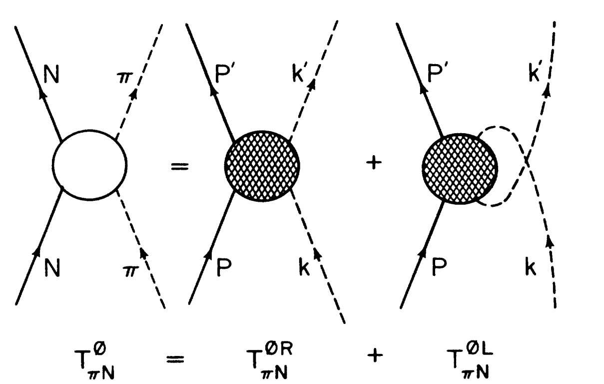

FIG. 1. Diagrams representing the decomposition of the crossing symmetric, free πN t matrix into right-hand and left-hand contributions.

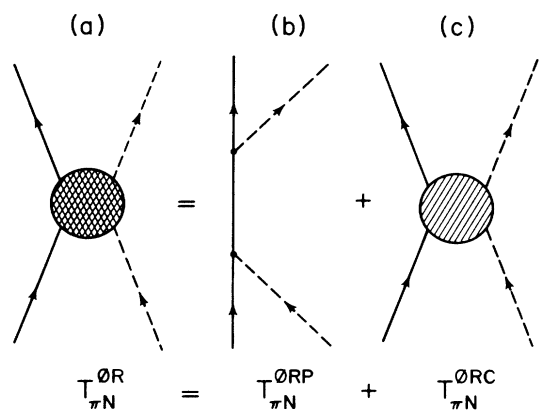

FIG. 2. Diagrams representing the decomposition of the direct amplitude into pole and cut contributions.

which are introduced by including the right-hand nucleon pole. This can be seen by noting that in the free amplitude the right-hand pole contribution is largely canceled by the crossed or left-hand pole contribution. Indeed, in the static limit the cancellation is exact. Therefore, if absorption channels are to be introduced through the right-hand pole contribution, the crossed or left-hand pole contribution must be included in order to provide a reasonably accurate description of the πN t matrix. No previous calculation of the self-consistent πN t matrix has contained a consistent treatment of the crossing symmetry of the πN t matrix.

An organization of the pion-nucleus many-body problem has been described in a series of papers.[7,9,11] This organization is based on a phenomenological description of the free πN t matrix derived using the Low expansion[12] and a Goldstone diagrammatic expansion of the many-body problem. From the standpoint of this paper, this approach has the advantage that the Low expansion of the πN t matrix naturally results in a separation of the t matrix into pole and cut contributions for both the direct and crossed parts of the t matrix. The Goldstone diagrammatic approach provides a natural means for embedding the free πN t matrix into the many-body system by means of a set of rules for evaluating the Goldstone diagrams. This approach guarantees that the various contributions to the free πN t matrix are evaluated at the correct energies and momenta in the many-body environment. This avoids the possibility of introducing spurious singularities into complex many-body processes by means of a naive application of the crossing relations of the free πN t matrix in the medium.

Reference 9 contains three parts. The first of these is the derivation of a nonstatic off shell representation of the free πN t matrix using the Low expansion approach. Only the right-hand contributions are derived, but the left-hand contributions can be found by simply applying the crossing relations to the right-hand pieces. The second part contains a discussion of the self-consistent t matrix equation in which the equation is separated into a pair of coupled equations, one which describes the self-consistent modification of the nucleon pole term and one which describes the self-consistent modification of the cut term. The third part is a calculation of a nonstatic self-consistent πN t matrix in nuclear matter. In the presentation of these last two parts, only the right-hand contributions to the t matrices were retained. The objective of this paper is to extend the discussion and calculations of Ref. 9 to include crossing symmetry. In the next section of this paper, the formalism of Ref. 9 is extended to include crossing symmetry. In the following section, the resulting expressions will be evaluated in nuclear matter and the results of this calculation are presented. Conclusions are then drawn from this calculation.

II. THEORY

In Ref. 9 it was demonstrated that it is convenient to separate the self-consistent t matrix equation into a pair of coupled equations. One of these modifies the cut contribution while the other modifies the pole contribution. This is done by dividing the self-consistent equation into parts which have cuts containing only a single nucleon and parts which contain cuts with either multiple nucleons or nucleons and pions. If crossing symmetry is to be included, the self-consistent equation can be divided into a set of four coupled integral equations. These are represented schematically by the equations

$$\begin{aligned} T^{RP}_{\pi N} = T^{0RP}_{\pi N} &+ T^{0RF}_{\pi N}(G_{\pi N}-G^{0}_{\pi N})(T_{\pi N}-T^{RP}_{\pi N}) \\ &+ (T^{0}_{\pi N}-T^{0RP}_{\pi N})(G_{\pi N}-G^{0}_{\pi N}) \\ &\times T^{RP}_{\pi N}+T^{0RP}_{\pi N}(G_{\pi N}-G^{0}_{\pi N})T^{RP}_{\pi N}) \end{aligned} \tag{3}$$

for the pole contributions, and

$$T^{RC}_{\pi N}=T^{0RC}_{\pi N}+(T^{0}_{\pi N}-T^{0RP}_{\pi N})(G_{\pi N}-G^{0}_{\pi N})(T_{\pi N}-T^{RP}_{\pi N}) \tag{4}$$

for the cut contributions, where

$$T^{0}_{\pi N}=T^{0RP}_{\pi N}+T^{0RC}_{\pi N}+T^{0LP}_{\pi N}+T^{0LC}_{\pi N} \tag{5a}$$

and

$$T_{\pi N}=T^{RP}_{\pi N}+T^{RC}_{\pi N}+T^{LP}_{\pi N}+T^{LC}_{\pi N} \ . \tag{5b}$$

Here the superscripts L and R refer to the left- and right-hand contributions, and the superscripts P and C refer to pole and cut contributions. The left-hand pole and cut contributions can be found by crossing the external variables in (3) and (4). For convenience we will refer to these crossed equations for $T^{LP}_{\pi N}$ and $T^{LC}_{\pi N}$ as (3 cross) and (4 cross), respectively.

The integral equations represented by (3) and (4) and the corresponding crossed equations can be evaluated provided some care is taken in determining the energy arguments of the πN amplitudes which appear internally in these expressions. This can be done correctly by using the πN amplitudes derived in Ref. 9. The right-hand contributions to these t matrices have the form (see Ref. 9)

$$T^{0Ri}_{\pi\mathrm{N}}(k'\beta,k\alpha;p',p)=-4\pi\sum_{IIJ} h^{0li}_{2I,2J}(\epsilon-\mathbf{L}^2/2m_R)\Lambda_{2I}(\beta,\alpha)\Omega^l_{2J}(\mathbf{k}'_{\mathrm{c.m.}},\mathbf{k}_{\mathrm{c.m.}})\phi(\mathbf{k}'_{\mathrm{c.m.}})\phi(\mathbf{k}_{\mathrm{c.m.}})(4\omega_k,\omega_k)^{-1/2}\ , \quad (6)$$

where $\epsilon=\omega+p^0$ is the starting energy of the πN pair, L is the momentum of the πN pair, m_R is the recoil mass, and the pion center of mass momenta are given by

$$\mathbf{k}'_{\mathrm{c.m.}}=\mathbf{k}'-\delta\mathbf{L} \text{ and } \mathbf{k}_{\mathrm{c.m.}}=\mathbf{k}-\delta\mathbf{L}\ , \quad (7)$$

where $\delta=(m_R-m)/m_R$. Λ and Ω are isospin and spin projection operators.

Applying the free crossing relations to this expression gives

$$T^{0Li}_{\pi\mathrm{N}}(k'\beta,k\alpha;p',p)=-4\pi\sum_{IIJ} h^{0li}_{2I,2J}(\epsilon-\bar{\mathbf{L}}^2/2m_R)\Lambda_{2I}(\alpha,\beta)\Omega^l_{2J}(-\bar{\mathbf{k}}_{\mathrm{c.m.}},-\bar{\mathbf{k}}'_{\mathrm{c.m.}})\phi(\mathbf{k}'_{\mathrm{c.m.}})\phi(\mathbf{k}_{\mathrm{c.m.}})(4\omega_k,\omega_k)^{-1/2}\ , \quad (8)$$

where

$$\bar{\mathbf{L}}=\mathbf{p}-\mathbf{k}',\ \ \bar{\mathbf{k}}'_{\mathrm{c.m.}}=\mathbf{k}'-\delta\bar{\mathbf{L}},\ \ \bar{\mathbf{k}}_{\mathrm{c.m.}}=\mathbf{k}-\delta\bar{\mathbf{L}}\ . \quad (9)$$

Note that the order of the arguments in the projection operators have been reversed and that the momenta appearing in the spin projection operator are no longer the correct c.m. momenta. These projection operators can be reexpressed as a linear combination of the projection operators appearing in (5) with the correct c.m. momenta. As a result, the left-hand amplitudes for a given spin-isospin channel is a linear combination of right-hand amplitudes from many channels evaluated at the crossed energy. This mixing of channels in crossing can be expressed in terms of a crossing matrix.

If the crossed t matrix is to be used as an element in a many-body diagram, the crossing of variables in the projection operators and form factors will be as in the free t matrix but the energy at which the amplitudes are to be evaluated will be determined by the position of the crossed t matrix in the many-body diagram. This determination can be made easily by using the Low expansion amplitudes (5) and simple Goldstone rules for time ordered diagrams. This follows from the form of the right-hand amplitudes. The right-hand pole amplitude is given by

$$h^{0IRP}_{2I,2J}(\epsilon-\mathbf{L}^2/2m)=-\frac{3g^2_\pi(0)}{16\pi m^2}\frac{1}{\epsilon-\mathbf{L}^2/2m-m+i\eta}\times\delta_{I,1/2}\delta_{J,1/2}\delta_{1,1} \quad (10)$$

and the right hand cut amplitude is given by

$$h^{0IRC}_{2I,2J}(\epsilon-\mathbf{L}^2/2m_\Delta)=-\frac{1}{\pi}\int_{m+m_\pi}^{\infty}dW\frac{\phi^{-2}(k''_{\mathrm{c.m.}})\mathrm{Im}h^{0I}_{2I,2J}(W)}{\epsilon-\mathbf{L}^2/2m_\Delta-W+i\eta}\ . \quad (11)$$

The right-hand pole amplitude clearly has the form of a factor times a nucleon propagator. The right-hand cut contribution has the form of a weighted integral over a propagator with a varying mass W. In the pole amplitude, the recoil mass is the nucleon mass, as expected, while in the cut contribution the recoil mass is the mass of the dominant resonance in each channel. Since the Δ isobar is by far the most important cut contribution in the region of interest in this paper, the Δ mass has been used in all channels for simplicity. The Goldstone rules[11] can be used to determine the appropriate energy to be used in evaluating the amplitudes by drawing time ordered diagrams with the amplitudes represented by a propagator for a particle with recoil mass m_R and mass M. M will then be the nucleon mass m for pole amplitudes and the integration variable W for cut amplitudes. The important rules for this application are those needed to determine the denominators of the global propagators which appear in time ordered perturbation theory. For propagators corresponding to each time cut across the diagram, the denominator is given by the rules

(i) Add the total initial asymptotic energy of the system.

(ii) Subtract the "on-shell" energy of each upward going line which is cut.

(iii) Add the "on-shell" energy of each downward going line which cut.

(iv) Add $i\eta$.

In the self-consistent t matrix equation, the crossed t matrix can appear in three places: as the first scattering, as the last scattering, or as part of a self-energy insertion on the intermediate pion propagator. The first of these can be represented as the diagram in Fig. 3. In this figure the open box represents either a pole or cut contribution to the πN amplitude. The Goldstone rules can be applied to evaluate the global propagator for the time cut represented by the dotted line giving

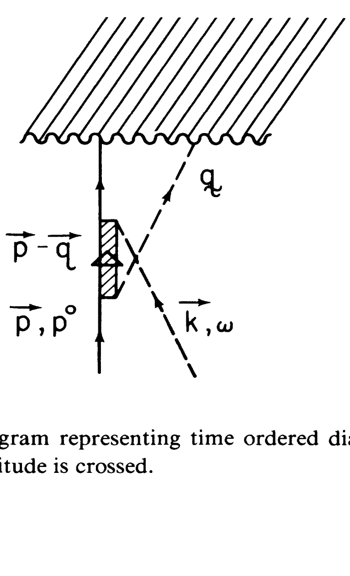

FIG. 3. Diagram representing time ordered diagrams where the initial amplitude is crossed.

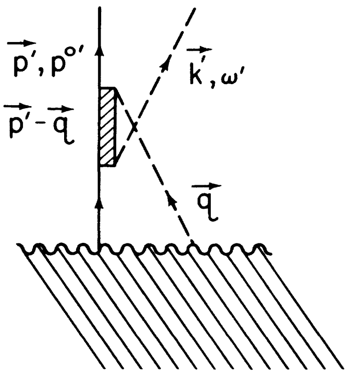

FIG. 4. Diagram representing time ordered diagrams where the final amplitude is crossed.

$$[\omega+p^0-(\mathbf{p}-\mathbf{q})^2/2m_R-M-\omega-\omega_q+i\eta]^{-1}$$
$$=[-\omega_q+p^0-(\mathbf{p}-\mathbf{q})^2/2m_R-M+i\eta]^{-1} . \quad (12)$$

The right-hand amplitudes which are combined to give the left-hand amplitude will therefore be evaluated at

$$-\omega_q+p^0-(\mathbf{p}-\mathbf{q})^2/2m_R . \quad (13)$$

The second type of contribution can be represented by the diagram in Fig. 4. As in the first case, the denominator can be determined easily to be

$$[\omega'+p'^0-(\mathbf{p}'-\mathbf{q})^2/2m_R-M-\omega'-\omega_q+i\eta]^{-1}$$
$$=[-\omega_q+p'^0-(\mathbf{p}'-\mathbf{q})^2/2m_R-M+i\eta]^{-1} , \quad (14)$$

where conservation of energy has been used in writing

$$\epsilon=p^0+\omega=p'^0+\omega' . \quad (15)$$

The evaluation of the third type of contribution is somewhat more complicated. Treatment of the third type of contribution, where the t matrix appears as part of a self-energy insertion, can be illustrated by consideration of the diagram in Fig. 5. The hatched area represents some general diagram to which the crossed insertion is attached. The points labeled $-M$ to N correspond to times T_{-M} to T_N which are determined by various interaction vertices in the body of the diagram. The points labeled U and L correspond to the final and initial times T_U and T_L of the self-energy interaction. The problem is that there are a large number of possible orderings of the times associated with the self-energy relative to the times defined by the main body of the diagram. All of the possible diagrams are subject to the limitation that $T_U>T_L$. Since the Goldstone rules require that all pions lines must be drawn with the arrow pointing upward, those time orderings where $T_U<T_0$ or $T_L>T_1$ will correspond to the creation or absorption of two pions which should lead to contributions which are suppressed relative to the other time orderings. All the remaining contributions where $T_U>T_0$ and $T_L<T_1$ can be summed to obtain a single simple result. If Δ is defined as

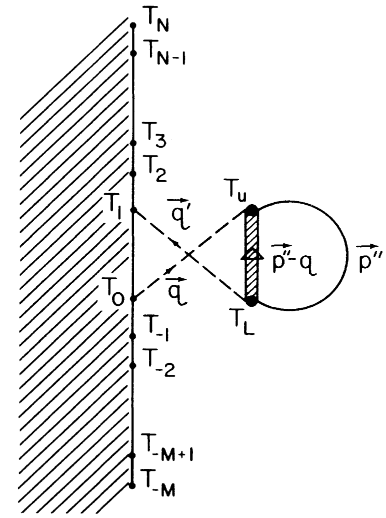

FIG. 5. Diagram representative of diagrams where the crossed contribution to the pion self-energy is included in dressing internal pion propagators.

$$\Delta=p''^0-(\mathbf{p}''-\mathbf{q})^2/2m_R-M , \quad (16)$$

and the contribution of the main body of the diagram to the energy of the propagator denominator for a cut between points $i+1$ and i is denoted by D_i where $-M<i<N-1$, then the result of this generalized time ordered sum is given by

$$\left[\prod_{i=1}^{M}\frac{1}{D_{-i}}\right]\left[\frac{1}{D_0-\omega_q}\frac{1}{\Delta-\omega_q}\frac{1}{D_0-\omega_{q'}}+\frac{1}{\Delta-\omega_q}\frac{1}{D_0-\omega_{q'}}\frac{1}{\Delta-\omega_{q'}}\right]\left[\prod_{j=1}^{N}\frac{1}{D_j}\right] . \quad (17)$$

The first term follows the pattern of the two previous cases in that it suggests that the amplitudes be evaluated at the energy

$$-\omega_q+(\mathbf{p}''-\mathbf{q})^2/2m_R . \quad (18)$$

The second term is a renormalization of the global propagator for the emission and reabsorption of a pion by the main body of the diagram. This procedure is virtually identical to that used in placing hole-line insertions on the energy shell in Brueckner theory.[13,14] Extension of this example to allow for a general crossed self-energy insertion is straightforward but tedious.

Neglecting the renormalization term, which has been found to give a modification to the propagators of at most 5% in the model presented below, the above results can be summarized by requiring that when the crossed πN t matrix is used as an internal part of a time ordered diagram it is of the form

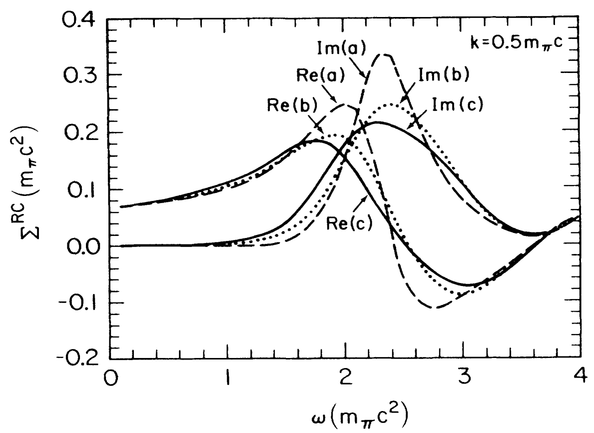

FIG. 6. Σ^{RC} calculated with the free nonstatic amplitude (curve a), the self-consistent amplitudes without crossing (curve b) and self-consistent amplitudes with crossing (curve c) as a function of pion energy ω at fixed pion momentum $k=0.5m_\pi c$.

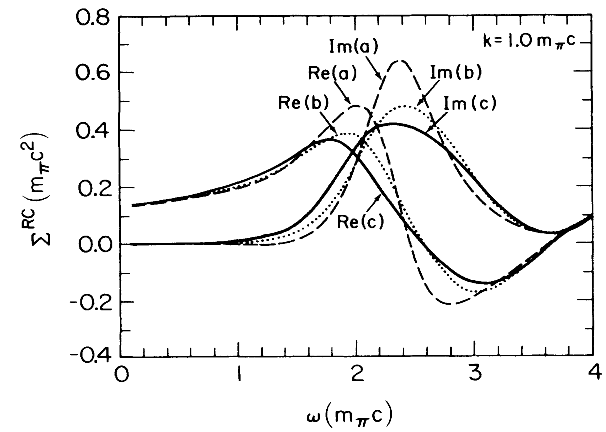

FIG. 7. As in Fig. 6, but with $k=1.0m_\pi c$.

$$T^{0Li}_{\pi N}(k'\beta,k\alpha;p',p)=-4\pi\sum_{lIJ} h^{0li}_{2I,2J}(-\omega_{k'}+p^0-\bar{\mathbf{L}}^2/2m_R)\Lambda_{2I}(\alpha,\beta)\Omega^l_{2J}(-\bar{\mathbf{k}}_{\mathrm{c.m.}},-\bar{\mathbf{k}}'_{\mathrm{c.m.}})\phi(\mathbf{k}'_{\mathrm{c.m.}})\phi(\mathbf{k}_{\mathrm{c.m.}})(4\omega_{k'}\omega_k)^{-1/2} . \qquad (19)$$

III. CROSSING SYMMETRIC t MATRIX IN NUCLEAR MATTER

The relative importance of the crossing corrections to the self-consistent equations can be estimated by calculating the effective πN t matrix in nuclear matter where the solution of these equations is relatively simple. The solution of the four coupled crossing symmetric self-consistent equations (3), (4), (3 cross), and (4 cross) can be further simplified by making the following observations:

1. In the calculation of Ref. 9, the pole contributions are little modified by the imposition of self-consistency. For this reason, we will ignore (3) and (3 cross) and use the free pole terms with Pauli blocking, $T^{0RP}_{\pi N}\theta(|\mathbf{L}|-k_F)$ and $T^{0LP}_{\pi N}\theta(|\bar{\mathbf{L}}|-k_F)$ for $T^{RP}_{\pi N}$ and $T^{LP}_{\pi N}$ in (4) and (4 cross).

2. When $T^{LC}_{\pi N}$ appears in (4) it is always evaluated on energy shell as shown in (19) in the preceding section. This on shell $T^{LC}_{\pi N}$ is related to $T^{RC}_{\pi N}$ evaluated at negative energy where the amplitude is relatively small and slowly varying. Since the self-consistent modifications are greatest in the vicinity of the resonance and become negligible at zero energy, the $T^{LC}_{\pi N}$ amplitude may be replaced by $T^{0LC}_{\pi N}$ where it appears in (4).

3. An additional simplification can be made by noting

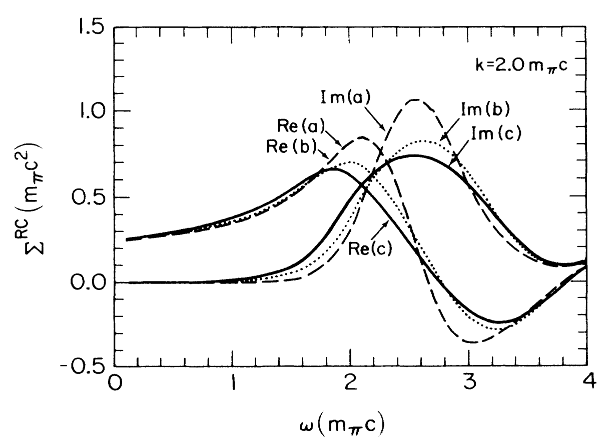

FIG. 8. As in Fig. 6, but with $k=2.0m_\pi c$.

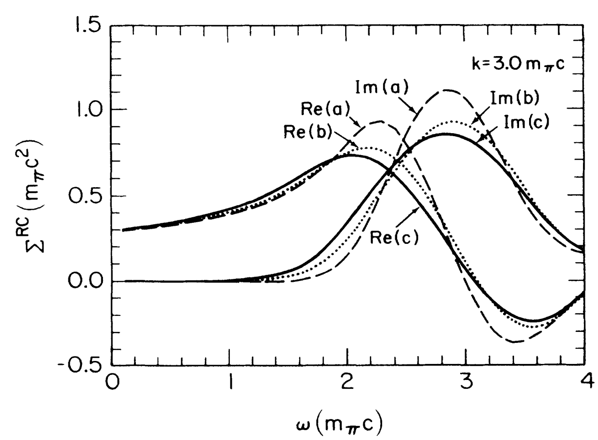

FIG. 9. As in Fig. 6, but with $k=3.0m_\pi c$.

that since $T^{0L}_{\pi N}$ is slowly varying when evaluated on shell, Fermi motion effects will not be very important for this amplitude so the nucleon momentum can be safely neglected in this amplitude in (4).

Using the approximations detailed in Ref. 9, and keeping only p-wave contributions to the various amplitudes, the self-consistent right-hand cut amplitude with crossing corrections is given by

$$H^{RC}_{2I}(\epsilon-\bar{\mathbf{L}}^2/2m_\Delta)=\frac{H^{0RC}_{2I}(\epsilon-\mathbf{L}^2/2m_\Delta)[1-F^{(1)}_{2I}(\epsilon,L)]-F^{(2)}_{2I}(\epsilon,L)}{1+H^{0RC}_{2I}(\epsilon-\mathbf{L}^2/2m_\Delta)F^{(0)}_{2I}(\epsilon,L)+F^{(1)}_{2I}(\epsilon,L)}\ , \tag{20}$$

where

$$H^{0RC}_{2I}=4\pi\sum_J (J+\tfrac{1}{2})h^{0RC}_{2I,2J}\ , \tag{21}$$

$$H^{RC}_{2I}=4\pi\sum_J (J+\tfrac{1}{2})h^{RC}_{2I,2J}\ , \tag{22}$$

and

$$F^{(n)}_{2I}(\epsilon,L)=\frac{1}{(2\pi)^3}\int\frac{d^3q}{2\omega_q}\frac{\mathbf{k}'_{\text{c.m.}}\cdot\mathbf{q}_{\text{c.m.}}\,\mathbf{q}_{\text{c.m.}}\cdot\mathbf{k}_{\text{c.m.}}}{\mathbf{k}'_{\text{c.m.}}\cdot\mathbf{k}_{\text{c.m.}}}$$
$$\times[H^{0LC}_{2I}(\omega_q+\mathbf{q}^2/2m_\Delta)+H^{0LP}_{2I}(\omega_q+\mathbf{q}^2/2m)\theta(|\mathbf{q}|-k_F)]^n(G_{\pi N}-G^0_{\pi N})\ . \tag{23}$$

The dressed and free πN propagators are given by

$$G_{\pi N}=\frac{\theta(|\mathbf{L}-\mathbf{q}|-k_F)}{\epsilon-E(\mathbf{L}-\mathbf{q})-\omega_q-\Sigma^R[\mathbf{q},\epsilon-E(\mathbf{L}-\mathbf{q})]-\Sigma^R(\mathbf{q},-\omega_q)+i\eta} \tag{24}$$

and

$$G^0_{\pi N}=\frac{1}{\epsilon-E(\mathbf{L}-\mathbf{q})-\omega_q+i\eta}\ , \tag{25}$$

where $\Sigma^R=\Sigma^{RP}+\Sigma^{RC}$.

Figures 6, 7, 8, and 9 show the right-hand cut contribution to the pion self-energy as a function of pion energy for constant pion three-momenta of $0.5m_\pi c$, $1.0m_\pi c$, $2.0m_\pi c$, and $3.0m_\pi c$, respectively. In each case the curves labeled a, and shown as dashed lines, represent the self-energy calculated with the nonstatic free πN t matrix. The dotted lines, which are labeled b, represent the self-consistent self-energy calculated using only the right-hand amplitudes, as calculated in Ref. 9. The solid lines labeled c are the result of the self-consistent t matrix with both direct and crossed amplitudes included, as presented above. In all of these figures the inclusion of the crossed amplitudes results in an additional spreading of the resonance peak of between 15 and 25 %, and a downward shift of the peak position of 20—30 MeV. Therefore, the proper treatment of crossing symmetry in the self-consistent t matrix equations has an appreciable effect on the resonance in the effective t matrix. Furthermore, the results remain essentially the same if the functions $F^{(i)}_{2I}$, $i=1,2,3$, are set equal to zero in Eq. (20). This implies that the effect of the crossed contributions comes in the self-energy dressing of the internal pion propagators.

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