$\Delta(1232)$ production via charge-changing weak currents: $e^- + p \to \Delta^0 + \nu_e$ and $e^+ + p \to \Delta^{++} + \bar{\nu}_e$

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We investigate $\Delta(1232)$ production via charge-changing weak currents $e^- + p \to \Delta^0 + \nu_e$ and $e^+ + p \to \Delta^{++} + \bar{\nu}_e$ at electron/positron beam energies in the range of a few GeV. A general formalism is introduced for the $N \to \Delta$ transition form factors, including the weak magnetism, weak quadrupole, and weak scalar form factors for the polar-vector current and the axial, pseudoscalar, recoil, and weak electric form factors for the axial current. The form factors related to the polar-vector current are related to $\Delta(1232)$ electroproduction on a nucleon target. For nucleons and deltas, we adopt in this calculation the flavor SU(6) wave functions, with quarks described as confined Dirac particles. The quark wave function adopted is of the form given by the MIT bag model, with or without the sharp boundary smoothened out. In the few GeV range, it is found that cross sections can be as large as 10^{-38} cm^2 and are sensitive to induced form factors such as the weak magnetism form factor. It is also found that, for a beam energy of around 4.0 GeV, the predicted cross sections depend sensitively on whether or not the sharp boundary in the quark wave function is smoothened out.

I. INTRODUCTION

The forthcoming construction of high energy (>1 GeV) and high intensity electron accelerators will make it feasible to study weak processes in a region where quark degrees of freedom are expected to be dominant, but where confinement effects cannot be neglected. The cross section for an exclusive semileptonic weak process necessarily falls off rapidly with q^2 due to hadronic form factors. Thus, exclusive semileptonic weak processes are best studied in the energy range of a few hundred MeV to a few GeV. In this paper, we consider $\Delta(1232)$ production via charge-changing weak currents, $e^- + p \to \Delta^0 + \nu_e$ and $e^+ + p \to \Delta^{++} + \bar{\nu}_e$, which complement isospin analog reactions such as $e^- + p \to n + \nu_e$ and offers an opportunity of probing the behavior of weak transition form factors.

Study of a weak exclusive reaction will remain a rather difficult experiment. Although detection of the two charged particles from the strong decay of a $\Delta(1232)$ helps reconstruction of the missing (neutrino) four-momentum, it remains to be seen whether the experiment is indeed feasible. Although the $N \to \Delta$ polar-vector transition form factors can be probed by electroexcitation of $\Delta(1232)$'s, axial-current and parity-violating form factors cannot be studied in this way and require experiments such as that being studied here. The transition form factors introduced here will be useful for other experiments such as those induced by neutral weak current. Furthermore, the reaction studied here provides further tests of quark models of nucleons and deltas.

The rest of this paper is organized as follows: In Sec. II, a general formalism is introduced for $N \to \Delta$ transition form factors, for both the polar-vector and axial-vector currents. In Sec. III we describe a quark-model calculation of these form factors. Sample numerical predictions are presented in Sec. IV, while questions related to experimental feasibility are discussed in Sec. V.

II. FORMULATION

In this section we outline a procedure which allows for a description of the reaction $e^- + p \to \Delta^0 + \nu_e$ or $e^+ + p \to \Delta^{++} + \bar{\nu}_e$ in the GeV range. To describe a spin-$\frac{3}{2}$ object such as $\Delta(1232)$, we introduce four-component Pauli spinors:

$$\chi_\Delta(J_z = \tfrac{3}{2}) \equiv \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \chi_\Delta(J_z = \tfrac{1}{2}) \equiv \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \quad \chi_\Delta(J_z = -\tfrac{1}{2}) \equiv \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \quad \chi_\Delta(J_z = -\tfrac{3}{2}) \equiv \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \tag{1}$$

We recall that, for the $(\frac{1}{2}\to\frac{1}{2})$ transition, we may use $\{1,\sigma_x,\sigma_y,\sigma_z\}$ as a complete set of operators in constructing the polar-vector or axial-vector matrix element. In the present case we may introduce three spin-vector S and five quadrupole Q 4×2 matrices (and their Hermitian conjugate matrices) which link the nucleon-spin space to the delta-spin space. Specifically, we take components of the transition spin matrices $\{S_+,S_-,S_0\}$, with $S_+\equiv -2^{-1/2}(S_x+iS_y)$, $S_-\equiv 2^{-1/2}(S_x-iS_y)$, and $S_0\equiv S_z$, as follows:

$$S_+\equiv\begin{bmatrix}1 & 0\\ 0 & 3^{-1/2}\\ 0 & 0\\ 0 & 0\end{bmatrix}, \tag{2a}$$

$$S_-\equiv\begin{bmatrix}0 & 0\\ 0 & 0\\ 3^{-1/2} & 0\\ 0 & 1\end{bmatrix}, \tag{2b}$$

$$S_0\equiv\begin{bmatrix}0 & 0\\ (\frac{2}{3})^{1/2} & 0\\ 0 & (\frac{2}{3})^{1/2}\\ 0 & 0\end{bmatrix}. \tag{2c}$$

Here we have fixed the normalization by equating $\chi_\Delta(m')^\dagger \mathbf{S}(m'')\chi_N(m)$ to $\langle\frac{1}{2},m;1,m''\mid\frac{3}{2},m'\rangle$. The remaining five 4×2 matrices will be referred to as "quadrupole matrices" since they allow the introduction of the $\Delta\to$N quadrupole-moment form factor. Specifically, we take

$$Q_{+2}\equiv\begin{bmatrix}0 & -(\frac{4}{5})^{1/2}\\ 0 & 0\\ 0 & 0\\ 0 & 0\end{bmatrix}, \tag{3a}$$

$$Q_{-2}\equiv\begin{bmatrix}0 & 0\\ 0 & 0\\ 0 & 0\\ (\frac{4}{5})^{1/2} & 0\end{bmatrix}, \tag{3b}$$

$$Q_{+1}\equiv\begin{bmatrix}5^{-1/2} & 0\\ 0 & -(\frac{3}{5})^{1/2}\\ 0 & 0\\ 0 & 0\end{bmatrix}, \tag{3c}$$

$$Q_{-1}\equiv\begin{bmatrix}0 & 0\\ 0 & 0\\ (\frac{3}{5})^{1/2} & 0\\ 0 & -5^{-1/2}\end{bmatrix}, \tag{3d}$$

$$Q_0\equiv\begin{bmatrix}0 & 0\\ (\frac{2}{5})^{1/2} & 0\\ 0 & -(\frac{2}{5})^{1/2}\\ 0 & 0\end{bmatrix}. \tag{3e}$$

The eight 4×2 matrices $\{S,Q\}$ just introduced form a complete basis for a spin $\frac{1}{2}\to\frac{3}{2}$ transition. In a Cartesian basis, we find

$$\begin{aligned}
&Q_{zz}=Q_0\ ,\\
&Q_{xx}=(\tfrac{3}{8})^{1/2}(Q_{+2}+Q_{-2})-Q_0/2\ ,\\
&Q_{yy}=-(\tfrac{3}{8})^{1/2}(Q_{+2}+Q_{-2})-Q_0/2\ ,\\
&Q_{yz}=Q_{zy}=i(\tfrac{3}{8})^{1/2}(Q_{+1}+Q_{-1})\ ,\\
&Q_{zx}=Q_{xz}=-(\tfrac{3}{8})^{1/2}(Q_{+1}-Q_{-1})\ ,\\
&Q_{xy}=Q_{yx}=-i(\tfrac{3}{8})^{1/2}(Q_{+2}-Q_{-2})\ .
\end{aligned} \tag{4}$$

We may now consider a three-vector defined by $\langle\Delta^0(p')\mid V_i(0)\mid \mathrm{p}(p)\rangle$ with $i=1$, 2, or 3. In the laboratory frame (where the target proton is at rest), the only other three-vector is the three-momentum transfer $\mathbf{q}$. Thus, we may introduce, in the laboratory frame, the most general form for V_i:

$$\begin{aligned}\langle\Delta^0(p')\mid V_i(0)\mid \mathrm{p}(p)\rangle=\chi_\Delta^\dagger\{&i(\mathbf{S}\times\mathbf{q}/\mid\mathbf{q}\mid)_i G_M\\ &+[(Q_{ij}q_j)/\mid\mathbf{q}\mid]G_Q\\ &+q_i[(Q_{jk}q_jq_k)/\mid\mathbf{q}\mid^3]G_S\}\chi_\mathrm{p}\ .\end{aligned} \tag{5a}$$

Here we have used $q_\lambda\equiv(p'-p)_\lambda$. We identify χ_Δ and χ_p as Pauli spinors defined in the hadron's own rest frame. Thus, effects due to the Wigner rotation caused by the nonzero velocity of Δ are absorbed into the definition of the form factors. It is clear that Eq. (5a) is the most general form for a spatial transition operator which is linear in S or Q.

Analogously, we may introduce the most general forms for V_0, A_i, and A_0 in the same frame:

$$\langle\Delta^0(p')\mid V_0(0)\mid \mathrm{p}(p)\rangle=\chi_\Delta^\dagger[(Q_{jk}q_jq_k)/\mid\mathbf{q}\mid^2]\chi_\mathrm{p}G_V\ , \tag{5b}$$

$$\begin{aligned}&\langle\Delta^0(p')\mid A_i(0)\mid \mathrm{p}(p)\rangle\\ &\quad=\chi_\Delta^\dagger\{-S_iG_A-q_i(\mathbf{S}\cdot\mathbf{q}/\mid\mathbf{q}\mid^2)G_P\\ &\qquad+[(i\epsilon_{ijk}Q_{jm}q_mq_k)/\mid\mathbf{q}\mid^2]G_R\}\chi_\mathrm{p}\ ,\end{aligned} \tag{5c}$$

$$\langle\Delta^0(p')\mid A_0(0)\mid \mathrm{p}(p)\rangle=\chi_\Delta^\dagger[(\mathbf{S}\cdot\mathbf{q})/\mid\mathbf{q}\mid]\chi_\mathrm{p}G_E\ . \tag{5d}$$

We shall refer to the form factors G_M, G_Q, G_S, G_A, G_P, G_R, and G_E, respectively, as the weak magnetism, weak quadrupole, weak scalar, axial, pseudoscalar, recoil, and weak electric form factors, following the convention of Ref. 1. Equations (5a)—(5d) can readily be written in a relativistic form. There are the same number of form factors in this case, but they are linear combinations of those which we have introduced. In this paper, we use the expressions of Eqs. (5a)—(5d) that are valid only in the laboratory frame and obtain expressions in any other frame of reference by Lorentz transformation. The polar vector current is conserved so that

$$\partial_\lambda V_\lambda(x)=0$$

or

$$\mathbf{q}\cdot\langle\Delta^0(p')\,|\,\mathbf{V}(0)\,|\,\mathrm{p}(p)\rangle-q_0\langle\Delta^0(p')\,|\,V_0(0)\,|\,\mathrm{p}(p)\rangle=0\ . \tag{6a}$$

This implies, in the laboratory frame,

$$|\,\mathbf{q}\,|(G_Q+G_S)-q_0G_V=0\ , \tag{6b}$$

and that there are three independent polar-vector form factors, which we choose as the magnetic (M), quadrupole (Q), and scalar (S) ones. Since q_0 is always different from zero, we may solve G_V in terms of G_Q and G_S.

It is also of importance to emphasize that, owing to the conserved-vector current (CVC) hypothesis,[1] the charge-changing weak polar-vector current is related to the hadronic electromagnetic current via an isospin rotation. To see this, we write

$$\begin{aligned}\langle\Delta^0\,|\,V_\lambda(0)\,|\,\mathrm{p}\rangle&=-\langle\Delta^0\,|\,[I^3_\lambda(0),I_-]\,|\,\mathrm{p}\rangle\\&=-\langle\Delta^0\,|\,I^3_\lambda(0)I_-\,|\,\mathrm{p}\rangle+\langle\Delta^0\,|\,I_-I^3_\lambda(0)\,|\,\mathrm{p}\rangle\\&=-\langle\Delta^0\,|\,I^3_\lambda(0)\,|\,\mathrm{n}\rangle+2\langle\Delta^+\,|\,I^3_\lambda(0)\,|\,\mathrm{p}\rangle\ .\end{aligned} \tag{7a}$$

Here we have used

$$J_+\,|\,jm\,\rangle=[(j-m)(j+m+1)]^{1/2}\,|\,j,m+1\,\rangle\ ,$$

a well-known formula in angular momentum algebra. The hadronic electromagnetic currents can be decomposed in the standard manner,

$$J_\lambda(x)=I^3_\lambda(x)+\tfrac{1}{2}Y_\lambda(x)\ , \tag{7b}$$

so that

$$\langle\Delta^0\,|\,V_\lambda(0)\,|\,\mathrm{p}\rangle=-\langle\Delta^0\,|\,J_\lambda(0)\,|\,\mathrm{n}\rangle+2\langle\Delta^+\,|\,J_\lambda(0)\,|\,\mathrm{p}\rangle\ . \tag{7c}$$

Analogously, we have

$$\begin{aligned}\langle\Delta^{++}\,|\,V^+_\lambda(0)\,|\,\mathrm{p}\rangle&=+\langle\Delta^{++}\,|\,[I^3_\lambda(0),I_+]\,|\,\mathrm{p}\rangle\\&=3^{1/2}\langle\Delta^+\,|\,J_\lambda(0)\,|\,\mathrm{p}\rangle\ .\end{aligned} \tag{7d}$$

The formalism which describes the weak processes $e^-+p\to\Delta^0+\nu_e$ and $e^++p\to\Delta^{++}+\bar\nu_e$ clearly includes the electromagnetic process $e^-+p\to e^-+\Delta^+$ as a special case. Specifically, we may introduce the $N\to\Delta$ electromagnetic transition form factors defined in the laboratory frame,

$$\begin{aligned}\langle\Delta^+(p')\,|\,J_i(0)\,|\,\mathrm{p}(p)\rangle=\chi^\dagger_\Delta\{&i(\mathbf{S}\times\mathbf{q}/|\,\mathbf{q}\,|)_iF_M\\&+[(Q_{ij}q_j)/|\,\mathbf{q}\,|]F_Q\\&+q_i[(Q_{jk}q_jq_k)/|\,\mathbf{q}\,|^3]F_S\}\chi_\mathrm{p}\ .\end{aligned} \tag{8}$$

Owing to current conservation [Eq. (6b)], there is no need to write down explicitly the expression for the time component. The weak polar-vector form factors for the $p\to\Delta^{++}$ transition are completely determined if the CVC hypothesis holds and the electromagnetic form factors specified by Eq. (8) are known. On the other hand, Eqs. (7c) and (7d) indicate that a measurement of the $p\to\Delta^0$ weak polar-vector transition form factors provides information on $\langle\Delta^0\,|\,J_\lambda(0)\,|\,\mathrm{n}\rangle$.

Having introduced the appropriate weak form factors, we may write the transition amplitude for the reaction $e^-+p\to\Delta^0+\nu_e$:

$$T(e^-+p\to\Delta^0+\nu_e)=i\,\bar u(p_\nu)\gamma_\lambda(1+\gamma_5)u(p_e)(2^{-1/2}G_F\cos\theta_C)\langle\Delta^0(p')\,|\,[V_\lambda(0)+A_\lambda(0)]\,|\,\mathrm{p}(p)\rangle\ , \tag{9}$$

with the matrix element $\langle\Delta^0(p')\,|\,[V_\lambda(0)+A_\lambda(0)]\,|\,\mathrm{p}(p)\rangle$ defined by Eqs. (7a)–(7d). Here, G_F and θ_C are, respectively, the Fermi coupling constant and the Cabibbo angle. Integrating over the unobserved neutrino three-momentum, we define the differential cross section $d\sigma/d\Omega_\Delta$ (in the laboratory frame), for the case of an unpolarized beam of electrons on an unpolarized hydrogen target with the final hadron polarization undetected,

$$\frac{d\sigma}{d\Omega_\Delta}=(2\pi)^{-2}E_\Delta p_\Delta\{E_\Delta/[E_\mathrm{p}+(E_e/p_\Delta)(p_\Delta-E_\Delta\cos\theta_\Delta)]\}\Sigma_\mathrm{av}\,|\,T\,|^2\ .$$

Thus, we have

$$\begin{aligned}\frac{d\sigma}{d\Omega_\Delta}=&[G_FE_\nu\cos\theta_C/(2\pi)]^2(p_\Delta/E_\nu)\{E_\Delta/[E_\mathrm{p}+(E_e/p_\Delta)(p_\Delta-E_\Delta\cos\theta_\Delta)]\}\\&\times\Sigma_\mathrm{av}\{[\mathbf{j}^*\cdot\mathbf{j}(1-\mathbf{e}\cdot\boldsymbol{\nu})+(\mathbf{e}\cdot\mathbf{j}^*\,\boldsymbol{\nu}\cdot\mathbf{j}+\mathbf{e}\cdot\mathbf{j}\,\boldsymbol{\nu}\cdot\mathbf{j}^*)-i(\mathbf{e}-\boldsymbol{\nu})\cdot\mathbf{j}^*\times\mathbf{j}]\\&\qquad+2\,\mathrm{Re}[-j_0^*(\mathbf{e}+\boldsymbol{\nu})\cdot\mathbf{j}+j_0^*i\mathbf{e}\times\boldsymbol{\nu}\cdot\mathbf{j}]+|\,j_0\,|^2(1+\mathbf{e}\cdot\boldsymbol{\nu})\}\ ,\end{aligned} \tag{10}$$

with

$$j_\lambda\equiv\langle\Delta^0(p')\,|\,[V_\lambda(0)+A_\lambda(0)]\,|\,\mathrm{p}(p)\rangle$$

[as defined by Eqs. (7a)–(7d)], $\mathbf{e}\equiv\mathbf{p}_e/|\,\mathbf{p}_e\,|$, and $\boldsymbol{\nu}\equiv\mathbf{p}_\nu/|\,\mathbf{p}_\nu\,|$. Once the input form factors G_M, G_Q, G_S, G_A, G_P, G_R, and G_E are given, we can evaluate the matrix element j_λ. Substituting the results into Eq. (10), we then obtain numerical predictions for the differential cross section. Although we can simplify Eq. (10) somewhat further by using Eqs. (5a)–(5d), we adopt a numerical method. Finally, we note that the charged weak reaction $e^+ + p \to \Delta^{++} + \bar{\nu}_e$ can be obtained by simply switching the sign of the term specified by $i(\mathbf{e}-\boldsymbol{\nu})\cdot\mathbf{j}^*\times\mathbf{j}$ [since antileptons appear in Eq. (9)]. Since the formulas suitable for Δ^{++} production can be obtained by substitutions, we use Δ^0 production as our basic example and simply mention the results for Δ^{++} as necessary.

III. A QUARK-MODEL CALCULATION OF THE N→Δ WEAK TRANSITION FORM FACTORS

In this section we wish to describe a quark-model calculation of the N→Δ weak transition form factors, assuming that nucleons and deltas are described by flavor SU(6) wave functions with quarks treated as Dirac particles. We consider in the next section two different possibilities for quark wave functions. The first possibility is the one given by the MIT bag model,[3] which is also used in chiral bag models with a sharp boundary. The second possibility on which we wish to concentrate a little more later in this paper is a quark wave function based on Dirac particles moving in a harmonic-oscillator confining potential (consisting of a Lorentz scalar and the time component of a Lorentz four-vector). As we shall see in the next section, the hadronic form factors are damped very rapidly as q^2 becomes large compared to $(1/R)^2$, where R is the confinement scale, so that cross sections arising from one-body currents become negligibly small for q^2 greater than, e.g., 50 fm^{-2}. As is well known in electronuclear physics (where Gaussian-type wave functions are used for nucleons), contributions from two-body currents (often referred to as meson-exchange currents in nuclear physics) and others become dominant at large q^2. On the other hand, the sharp boundary in the original MIT quark wave function gives rise to a slowly damped oscillating behavior of the form factors at large q^2 and the predicted cross section due to one-body currents remains fairly sizable at large q^2.

We recall that the matrix elements which we need to determine for the reaction $e^- + p \to \Delta^0 + \nu_e$ are

$$v_\lambda = \langle \Delta^0(p') \mid V_\lambda(0) \mid p(p) \rangle \tag{11a}$$

and

$$a_\lambda = \langle \Delta^0(p') \mid A_\lambda(0) \mid p(p) \rangle \ . \tag{11b}$$

To carry out a quark-model calculation of these matrix elements, we need to know (1) the initial and final baryon wave functions expressed in terms of quarks and other constituents (including the confining field), and (2) the operators which characterize the reaction mechanism at the quark level. For the purpose of this paper, we shall assume that the quark wave functions of the initial or final baryons at rest are determined by the flavor SU(6) symmetry. For instance, the proton wave function at rest is given by[2]

$$\begin{aligned} \mid p(\uparrow)\rangle_Q = 18^{-1/2}[&2u^{(1)}(+)u^{(2)}(+)d^{(3)}(-) \\ &-u^{(1)}(+)u^{(2)}(-)d^{(3)}(+) \\ &-u^{(1)}(-)u^{(2)}(+)d^{(3)}(+) \\ &+(1\leftrightarrow 3)+(2\leftrightarrow 3)] \ . \end{aligned} \tag{12a}$$

Analogously, the quark part of the Δ^0 wave function at rest is specified by[2]

$$\begin{aligned} \mid \Delta^0(J_z=\tfrac{3}{2})\rangle_Q = 3^{-1/2}[&u^{(1)}(+)d^{(2)}(+)d^{(3)}(+) \\ &+d^{(1)}(+)u^{(2)}(+)d^{(3)}(+) \\ &+d^{(1)}(+)d^{(2)}(+)u^{(3)}(+)] \ . \end{aligned} \tag{12b}$$

Here and in what follows, we suppress color indices wherever possible and use a shorthand notation, such as

$$u^{(a)}(+) \equiv \psi(r^{(a)}; s_z=\tfrac{1}{2}, I_3=\tfrac{1}{2})$$

and

$$u^{(a)}(-) \equiv \psi(r^{(a)}; s_z=-\tfrac{1}{2}, I_3=\tfrac{1}{2}) \ .$$

More specifically, the quark wave function which we adopt in this paper is of the relativistic form[3]

$$\psi(\mathbf{r};s) = \begin{bmatrix} u(r) \\ i\boldsymbol{\sigma}\cdot\hat{\mathbf{r}}v(r) \end{bmatrix} \chi_s \ , \tag{13}$$

with $r \equiv |\mathbf{r}|$ and $\mathbf{r}$ is the quark coordinate expressed relative to the center of the bag. χ_s is the Pauli spinor with s the z component of spin.

It is of importance to emphasize that the baryon wave functions in Eq. (12) do not give any information beyond the quark part, such as the gluonic components or the confining field. It is also of importance to note that only baryon wave functions at rest are given. Since the hadron energy momentum is not carried entirely by the quark constituents (as suggested, for instance, by high-energy deep inelastic lepton-proton scattering experiments), we wish in this paper to set aside the relativistic center-of-mass problem and to obtain the quark part of the hadron wave function in motion as though quarks were free (as suggested by the asymptotic-free nature of QCD, as well as by general successes of bag models such as the simple MIT bag[3] at low energies). Although the specific way which we choose to handle recoil effects requires detailed justifications, we do not do so here and refer the interested reader to Ref. 4 on the treatment of recoil effects in bag models.

The reaction $e^- + p \to \Delta^0 + \nu_e$ involves the one-body charge-lowering quark currents:[5]

$$V_\lambda(\mathbf{x}) = \sum_{a=1}^{3} \{i\tau_-\gamma_4\gamma_\lambda\}^{(a)}\delta^{(3)}(\mathbf{x}-\mathbf{r}^{(a)}) \tag{14a}$$

and

$$A_\lambda(\mathbf{x})=\sum_{a=1}^{3}\{i\tau_-\gamma_4\gamma_\lambda\gamma_5\}^{(a)}\delta^{(3)}(\mathbf{x}-\mathbf{r}^{(a)}) , \tag{14b}$$

which takes this simple form since quarks are pointlike Dirac particles. It is often assumed[5] that the impulse approximation in terms of Eqs. (14a) and (14b) holds only in the Breit frame, in which the initial and final hadron three-momenta are equal in magnitude but opposite in sign. This assumption is justified[5] in, e.g., the determination of the proton charge and anomalous magnetic form factors $e_p(q^2)$ and $\mu_p(q^2)$ where adoption of the Breit frame ensures both that $e_p(q^2)$ and $\mu_p(q^2)$ are functions of q^2 alone and that there is not any additional spurious form factor. Thus, we take the same assumption for the sake of consistency whenever Eqs. (14a) and (14b) are used. Using Eqs. (14a) and (14b), we obtain[5] (noting that the sign of our $\mathbf{q}_\lambda$ is different from that in Ref. 5)

$$\langle \Delta^0(\mathbf{p}')\,|\,V_\lambda(0)\,|\,\mathrm{p}(\mathbf{p})\rangle =\langle \Delta^0(\mathbf{p}')\,|\sum_{a=1}^{3}\exp(+i\mathbf{q}\cdot\mathbf{r}^{(a)})\{i\tau_-\gamma_4\gamma_\lambda\}^{(a)}\,|\,\mathrm{p}(\mathbf{p})\rangle , \tag{15a}$$

$$\langle \Delta^0(\mathbf{p}')\,|\,A_\lambda(0)\,|\,\mathrm{p}(\mathbf{p})\rangle =\langle \Delta^0(\mathbf{p}')\,|\sum_{a=1}^{3}\exp(+i\mathbf{q}\cdot\mathbf{r}^{(a)})\{i\tau_-\gamma_4\gamma_\lambda\gamma_5\}^{(a)}\,|\,\mathrm{p}(\mathbf{p})\rangle . \tag{15b}$$

Equations (15a) and (15b) indicate that, in the impulse approximation, it is adequate to know only the quark part of the baryon wave function, such as Eq. (12a) or (12b). Specifically, we find, with η the overlap integral for a spectator quark,

$$\langle \Delta^0(\mathbf{p}';J_z=\tfrac{3}{2})\,|\,V_\lambda(0)\,|\,\mathrm{p}(\mathbf{p};J_z=\tfrac{1}{2})\rangle=-(\tfrac{2}{3})^{1/2}\eta^2\int d^3r\,\psi_d^\dagger(\mathbf{r};+)\exp(+i\mathbf{q}\cdot\mathbf{r})S_f^\dagger\{i\gamma_4\gamma_\lambda\}S_i\psi_u(\mathbf{r};-) , \tag{16a}$$

$$\langle \Delta^0(\mathbf{p}';J_z=\tfrac{1}{2})\,|\,V_\lambda(0)\,|\,\mathrm{p}(\mathbf{p};J_z=\tfrac{1}{2})\rangle=(2^{1/2}/3)\eta^2\int d^3r[\psi_d^\dagger(\mathbf{r};+)\exp(+i\mathbf{q}\cdot\mathbf{r})S_f^\dagger\{i\gamma_4\gamma_\lambda\}S_i\psi_u(r;+) -\psi_d^\dagger(r;-)\exp(+i\mathbf{q}\cdot\mathbf{r})S_f^\dagger\{i\gamma_4\gamma_\lambda\}S_i\psi_u(r;-)] , \tag{16b}$$

$$\langle \Delta^0(\mathbf{p}';J_z=-\tfrac{1}{2})\,|\,V_\lambda(0)\,|\,\mathrm{p}(\mathbf{p};J_z=\tfrac{1}{2})\rangle=(2^{1/2}/3)\eta^2\int d^3r\,\psi_d^\dagger(\mathbf{r};-)\exp(+i\mathbf{q}\cdot\mathbf{r})S_f^\dagger\{i\gamma_4\gamma_\lambda\}S_i\psi_u(r;+) . \tag{16c}$$

The corresponding formulas for the charge-lowering axial current $A_\lambda(x)$ are identical with Eqs. (16a)—(16c), except that the operator $i\gamma_4\gamma_\lambda$ should be replaced by $i\gamma_4\gamma_\lambda\gamma_5$. Here the boost operators S_f and S_i are introduced such that $S_f\psi(\mathbf{r})$ and $S_i\psi(\mathbf{r})$, with $\psi(\mathbf{r})$ the quark wave function in the rest frame of the hadron, are, respectively, the final and initial quark wave functions as seen in the Breit frame.

To describe the charged weak reaction $e^+ + p \rightarrow \Delta^{++}\bar{\nu}_e$, we need to replace Eqs. (16a)—(16c) by following formulas:

$$\langle \Delta^{++}(\mathbf{p}';J_z=\tfrac{3}{2})\,|\,V_\lambda^+(0)\,|\,\mathrm{p}(\mathbf{p};J_z=\tfrac{1}{2})\rangle=2^{1/2}\eta^2\int d^3r\,\psi_u^\dagger(\mathbf{r};+)\exp(+i\mathbf{q}\cdot\mathbf{r})S_f^\dagger\{i\gamma_4\gamma_\lambda\}S_i\psi_d(\mathbf{r};-) , \tag{17a}$$

$$\langle \Delta^{++}(\mathbf{p}';J_z=\tfrac{1}{2})\,|\,V_\lambda^+(0)\,|\,\mathrm{p}(\mathbf{p};J_z=\tfrac{1}{2})\rangle =-(\tfrac{2}{3})^{1/2}\eta^2\int d^3r[\psi_u^\dagger(\mathbf{r};+)\exp(+i\mathbf{q}\cdot\mathbf{r})S_f^\dagger\{i\gamma_4\gamma_\lambda\}S_i\psi_d(\mathbf{r};+)-\psi_u^\dagger(\mathbf{r};-)\exp(+i\mathbf{q}\cdot\mathbf{r})S_f^\dagger\{i\gamma_4\gamma_\lambda\}S_i\psi_d(\mathbf{r};-)] , \tag{17b}$$

$$\langle \Delta^{++}(\mathbf{p}';J_z=-\tfrac{1}{2})\,|\,V_\lambda^+(0)\,|\,\mathrm{p}(\mathbf{p};J_z=\tfrac{1}{2})\rangle=-(\tfrac{2}{3})^{1/2}\eta^2\int d^3r\,\psi_u^\dagger(\mathbf{r};-)\exp(+i\mathbf{q}\cdot\mathbf{r})S_f^\dagger\{i\gamma_4\gamma_\lambda\}S_i\psi_d(\mathbf{r};+) . \tag{17c}$$

And, as before, similar expressions hold for the charge-raising axial current $A_\lambda^+(x)$. Noting that the matrix elements given by Eqs. (17a)—(17c) are related to those given by Eqs. (16a)—(16c) by an overall multiplying factor, we conclude that, in the quark-only impulse approximation, the form factors in the Δ^{++} case are exactly $-3^{1/2}$ times those in the Δ^0 case. This does not mean that cross sections in the former case are 3 times those for the latter, since the sign of an interference term in Eq. (10) should be switched in going from one case to the other.

To proceed further, we assume that the boost operators can be parametrized as follows:

$$S_i=a_i+b_i\boldsymbol{\alpha}\cdot(\mathbf{q}/|\,\mathbf{q}\,|) , \tag{18a}$$

$$S_f=a_f+b_f\boldsymbol{\alpha}\cdot(\mathbf{q}/|\,\mathbf{q}\,|) . \tag{18b}$$

For example, if the free boost operators are used, we find

$$a_i=[(E_i^B+M_i)/(2M_i)]^{1/2} , \tag{19a}$$

$$b_i=-[(E_i^B-M_i)/(2M_i)]^{1/2} , \tag{19b}$$

$$a_f=[(E_f^B+M_f)/(2M_f)]^{1/2} , \tag{19c}$$

$$b_f=[(E_f^B-M_f)/(2M_f)]^{1/2} . \tag{19d}$$

Here and in what follows, we use the superscript B to indicate explicitly those quantities which are defined in the Breit frame. It is useful to recall that a Lorentz four-vector $(\mathbf{v}^B,iv_0^B)$ in the Breit frame is related to $(\mathbf{v},iv_0)$ (in the laboratory frame),

$$\mathbf{v}^B=\mathbf{v}+\boldsymbol{\beta}(\boldsymbol{\beta}\cdot\mathbf{v})(\gamma-1)/|\,\boldsymbol{\beta}\,|^2-\boldsymbol{\beta}\gamma v_0 , \tag{20a}$$

$$v_0^B=\gamma(v_0-\boldsymbol{\beta}\cdot\mathbf{v}) , \tag{20b}$$

with

$$\boldsymbol{\beta}=\mathbf{q}/(E_\Delta+m_p) , \quad \gamma=(1-\boldsymbol{\beta}^2)^{-1/2} . \tag{20c}$$

This transformation can be applied to the matrix elements defined in Eqs. (5a)—(5d), so that a comparison with the microscopic model can be made. (Note that the Breit-frame matrix elements obtained in this way are expressed in terms of quantities defined in the laboratory frame.) The results are summarized immediately below. We define, with $\mathbf{q}$ fixed in the z direction,

$$I_0 \equiv \langle \Delta(\mathbf{q}^B/2, J_z'=\tfrac{1}{2}) \mid V_0(0) \mid \mathrm{p}(-\mathbf{q}^B/2, J_z=\tfrac{1}{2} \rangle \;,$$
$$I_+ \equiv \langle \Delta(\mathbf{q}^B/2, J_z'=\tfrac{3}{2}) \mid V_+(0) \mid \mathrm{p}(-\mathbf{q}^B/2, J_z=\tfrac{1}{2})\rangle \;, \quad (21)$$
$$I_- \equiv \langle \Delta(\mathbf{q}^B/2, J_z'=-\tfrac{1}{2} \mid V_-(0) \mid \mathrm{p}(-\mathbf{q}^B/2, J_z=\tfrac{1}{2})\rangle \;.$$

Then we have

$$G_M=\tfrac{3}{4}(I_+ - 3^{-1/2}I_-) \;, \quad (22a)$$
$$G_Q=(15^{1/2}/6)(I_+ + 3^{1/2}I_-) \;, \quad (22b)$$
$$G_S=[(E_\Delta^2-m_\mathrm{p}^2)/(\gamma 2m_\mathrm{p} \mid \mathbf{q} \mid)](\tfrac{5}{2})^{1/2}I_0 - G_Q \;. \quad (22c)$$

Analogously, we define, again with $\mathbf{q}$ in the z direction,

$$K_0 \equiv \langle \Delta(\mathbf{q}^B/2, J_z'=\tfrac{1}{2}) \mid A_0(0) \mid \mathrm{p}(-\mathbf{q}^B/2, J_z=\tfrac{1}{2})\rangle \;,$$
$$K_3 \equiv \langle \Delta(\mathbf{q}^B/2, J_z'=\tfrac{1}{2}) \mid A_3(0) \mid \mathrm{p}(-\mathbf{q}^B/2, J_z=\tfrac{1}{2})\rangle \;,$$
$$K_+ \equiv \langle \Delta(\mathbf{q}^B/2, J_z'=\tfrac{3}{2}) \mid A_+(0) \mid \mathrm{p}(-\mathbf{q}^B/2, J_z=\tfrac{1}{2})\rangle \;, \quad (23)$$
$$K_- \equiv \langle \Delta(\mathbf{q}^B/2, J_z'=-\tfrac{1}{2}) \mid A_-(0) \mid \mathrm{p}(-\mathbf{q}^B/2, J_z=\tfrac{1}{2})\rangle \;.$$

We have

$$G_A=-\tfrac{3}{4}(K_+ + 3^{-1/2}K_-) \;, \quad (24a)$$
$$G_R=(15^{1/2}/6)(K_+ - 3^{1/2}K_-) \;, \quad (24b)$$
$$G_P=-(\tfrac{3}{2})^{1/2}\gamma(K_3+\beta K_0)-G_A \;, \quad (24c)$$
$$G_E=(\tfrac{3}{2})^{1/2}\gamma(K_0+\beta K_3) \;. \quad (24d)$$

Using Eqs. (13) and (16a)—(16c), we obtain, with $\rho \equiv \mid \mathbf{q}^B \mid r$,

$$G_Q=0 \;, \quad (25a)$$
$$G_S=0 \;, \quad (25b)$$

$$G_M=\left[\frac{2}{\sqrt{3}}\right]\eta^2\int_0^\infty dr\, 4\pi r^2((a_f a_i - b_f b_i)j_1(\rho)[u_f(r)v_i(r)+v_f(r)u_i(r)] + (b_f a_i - a_f b_i)\{j_0(\rho)[u_f(r)u_i(r)-\tfrac{1}{3}v_f(r)v_i(r)]+j_2(\rho)v_f(r)v_i(r)\}) \;. \quad (25c)$$

Analogously, we have, for the axial current,

$$G_A=\left[\left[\frac{2}{\sqrt{3}}\right](a_f a_i - b_f b_i) - 3^{-1/2}(a_f b_i - b_f a_i)\right]\eta^2\int_0^\infty dr\, 4\pi r^2\{j_0(\rho)[u_f(r)u_i(r)-\tfrac{1}{3}v_f(r)v_i(r)]+j_2(\rho)v_f(r)v_i(r)\} \;, \quad (26a)$$

$$G_R=\tfrac{2}{3}5^{1/2}(a_f b_i - b_f a_i)\eta^2\int_0^\infty dr\, 4\pi r^2\{j_0(\rho)[u_f(r)u_i(r)-\tfrac{1}{3}v_f(r)v_i(r)]+j_2(\rho)v_f(r)v_i(r)\} \;, \quad (26b)$$

$$K_0=\tfrac{2}{3}2^{1/2}\eta^2\int_0^\infty dr\, 4\pi r^2((a_f a_i + b_f b_i)j_1(\rho)[u_f(r)v_i(r)-v_f(r)u_i(r)] - (a_f b_i + b_f a_i)\{j_0(\rho)[u_f(r)u_i(r)-\tfrac{1}{3}v_f(r)v_i(r)]-2j_2(\rho)v_f(r)v_i(r)\}) \;, \quad (26c)$$

$$K_3=\tfrac{2}{3}2^{1/2}\eta^2\int_0^\infty dr\, 4\pi r^2((a_f b_i + b_f a_i)j_1(\rho)[u_f(r)v_i(r)-v_f(r)u_i(r)] - (a_f a_i + b_f b_i)[j_0(\rho)[u_f(r)u_i(r)-\tfrac{1}{3}v_f(r)v_i(r)]-2j_2(\rho)v_f(r)v_i(r)]) \;. \quad (26d)$$

We note that spherical symmetry of the adopted S-wave quark wave function [Eq. (13)] makes the transition quadrupole form factor G_Q vanish identically. We also note that, without recoil corrections, the form factor G_R vanishes identically. This is why we choose to call it the "recoil form factor." We also note that, without introduction of a D-state component in the nucleon or delta wave function, it seems very difficult to make the weak quadrupole form factor G_Q different from zero.

In concluding this section, we wish to note that the polar-vector form factors G_M, G_Q, and G_S are to be evaluated via Eqs. (25a)—(25c), while the axial-vector form factors G_A, G_R, G_P, and G_E are determined through Eqs. (26a)—(26d) and (24c)—(24d). With the results as input, we then use the formulas in Sec. II to make predictions on cross sections.

IV. NUMERICAL PREDICTIONS

In this section we present selected numerical predictions for the reactions $e^- + p \rightarrow \Delta^0 + \nu_e$ and $e^+ + p \rightarrow \Delta^{++} + \bar{\nu}_e$, using the formalism developed in Secs. II and III. As indicated earlier, we wish to consider two choices for the quark wave functions, both of which are in the form of Eq. (13). The first choice is from the MIT bag model.[3] The upper and lower components $u(r)$ and $v(r)$ are given by

$$u(r)=\begin{cases} N[(\omega+m)/(4\pi\omega)]^{1/2}j_0(xr/R) & \text{for } r \le R \;, \\ 0 \quad \text{for } r > R \;, \end{cases} \quad (27a)$$

$$v(r)=\begin{cases} N[(\omega-m)/(4\pi\omega)]^{1/2}j_1(xr/R) & \text{for } r \le R \;, \\ 0 \quad \text{for } r > R \;, \end{cases} \quad (27b)$$

$$(N^2R^3)^{-1}=j_0^2(x)\{2\omega[\omega-(1/R)]+(m/R)\}[\omega(\omega-m)]^{-1}, \quad (27c)$$

with x and ω determined by

$$\tan x = x\{1-mR-[x^2+(mR)^2]^{1/2}\}^{-1} \;, \quad (27d)$$
$$\omega=(1/R)[x^2+(mR)^2]^{1/2} \;. \quad (27e)$$

This quark wave function has also been used in most chiral bag models where a sharp boundary is used. It gives rise to a slowly damped oscillating behavior in the form factors and the resultant cross section does not fall very rapidly as q^2 increases. As an alternative, we have also considered the case in which the quark (as a Dirac particle) is confined by a potential $(1+\gamma_4)(kr^2/4)+(a+b\gamma_4)$. The solution is then given by

$$u(r)=c\exp(-r^2/R^2)\ , \tag{28a}$$

$$v(r)=\xi rc\exp(-r^2/R^2)\ , \tag{28b}$$

with

$$\xi=kR^2/4\ , \tag{28c}$$

$$c^2R^3(\pi/2)^{3/2}=(1+\tfrac{3}{4}\xi^2R^2)^{-1}\ . \tag{28d}$$

Here we may adjust the constants a and b so that both the mass and the eigenenergy can be fixed to desired values. In addition, the strength of the lower component relative to the upper one can be adjusted by choosing a value for kR^2 [cf. Eq. (28c)].

In Table I we attempt to relate the two models by adjusting the parameters $\{\xi,R\}$ in the Dirac harmonic oscillator model (DHOM) to reproduce the same $\langle r^2\rangle_N$ and G_A as in the MIT bag model. We use Eqs. (27) and (28) to evaluate the various form factors by choosing $|\mathbf{q}^B|=4.0\times10^{-6}$ fm^{-1} and neglecting the boost operators (i.e., $a_f=a_i=1$ and $b_f=b_i=0$). We also use, for the bag model calculation, $R_N=0.987$ fm, $R_\Delta=1.081$ fm, and $m=10$ MeV. Correspondingly, we use $R_N=0.763$ fm and $R_\Delta=0.836$ fm in the Dirac harmonic oscillator model. It is clear that the DHOM with $\xi=1.6$ fm^{-1} yields results very close to the MIT bag.

TABLE I. Predictions on the various form factors. Here we have used $|\mathbf{q}^B|=4.0\times10^{-6}$ fm^{-1}, $a_i=a_f=1$, and $b_i=b_f=0$. The three entries for Dirac harmonic oscillator are (from top to bottom) for $\xi=1.1$, 1.6, and 2.1 fm^{-1}, respectively.

	Dirac harmonic oscillator	MIT bag model
$\langle r^2\rangle_N$	0.482	
	0.517	0.517
	0.553	
G_M	0.662×10^{-6}	
	0.787×10^{-6}	0.570×10^{-6}
	0.829×10^{-6}	
G_A	1.106	
	0.818	0.807
	0.562	
G_R	0	
	0	0
	0	
G_P	0.036	
	0.027	0.027
	0.019	
G_E	−0.286	
	−0.212	−0.209
	−0.146	

Once the various form factors have been determined, we use the formulas given in Sec. II to make predictions on cross sections. In what follows, we present sample numerical results. For the sake of clarity, we divide our results into two subsections according to the specific reaction of interest.

A. $e^-+p\rightarrow\Delta^0+\nu_e$

In Table II we present numerical predictions as functions of the recoiling Δ^0 angle (in the laboratory frame) for an electron beam energy of 0.5 GeV. In this figure and what follows, the MIT bag radius of 0.987 fm is adopted for $\{R_N\}_B$ and the quark mass is taken arbitrarily to be 10 MeV. The three entries under the Dirac harmonic oscillator model (from top to bottom) are obtained by choosing ξ to be 1.1, 1.6, and 2.1 fm^{-1}, respectively. For the sake of simplicity, we have adopted the boost operators as specified by Eqs. (18) and (19), so that the interaction effects have been neglected from this calculation. Note that the predictions from the Dirac harmonic oscillator model depend sensitively on the value chosen for ξ. The sensitivity to the value of ξ is far more dramatic than what may be caused by a choice of the detailed boost operators. Thus, we wish to set aside the question of the boost operators[4] and choose $\xi=1.6$ fm^{-1} in the rest of this paper.

TABLE II. The predictions for the electron capture reaction $e^-+p\rightarrow\Delta^0+\nu_e$ at $E_e=0.5$ GeV.

θ_Δ (deg)	$d\sigma/d\Omega_\Delta$ (10^{-40} cm^2) Dirac harmonic oscillator	MIT bag model	q^2 (fm^{-2})
2	17.13		
	15.05	10.48	3.95
	12.47		
4	17.18		
	15.04	10.51	3.85
	12.42		
6	17.31		
	15.06	10.58	3.69
	12.37		
8	17.61		
	15.16	10.74	3.44
	12.35		
10	18.32		
	15.54	11.14	3.11
	12.50		
12	20.49		
	16.96	12.40	2.63
	13.35		
14	51.84		
	40.23	31.07	1.75
	29.97		
16*	kinematically forbidden		

TABLE III. The predictions for the electron capture reaction $e^- + p \to \Delta^0 + \nu_e$ at $E_e = 4.0$ GeV.

θ_Δ (deg)	$d\sigma/d\Omega_\Delta$ (10^{-40} cm^2) Dirac harmonic oscillator	MIT bag model	q^2 (fm^{-2})
4	3.9×10^{-6}	0.041	155.3
8	1.0×10^{-5}	0.160	147.8
12	4.6×10^{-5}	0.411	136.3
16	2.9×10^{-4}	0.567	122.0
20	2.2×10^{-3}	0.294	106.1
24	1.6×10^{-2}	0.009	89.74
28	0.112	0.534	73.61
32	0.643	1.040	58.23
36	3.045	0.208	43.79
40	12.25	2.017	30.06
44	108.7	59.94	13.96
46*	kinematically forbidden		

In Table III we present numerical predictions as functions of the recoiling Δ^0 angle (in the laboratory frame) for an electron beam energy of 4.0 GeV. It is clear that, for 0.5 GeV electron beam, the available phase space is rather limited. Thus, the major message reflected by Table II is the possible dependence on the value chosen for ξ. On the other hand, the range for the allowed q^2 is considerably enlarged at $E_e = 4.0$ GeV, so that both the rapid Gaussian falloff of the predicted cross section for the harmonic-oscillator model and its oscillatory behavior (and the slow falloff behavior) for the bag model are clearly displayed in Table III. In fact, predictions for the two models differ considerably from each other, although they agree qualitatively at $E_e = 0.5$ GeV. We believe that an experiment at 4.0 GeV can distinguish between the two models and indicate whether the sharp surface is indeed present in the baryon structure.

It should be pointed out that, just like electron-nucleus scattering, the cross section is dominated by two-body currents or even by three-body contributions rather than by one-body currents, as q^2 becomes sufficiently large. Therefore, it is expected that the present DHOM predictions should be modified beyond a certain q^2, but this is not an indication of the failure of the model. Rather, the smallness of the large-q^2 one-body predictions in the DHOM makes room for two-body contributions and eventually for the perturbative QCD behavior at sufficiently large q^2. (It is not clear whether we can do the same thing with a bag model.)

The sensitivity of our predictions to each of the weak form factors are displayed in Table IV, where all the form factors except the one indicated explicitly are set to zero by hand. The first entry is the DHOM prediction with $\xi = 1.6$ fm^{-1}, while the second entry is the MIT bag model prediction. It is clear from this table that contributions from each of the form factors are of numerical importance and should not be neglected in the calculation. At a fixed q^2, one may perform experiments at several

TABLE IV. The predictions for the electron capture reaction $e^- + p \to \Delta^0 + \nu_e$ at $E_e = 4.0$ GeV.

θ_Δ (deg)	$d\sigma/d\Omega_\Delta$ (10^{-40} cm^2) $G_M \neq 0$	$G_A \neq 0$	$G_P \neq 0$	$G_R \neq 0$	$G_E \neq 0$
4	9.4	9.6	1.3	2.1	0.16×10^{-7}
	0.015	0.006	0.021	0.001	0.014
8	2.6	3.0	1.3	0.55	0.16×10^{-6}
	0.076	0.018	0.085	0.003	0.048
12	1.2	1.6	1.2	0.25	0.14×10^{-5}
	0.219	0.047	0.134	0.007	0.057
16	8.0	11.7	11.4	1.5	1.21×10^{-5}
	0.328	0.068	0.058	0.009	0.008
20	6.4	10.0	10.8	1.1	1.03×10^{-4}
	0.186	0.034	0.019	0.004	0.004
24	5.3	8.3	9.2	0.84	0.74×10^{-3}
	0.000 04	0.003	0.345	0.0003	0.220
28	0.039	0.061	0.064	0.006	0.004
	0.302	0.137	0.576	0.012	0.201
32	0.245	0.362	0.342	0.030	0.015
	0.642	0.262	0.151	0.022	0.006
36	1.25	1.75	1.39	0.128	0.029
	0.127	0.033	0.226	0.002	0.176
40	5.41	7.19	4.38	0.440	0.010
	1.06	0.880	2.35	0.054	0.452
44	50.44	72.23	23.75	2.77	0.720
	30.60	35.22	12.27	1.35	0.019

TABLE V. The predictions for the positron capture reaction $e^+ + p \to \Delta^{++} + \bar{\nu}_e$ at $E_e = 0.5$ GeV.

	$d\sigma/d\Omega_\Delta$ (10^{-40} cm^2)		
θ_Δ (deg)	Dirac harmonic oscillator	MIT bag model	q^2 (fm^{-2})
2	0.584	0.617	3.95
4	0.764	0.781	3.85
6	1.086	1.079	3.69
8	1.600	1.566	3.44
10	2.437	2.388	3.11
12	4.102	4.097	2.63
14	19.08	20.27	1.75
16	kinematically forbidden		

different electron beam energies, so that the measurements may be inverted to yield different form factors.

B. $e^+ + p \to \Delta^{++} + \bar{\nu}_e$

Although the form factors in the $p \to \Delta^{++}$ case can be obtained from those in the $p \to \Delta^0$ case by a simple rescaling [Eqs. (16a)–(16c) and (17a)–(17c)], the interference term between the axial current and polar-vector current, as appears in Eq. (10), changes its sign, so that the cross section in the $p \to \Delta^{++}$ case, when combined with that in the $p \to \Delta^0$ case, allows one to probe the size of this interference term. In Table V we show the predicted cross section for the positron capture reaction $e^+ + p \to \Delta^{++} + \bar{\nu}_e$ at the positron beam energy of 0.5 GeV. As before, the DHOM prediction is made with $\xi = 1.6$ fm^{-1}. As in the case of Table II for the $p \to \Delta^0$ transition, the predictions from the two models are rather similar at these energies.

In Table VI we show the predicted cross section at a positron beam energy of 4.0 GeV. The information revealed by this table is very similar to that in Table III. However, it is important to keep in mind that a measurement on the $p \to \Delta^{++}$ reaction provides information independent of that given by the $p \to \Delta^0$ reaction.

TABLE VI. The predictions for the positron capture reaction $e^+ + p \to \Delta^{++} + \bar{\nu}_e$ at $E_e = 4.0$ GeV.

	$d\sigma/d\Omega_\Delta$ (10^{-40} cm^2)		
θ_Δ (deg)	Dirac harmonic oscillator	MIT bag model	q^2 (fm^{-2})
4	7.0×10^{-7}	0.012	155.3
8	2.5×10^{-6}	0.091	147.8
12	1.6×10^{-5}	0.329	136.3
16	1.5×10^{-4}	0.578	122.0
20	1.5×10^{-3}	0.387	106.1
24	1.5×10^{-2}	0.025	89.74
28	0.130	0.745	73.61
32	0.914	1.721	58.23
36	5.104	0.447	43.79
40	23.72	4.039	30.06
44	247.7	137.1	13.96
46*	kinematically forbidden		

Although we have made calculations of partial cross sections with the initial proton target polarized and/or the final Δ polarization detected, we shall not present the results here since general characteristics of these numerical results are very similar.

V. DISCUSSION

The sample numerical predictions presented in the preceding section indicate the following.

(a) The predicted differential cross section $d\sigma/d\Omega_\Delta$ can be of the order of 10^{-39}–10^{-38} cm^2, which is fairly sizable.

(b) The predicted cross section is sensitive to the q^2 dependence of the weak form factors and can thus be used as a source of information for them.

(c) Except for near the maximum hadron recoiling angles (which correspond to very small q^2), the differential cross section is dominated by contributions due to the induced form factors such as the weak magnetic form factor G_M. (Note that G_A is the only form factor that is not "induced.")

To see why cross sections of this size are worthy of serious consideration for experimentation, we may consider a conceptual design which allows for detection of deltas of kinetic energy of around 200 MeV with an energy resolution of about 1 MeV. It should not be difficult to acquire such energy resolution by reconstructing from the observed four-momenta of the two detected charge particles a proton and a charged pion. As we shall see shortly, it is also essential to acquire an angular resolution in the vicinity of 0.1°. For the moment, we assume the solid angle subtended by the detector to be

$$\Delta\Omega = 2\pi[\Delta\theta_\Delta(\text{in deg})](\pi/180)\sin\theta_\Delta$$
$$= 0.1097[\Delta\theta_\Delta(\text{in deg})]\sin\theta_\Delta\ .$$

Here we have assumed that the detector essentially covers 2π in azimuthal angle. For the sake of illustration, we take a luminosity of 10^{37} cm^{-2}s^{-1}, less than a tenth of a typical value expected at the forthcoming CEBAF, for instance. With a cross section of 10^{-39} cm^2, we estimate the counting rate as follows:

$$\text{counts/h} = 0.1097[\Delta\theta_\Delta(\text{in deg})]\sin\theta_\Delta \times 10^{37}\ \text{cm}^{-2}\text{s}^{-1} \times (10^{-39}\ \text{cm}^2) \times 3600\ \text{s/h} \times (\text{detector efficiency})$$
$$= (4.0\ \text{counts/h}) \times [\Delta\theta_\Delta(\text{in deg})]\sin\theta_\Delta \times (\text{detector efficiency})\ .$$

It is clear that the number of counts per hour is high enough to warrant further serious consideration for experimentation. One problem is connected to sources for producing a huge background which we may have to veto.

To observe the charge-changing weak reaction $e^- + p \to \Delta^0 + \nu_e$ experimentally, we may use the immediate decay products $p + \pi^-$ as a unique signature. It is different from the $e^- + p \to \Lambda + \nu_e$ channel since Λ travels over a distance greater than the size of the beam before it decays. On the other hand, reconstruction of the vertex should indicate that, in the weak production of $\Delta(1232)$'s, the vertex is generally located within the beam since a Δ decays into $p+\pi$ through strong interactions. It is important to use the missing mass plot to make certain that the detected $(p\pi^-)$ pair does not come from the double-pion production process $e^-+p\to e^-+p+\pi^-+\pi^+$. In the Λ case, the mass is also well defined, so that the invariant mass of the $p+\pi^-$ provides a simple vetoing mechanism. We cannot have this vetoing procedure in the Δ case since Δ has a broad width.

For any potential experimental design, it is useful to allow for using the missing mass plot as a vetoing mechanism, as already suggested in Ref. 8. Note that the missing mass squared for the double pion production process $e^-+p\to e^-+p+\pi^-+\pi^+$ is given by

$$\begin{aligned} m_X^2 &\equiv -(p_e+p_p-p_\Delta)^2 \\ &= -(p_\pi+p_e')^2 \\ &= m_\pi^2+m_e^2+2(E_\pi E_e'-\mathbf{p}_\pi\cdot\mathbf{p}_e') \\ &> m_\pi^2 . \end{aligned}$$

It may be possible that the four-momentum of Δ^0 can be reconstructed well enough by a high-resolution measurement of the decaying products $p+\pi^-$. Specifically, we may rewrite the missing mass squared as follows:

$$\begin{aligned} m_X^2 = m_e^2+m_p^2+m_\Delta^2+2E_e m_p-2E_\Delta m_p \\ +2(E_e p_\Delta\cos\theta_\Delta-E_e E_\Delta) , \end{aligned}$$

so that

$$\begin{aligned} \delta m_X^3 = (2m_p-2E_\Delta+2p_\Delta\cos\theta_\Delta)\delta E_e \\ -[2m_p+2E_e-2E_e(E_\Delta/p_\Delta)\cos\theta_\Delta]\delta E_\Delta \\ -2E_e p_\Delta\sin\theta_\Delta\delta\theta_\Delta . \end{aligned}$$

To allow for a clear separation of genuine events from the pion associated produced events, we may assume

$$\begin{aligned} |[2m_p+2E_e-2E_e(E_\Delta/p_\Delta)\cos\theta_\Delta]\delta E_\Delta| < m_\pi^2/2 , \\ |2E_e p_\Delta\sin\theta_\Delta\delta\theta_\Delta| < m_\pi^2/2 , \end{aligned}$$

which allows us to determine the requirement on the energy and angular resolutions. Sample results for an electron beam of 4 GeV are presented in Table VII, where we have put an equal sign in the above equations in computing δE_Δ and $\delta\theta_\Delta$. It is clear that there is a fairly wide range in θ_Δ in which the predicted cross section (in Table III) is greater than 10^{-39} cm and yet it might be possible to achieve the resolution requirement recorded in Table VII. Since pions are lightest mesons that can be associatively produced, the same resolution requirement also allows for separation of genuine events from other associated produced background events, including more-than-three-pion events. Thus, the missing-mass plot can be used as an important vetoing mechanism in the proposed experiment.

To conclude this paper, we wish to mention the following.

TABLE VII. The resolution requirement for the electron capture reaction $e^-+p\to\Delta^0+\nu_e$ at $E_e=4$ GeV.

θ_Δ (deg)	T_Δ (MeV)	δE_Δ (MeV)	$\delta\theta_\Delta$ (deg)
4	3268	6.168	0.2310
8	3113	6.030	0.1203
12	2875	5.820	0.0857
16	2578	5.566	0.0702
20	2248	5.297	0.0627
24	1908	5.044	0.0594
28	1573	4.836	0.0590
32	1254	4.715	0.0610
36	955	4.768	0.0657
40	670	5.317	0.0749
44	336	17.10	0.1036
46*	kinematically forbidden		

(1) We have developed a simple formalism for investigating the spin $(\frac{1}{2}\to\frac{3}{2})$ transitions, such as $\Delta(1232)$ production via charge-changing weak currents, $e^-+p\to\Delta^0+\nu_e$ and $e^++p\to\Delta^{++}+\bar{\nu}_e$. The formalism can easily be generalized to describe transitions involving different initial and final spins, such as production of high spin baryon resonances in the few GeV range.

(2) In the absence of experimental information, we have adopted two different quark models, the MIT bag model[3] and the Dirac harmonic oscillator model, in the impulse approximation [Eqs. (14a) and (14b)], to calculate the various weak form factors and to make predictions on the differential cross sections. Although our present predictions are subject to corrections such as pion cloud effects, we are able to demonstrate that the predicted cross sections at the beam energy of about 4 GeV vary significantly with the model. Further, the predicted cross sections can be as large as 10^{-38} cm^2, which warrants serious consideration for actual experimentation.

(3) Although at present we know nothing about the weak transition form factors introduced in this paper, an experimental investigation of the reactions studied here will allow determination of some of the form factors, depending on the kinematic region that one chooses to work with. Since cross sections are often dominated by contributions due to the induced form factors, it is not more difficult to measure the induced form factors—such as the weak magnetism one—than the axial form factor. Comparing the extracted G_M with what we learn from $e+p\to e+\Delta^+$ constitutes a test of the CVC hypothesis in the few GeV range. It is clear that information on the other form factors will allow tests of other important aspects related to the standard $SU(3)_c\times SU(2)_w\times U(1)$ model of strong, electromagnetic, and weak interactions.

Noted added. We have been informed by Dr. Paul Singer (Technion, Haifa, Israel) of the fact that Eeg[9] has also adopted the 4×2 matrices for the $\Omega^-\to\Xi^-\gamma$ studies. Although our $S_i[Q_{ij}]$ agrees with Eeg's $\lambda_i[K_{ij}]$ up to an overall normalization factor, Eeg did not discuss how the most general forms for V_i, V_0, A_i, and A_0 can be introduced, nor did he describe how these form factors at large q^2 can be determined from a quark model.

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