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Quark model for nucleus-nucleus collisions

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We present a simple quark model for nucleus-nucleus collisions which employs the Glauber multiple scattering theory. The model is first used to fit a quark-quark scattering amplitude to high energy nucleon-nucleon elastic scattering data. This amplitude is then used to calculate high energy αp and $\alpha\alpha$ elastic scattering cross sections. The nucleus-nucleus calculations are performed using both a simple first-order optical phase shift function (essentially a $t\rho$ approximation) and an optical phase shift function calculated through fifth order. We also calculate the same cross sections using a conventional nuclear model together with Glauber theory. Comparisons of the results of the different theoretical approaches and the data are presented.

INTRODUCTION

During the past decade the study of nucleus-nucleus collisions at medium and high energies has become a useful means of investigating nuclei and nucleons. The traditional approaches to the description of the measurements have included those given in terms of nuclei composed of nucleons. It would be interesting to investigate the effects of the substructure of nucleons on such nucleus-nucleus collisions. As a step in this direction we present a simple model for nucleus-nucleus scattering in which the nucleons are composed of constituent entities (e.g., quarks). In Sec. I we describe the model. In Sec. II we use the model to fit high energy nucleon-nucleon scattering data. In Sec. III we apply our model to high energy alpha-proton scattering. In Sec. IV we apply our model to high energy alpha-alpha scattering. In Sec. V we calculate the alpha-proton and alpha-alpha cross sections using a conventional description of the ^{4}He nucleus. We then give some concluding remarks.

I. DESCRIPTION OF A SIMPLE QUARK MODEL FOR NUCLEUS-NUCLEUS COLLISIONS

To secure an estimate of the effects of the constituent substructure of nucleons on nucleus-nucleus collisions we consider a simple model in which nuclei of mass numbers A and B consist of nA and nB constituents, respectively, where n is a positive integer. Let the constituents be labeled Q. We will, in this model, assume that the constituents (assumed for definiteness to be quarks, but not restricted to be such) interact and that we can represent the collision between two such constituents (one in nucleus A and the other in nucleus B) by a scattering amplitude f_Q. As far as this amplitude is concerned, in this work all constituents involved in the collision are considered to be identical. The form of the amplitude is assumed to be

$$f_Q(q)=\frac{ik_Q}{4\pi}\sigma_Q(1-i\rho_Q)e^{-(1/2)a_Q q^2}\ , \tag{1.1}$$

where $\hbar k$ is the momentum of a constituent in the incident nucleus and $\hbar^2 q^2=-t$ is the squared momentum transfer. The total cross section (σ_Q), the slope parameter (a_Q), and the ratio of real to imaginary parts of the forward amplitude (ρ_Q) are initially taken to be free parameters which are constrained, however, to fit *nucleon-nucleon* (NN) measurements in the following manner.

We consider Glauber theory in order to calculate the elastic scattering amplitude for two systems (nucleons), each consisting of n constituents. The amplitude is given by[1,2]

$$F_{\rm NN}(q)=\frac{ik_{\rm N}}{2\pi}K(q)\int e^{i\mathbf{q}\cdot\mathbf{b}}d^2b\,\langle\psi_{\rm N}(\mathbf{r}_1,\ldots,\mathbf{r}_n)\psi_{\rm N}(\mathbf{r}'_1,\ldots,\mathbf{r}'_n)\,|\times\left\{1-\prod_{i=1}^{n}\prod_{j=1}^{n}[1-\Gamma_{ij}(\mathbf{b}-\mathbf{s}_i+\mathbf{s}'_j)]\right\}|\,\psi_{\rm N}(\mathbf{r}_1,\ldots,\mathbf{r}_n)\psi_{\rm N}(\mathbf{r}'_1,\ldots,\mathbf{r}'_n)\rangle\ , \tag{1.2}$$

where $\hbar k_{\rm N}$ is the incident nucleon momentum and $\psi_{\rm N}$ is the ground state wave function of the nucleon. For simplicity we assume a single particle density given by

$$\rho(r)=(\pi R_{\rm N}^2)^{-3/2}e^{-r^2/R_{\rm N}^2}\ , \tag{1.3}$$

with

$$R_{\rm N}^2=\frac{\frac{2}{3}(\langle r_{\rm N}^2\rangle-\langle r_Q^2\rangle)}{1-n^{-1}}\ , \tag{1.4}$$

where $1-n^{-1}$ is a center of mass effect, $\langle r_Q^2\rangle$ is the mean square radius of the dressed quarks, and $\langle r_N^2 \rangle$ is the mean square radius of the nucleon. (We do not distinguish between neutron and proton here.) The factor $K(q)$ is the center of mass correlation function

$$K(q)=e^{R_N^2 q^2/2n} . \quad (1.5)$$

The functions Γ_{ij} can be obtained in terms of the amplitudes f_Q for scattering between two constituents by the relation

$$\Gamma_{ij}(\mathbf{b})=(2\pi i k_Q)^{-1}\int e^{-i\mathbf{q}\cdot\mathbf{b}} f_Q(\mathbf{q}) d^2q , \quad (1.6)$$

$$=\frac{\sigma_Q(1-i\rho_Q)}{4\pi a_Q} e^{-b^2/2a_Q} . \quad (1.7)$$

When ρ, K, and Γ_{ij} of Eqs. (1.3), (1.5), and (1.7) are substituted into Eq. (1.2), the integration may be performed analytically for a given value of n. The parameters σ_Q, ρ_Q, and a_Q are then adjusted to fit the NN (i.e., the proton-proton) total cross section data and elastic scattering data at small momentum transfers by means of $F_{NN}(q)$ as given by Eq. (1.2). (It seems not unreasonable to fit the NN data for small momentum transfers only, since we know from Glauber theory that the multiple scattering effects in nucleus-nucleus scattering at a given t depend mainly on the basic constituent amplitudes at approximately t/μ^2, where μ is the order of multiple scattering and that at larger momentum transfers it is the higher order multiple scattering effects which dominate.)

In addition to the constraints on the quark-quark amplitude imposed by the NN elastic scattering and total cross section measurements, unitarity imposes a further constraint, namely

$$\sigma_Q(1+\rho_Q^2)\le 8\pi a_Q , \quad (1.8)$$

and therefore

$$a_Q \ge \sigma_Q/8\pi . \quad (1.9)$$

Assuming that shadowing effects tend to diminish the NN total cross section relative to the sum of the quark-quark cross sections, we can expect $n^2\sigma_Q > \sigma_{NN}$, so that

$$a_Q > \sigma_{NN}/8\pi n^2 . \quad (1.10)$$

For high energy NN collisions (e.g., $\sqrt{s}\approx 30-50$ GeV), $\sigma_{NN}\approx 40$ mb. If we take $n=3$, unitarity would then give the additional requirement that $a_Q \gtrsim 0.018$ fm^2.

Next we use Glauber theory to calculate the elastic scattering amplitude for two systems containing nA and nB particles, respectively. Although the resulting expression can, in principle, be obtained analytically and thereby numerically, in practice this is not feasible since for nA and $nB \gtrsim 6$ the calculation of the orbits, i.e., the different types of multiple scattering contributions, becomes very tedious and lengthy. Consequently we will use an optical phase shift function method[3] and write the amplitude for nucleus-nucleus scattering as

$$F_{AB}(q)=(ik/2\pi)\int e^{i\mathbf{q}\cdot\mathbf{b}}(1-e^{i\bar{\chi}_{\rm opt}(b)})d^2b , \quad (1.11)$$

where $\bar{\chi}_{\rm opt}(b)$ is a phase shift function for which we have developed a series expansion.[3] For given nA and nB, the input for F_{AB} consists of the constituent scattering parameters σ_Q, ρ_Q, a_Q (which we will have obtained from the fit to the pp data) and the rms radii of the nuclei A and B which are obtained from nuclear measurements. So once the three constituent scattering parameters are obtained from the NN data, the nucleus-nucleus calculations contain no free parameters.

II. NUCLEON-NUCLEON COLLISIONS

The use of Glauber theory with a quark model to describe NN collisions has a relatively long history. The early works were published almost 20 years ago.[4–6]

In the present work we take n, the number of constituents in the nucleon, to be 3, in which case the NN scattering amplitude contains multiple scattering terms up through ninth-order multiple collisions. There are 25 different types ("orbits") of such multiple collisions, each type being possible in between one (for the ninth-order collision) and 36 ways (for some of the orbits of the third-, fourth-, fifth-, and sixth-order collisions). We calculate all 25 different contributions to the NN amplitude analytically.

With $n=3$, R_N^2 in Eq. (1.4) becomes

$$R_N^2=\langle r_N^2\rangle-\langle r_Q^2\rangle . \quad (2.1)$$

For $\langle r_N^2\rangle$ we use the measured[7] value $\langle r_p^2\rangle^{1/2}=0.81$ fm. For the mean square radius of the dressed quark, we use the estimate[8] $\langle r_Q^2\rangle\approx\frac{1}{5}\langle r_N^2\rangle$. (We note here that the results of our calculations will be rather insensitive to the precise value of $\langle r_Q^2\rangle$ for $\langle r_Q^2\rangle \ll \langle r_N^2\rangle$. Even the choice $\langle r_Q^2\rangle=0$ will not alter our results significantly.) The value of R_N^2 is thereby fixed at 0.525 fm^2.

We fit the pp data at $\sqrt{s}=30.7$ GeV and $\sqrt{s}=44.7$ GeV. At 30.7 GeV the values $\sigma_Q=5.352$ mb, $\rho_Q=0.045$, and $a_Q=0.083$ fm^2 lead to a theoretical NN amplitude with a total cross section of $\sigma_N=40.14$ mb, a ratio of real to imaginary part of the forward NN amplitude of $\rho_N=0.037$, and a NN slope parameter of $b_N=12.2$ (GeV/c)$^{-2}$. This agrees well with the measured[9,10] values $\sigma_N=40.14\pm0.10$ mb, $\rho_N=0.037\pm0.006$, and $b_N=12.2\pm0.2$ (GeV/c)$^{-2}$. This agreement, of course, is no great feat since we had three free parameters to adjust. Our aim here has been simply to fix the quark parameters so that the resulting theoretical NN scattering amplitude agrees with the measurements at small $|t|$. We see that this can be done. At 44.7 GeV the values $\sigma_Q=5.555$ mb, $\rho_Q=0.075$, and $a_Q=0.105$ fm^2 lead to a theoretical NN amplitude with $\sigma_N=41.79$ mb, $\rho_N=0.062$, and $b_N=12.8$ (GeV/c)$^{-2}$, in good agreement with the measured[9,10] values $\sigma_N=41.79\pm0.16$ mb, $\rho_N=0.062\pm0.011$, and $b_N=12.8\pm0.2$ (GeV/c)$^{-2}$. At both energies the unitarity requirement (1.8) is satisfied. We note that the quark-quark total cross section is somewhat larger than $\sigma_N/9$, a result which is due to the shadowing corrections that arise from Glauber theory. The ratio ρ_Q is slightly larger than ρ_N, and the quark slope parameter is approximately five to six times smaller than the NN slope parameter.

In Fig. 1 we show our fits to $d\sigma/dt$ for NN scattering at $\sqrt{s}=30.7$ GeV and $\sqrt{s}=44.7$ GeV over the range $0\le -t\le 0.1$ (GeV/c)^2. We note that the resulting calcu-

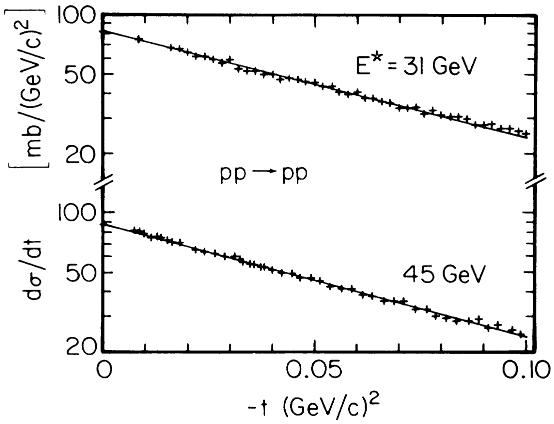

FIG. 1. Fit of quark-model NN amplitude to the pp elastic scattering data (Ref. 11) (crosses).

lated NN elastic scattering differential cross sections $d\sigma/dt$ are in excellent agreement with the measurements[11] over the range $0 \leq -t \leq 0.1$ (GeV/c)^2. But for $-t \gtrsim 0.1$ (GeV/c)^2 the theoretical results do not agree with the data,[11] differing significantly for $-t \gtrsim 0.2$ (GeV/c)^2, and even qualitatively for $-t \gtrsim 0.5$ (GeV/c)^2.

III. ALPHA-PROTON COLLISIONS

In this section we use the theoretical NN amplitude obtained in Sec. II for NN collisions at $\sqrt{s_{NN}} = 44.7$ GeV to calculate the αp cross section. This NN energy corresponds to an αp energy of $\sqrt{s_{\alpha p}} = 89$ GeV for which measurements exist.[12–14]

For αp collisions in our quark model the full Glauber multiple scattering series in the form of Eq. (1.2) will contain 36 orders of multiple collisions, each order in general containing a number of different types of such collisions, and each type being possible in a number of ways. To avoid such a lengthy and time consuming calculation we use an alternative approach, namely that of the optical phase shift function $\overline{\chi}_{\rm opt}(b)$, and calculate the scattering amplitude by means of Eq. (1.11).

We assume a Gaussian model for the 12 quarks in ^{4}He in which the quarks are allowed to wander throughout the nucleus,

$$\rho_\alpha(r) = (\pi R_\alpha^2)^{-3/2} e^{-r^2/R_\alpha^2} , \quad (3.1)$$

with

$$R_\alpha^2 = \frac{\frac{2}{3}[\langle r_\alpha^2 \rangle - \langle r_Q^2 \rangle]}{1 - n_\alpha^{-1}} . \quad (3.2)$$

The simplest approximation for $\overline{\chi}_{\rm opt}$ is the "tp" approximation or what is essentially the first term $\overline{\chi}_1$ in the expansion of $\overline{\chi}_{\rm opt}$ given in Ref. 3. This approximation is most commonly used in nucleus-nucleus calculations. Here it is given by

$$i\overline{\chi}_1 = -18(\sigma_Q/\pi R^2)(1 - i\rho_Q) e^{-b^2/R^2} , \quad (3.3)$$

where

$$R^2 = R_N^2(1 - n_N^{-1}) + R_\alpha^2(1 - n_\alpha^{-1}) + 2a_Q , \quad (3.4)$$

$$= \tfrac{2}{3} R_N^2 + \tfrac{11}{12} R_\alpha^2 + 2a_Q , \quad (3.5)$$

in which we have taken the number of constituents in the nucleon and α particle, n_N and n_α, to be 3 and 12, respectively. (The tp approximation sometimes includes the additional approximation $a_Q = 0$.) We have used $R_\alpha^2 = 1.945$ fm^2, corresponding to the measured value[15] of 1.675 fm for the rms radius of ^{4}He. The parameters R_N^2, σ_Q, ρ_Q, and a_Q have been fixed in Sec. II by measurements on the nucleon. In Fig. 2 we present the results of our calculation of $d\sigma/dt$ for αp collisions (dotted curve) together with the measurements.

It is known[3] that the first order term $\overline{\chi}_1$ for the optical phase shift does not lead to an accurate approximation to the full Glauber multiple scattering series in nucleus-nucleus collisions. For example, the calculation[3] for α-^{12}C scattering (which is a system of 4 plus 12 nucleons) leads to an inaccuracy which increases from $\sim 7\%$ in the forward direction to factors of more than 2 at larger angles. Similar inaccuracies should be expected in the present calculations involving 3 plus 12 quarks, and it is necessary therefore to calculate high-order contributions to the phase shift function $\overline{\chi}_{\rm opt}(b)$. The series developed[3] for $\overline{\chi}_{\rm opt}$ converges quite well for the present calculation. The third-order result for $\overline{\chi}_{\rm opt}$ leads to calculated differential cross sections that are within 1% of the exact results

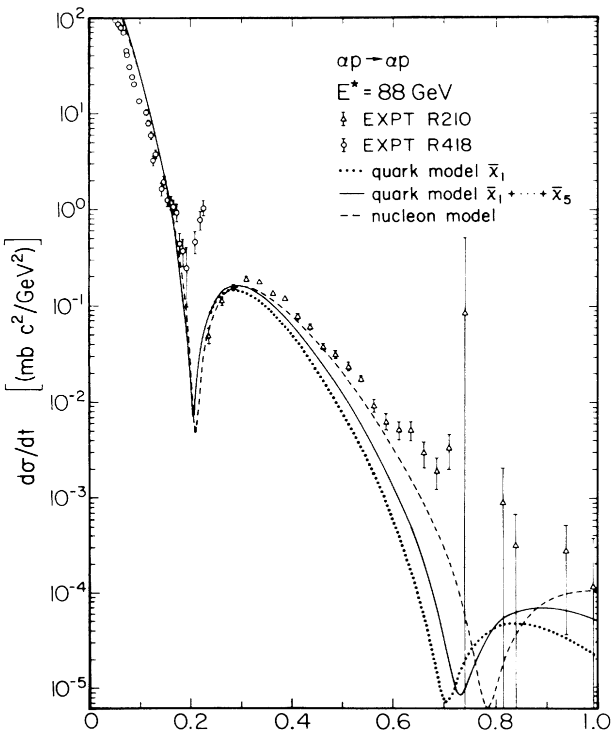

FIG. 2. Alpha-proton elastic scattering at $\sqrt{s} = 88$ GeV. The data are from Refs. 12 and 13. The dotted curve is the quark model result using only the first-order optical phase shift function $\overline{\chi}_1$. The solid curve is the quark model result using the first-order through fifth-order phase shift functions, $\overline{\chi}_1, \ldots, \overline{\chi}_5$. The dashed curve is the conventional nucleon model result.

at the minima, which are the most sensitive points. The fourth- and fifth-order calculations are even more accurate. In Fig. 2 we present the results of the fifth-order calculation (solid curve). Beyond the secondary maximum, $-t \gtrsim 0.29$ (GeV/c)^2, the results differ significantly from the calculation using only $\overline{\chi}_1$, being as much as a factor of 5 different. This illustrates the danger in using the simple first-order or $t\rho$ approximation for $\overline{\chi}_{\rm opt}$.

Although the two measurements[12,13] do not overlap in t, some of the measurements in the range $0.2 \le -t \le 0.3$ (GeV/c)^2 seem, in the opinion of this author, likely to be incorrect. (The errors shown are statistical only. Overall scale errors are discussed in Refs. 12 and 13.) The theoretical results of this very simple model are, apart from the depths of the minima, in rough qualitative agreement with the measurements. In the columns labeled "Quark model" and "Expt." in Table I we present a comparison of the theoretical and experimental total cross section, positions of the minimum and secondary maximum, and the differential cross section at the maximum. For those data which are obtained only from plots, the values are estimated from those plots. The agreement between theory and experiment is good.

IV. ALPHA-ALPHA COLLISIONS

In this section we use the theoretical NN amplitude obtained in Sec. II for NN collisions at $\sqrt{s_{\rm NN}} = 30.7$ GeV to calculate the $\alpha\alpha$ cross section. This NN energy corresponds to an $\alpha\alpha$ energy of $\sqrt{s_{\alpha\alpha}} = 123$ GeV. Measurements exist[12–14,16,17] at nearby $\alpha\alpha$ energies of 125 and 126 GeV.

For $\alpha\alpha$ collisions in our quark model (12 plus 12 quarks) the full Glauber multiple scattering series in the form of Eq. (1.2) will contain 144 orders of multiple collisions, each order in general containing many different types of such collisions, and each type being possible in a number of ways. We again avoid the corresponding lengthy and time consuming calculation by using the optical phase shift approach and calculate the scattering amplitude by means of Eq. (1.11).

The simplest approximation for $\overline{\chi}_{\rm opt}$ is the $t\rho$ approximation or the first term $\overline{\chi}_1$ in the expansion of $\overline{\chi}_{\rm opt}$ given in Ref. 3. It is given by

$$i\overline{\chi}_1 = -72(\sigma_Q/\pi R^2)(1-i\rho_Q)e^{-b^2/R^2} , \tag{4.1}$$

where

$$R^2 = \tfrac{11}{6} R_\alpha^2 + 2a_Q . \tag{4.2}$$

The parameters have all been fixed in Secs. II and III by measurements on ^{4}He and on the nucleon. In Fig. 3 we present the results of our calculation of $d\sigma/dt$ for $\alpha\alpha$ collisions (dotted curve) together with the measurements.

The warning of Sec. III regarding the inaccuracy of $t\rho$ type of approximations is even stronger here. For example, the calculation[3] for ^{12}C-^{12}C scattering (which is also a system of 12 plus 12 "particles") leads to an inaccuracy which increases from $\sim 6\%$ in the forward direction to factors of more than 5 at larger angles. Similar inaccuracies should be expected in the present calculation involving 12 plus 12 quarks, and it is therefore necessary to calculate higher-order contributions to the phase shift function $\overline{\chi}_{\rm opt}(b)$. The series developed[3] for $\overline{\chi}_{\rm opt}$ converges

TABLE I. Comparison of theoretical and experimental total cross sections, positions of minima and maxima, and differential cross sections at maxima for αp collisions at $\sqrt{s} = 88$ GeV and for $\alpha\alpha$ collisions at $\sqrt{s} = 125$ GeV.

	αp Quark model	αp Nucleon model	αp Expt.	$\alpha\alpha$ Quark model	$\alpha\alpha$ Nucleon model	$\alpha\alpha$ Expt.
$\sigma_{\rm tot}$ (mb)	147	141	130 ± 20[a]	408	383	295 ± 40[a] 250 ± 50[a] 280 ± 70[b] 315 ± 18[e]
$-t_{\rm min,1}$ (GeV/c)^2	0.21	0.21	0.20 ± 0.02[a]	0.088	0.093	0.10 ± 0.01[a] 0.098 ± 0.002[b]
$-t_{\rm max,1}$ (GeV/c)^2	0.29	0.30	0.31 ± 0.02[c]	0.13	0.14	0.12 ± 0.02[a] 0.14 ± 0.02[b]
$\sigma(t_{\rm max,1})$ (mb c^2/GeV^2)	0.17	0.16	0.19 ± 0.01[c]	9.2	6.7	9.2 ± 0.2[a] 7.4 ± 0.3[b]
$-t_{\rm min,2}$ (GeV/c)^2				0.32	0.35	0.38 ± 0.02[a] 0.39 ± 0.03[c,d]
$-t_{\rm max,2}$ (GeV/c)^2				0.41	0.46	0.44 ± 0.03[a] 0.49 ± 0.02[c,d]
$\sigma(t_{\rm max,2})$ (mb c^2/GeV^2)				0.0327	0.0176	0.068 ± 0.012[a] 0.0076 ± 0.0014[c,d]

[a]Reference 12.
[b]Reference 16.
[c]Reference 13.
[d]Reference 14.
[e]Reference 17.

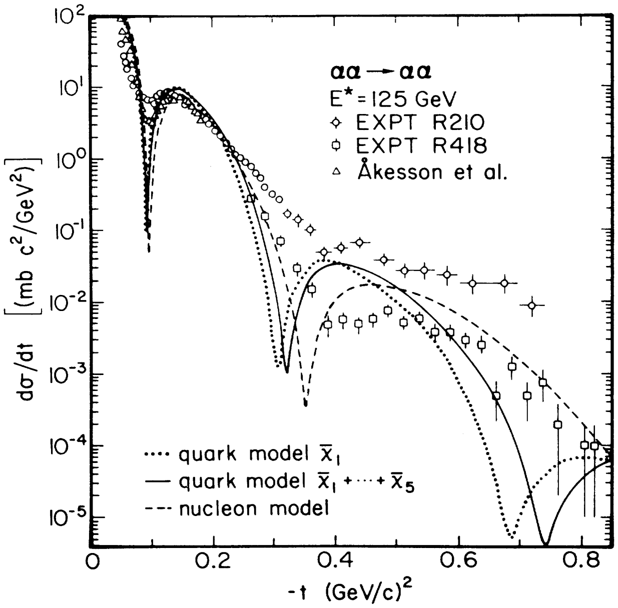

FIG. 3. Alpha-alpha elastic scattering at $\sqrt{s}=125$ GeV. The data are from Refs. 12, 13, and 16. The dotted curve is the quark model result using only the first-order optical phase shift function $\bar{\chi}_1$. The solid curve is the quark model result using the first-order through fifth-order phase shift functions $\bar{\chi}_1, \ldots, \bar{\chi}_5$. The dashed curve is the conventional nucleon model result.

quite well for the present calculation. The third order result for $\bar{\chi}_{\text{opt}}$ leads to calculated differential cross sections that are within 1.5% or better of the exact results at the minima. The fourth- and fifth-order results are even more accurate. In Fig. 2 we present the results of the fifth-order calculation (solid curve). Beyond the first minimum, $-t \gtrsim 0.09$ $(\text{GeV}/c)^2$, the results differ significantly from the calculation using only $\bar{\chi}_1$, being as much as an order of magnitude different. Again this illustrates the danger in using the simple first-order or $t\rho$ approximation for $\bar{\chi}_{\text{opt}}$.

The $\alpha\alpha$ measurements[12,13,16] are not consistent with one another. (The overall scale errors are discussed in Refs. 12, 13, and 16.) The theoretical results in this very simple model are, apart from the depths of the minima, in rough qualitative agreement with the measurements. In the columns labeled "Quark model" and "Expt." in Table I we present a comparison of the theoretical and experimental total cross section, positions of the minima and maxima, and the differential cross sections at the maxima. For those data which are obtained only from plots, the values are estimated from those plots. The agreement between theory and experiment is only qualitative.

V. CONVENTIONAL NUCLEAR MODEL

In this section we analyze αp and $\alpha\alpha$ elastic scattering in terms of the conventional Glauber theory in which the ^{4}He nucleus consists of four nucleons. The NN scattering amplitude used is of the form

$$f_N(q)=(ik_N/4\pi)\sigma_N(1-i\rho_N)e^{-(1/2)b_N q^2}, \qquad (5.1)$$

where σ_N, ρ_N, and b_N are taken from the NN measurements[9,10] and are quoted in Sec. II. Thus the NN amplitude used as input in this calculation and the NN amplitude obtained in Sec. II from the quark model both fit the NN data for $-t \leq 0.1$ $(\text{GeV}/c)^2$ very well and lead to identical values of σ_N, ρ_N, and b_N, namely the measured values.

The αp cross section is easily calculated, containing only four terms, one for each order of multiple scattering through quadruple scattering. The results are shown in Table I in the column labeled "nucleon model" and in Fig. 2 (dashed curve). The conventional Glauber calculation is in better agreement with the data than the quark model calculation, although it is somewhat too high near the forward direction and somewhat too low beyond the secondary maximum. The major effect of the quark model in αp collisions is to make the diffraction pattern narrower without appreciably changing the magnitude of the intensity at the maximum.

The $\alpha\alpha$ cross section is more difficult to calculate. There are 16 orders of multiple scattering and 191 different orbits (i.e., different types of multiple collisions). The full scattering amplitude is published in detail elsewhere.[18] The results are shown in Table I in the column labeled "Nucleon model" and in Fig. 3 (dashed curve). The conventional Glauber calculation again is in better agreement with the data than the quark model calculation, although it is somewhat too high near the forward direction and the depths of the two minima are too great. The major effect of the quark model in $\alpha\alpha$ collisions is to make the diffraction pattern significantly narrower and to appreciably increase the intensities at the maxima.

CONCLUDING REMARKS

We have presented a simple quark model for nucleus-nucleus (and nucleon-nucleus) collisions which uses Glauber multiple scattering theory. The quark parameters were fixed by fitting the model to NN elastic scattering measurements at small $|t|$. The applications to αp and $\alpha\alpha$ collisions then had no free parameters. We showed that the first-order optical phase shift $\bar{\chi}_1$, or the $t\rho$ approximation, is inaccurate in αp (12 plus 3 quarks) and $\alpha\alpha$ (12 plus 12 quarks) collisions, and we presented fifth-order results for $\bar{\chi}_{\text{opt}}$ in these cases. For comparison, conventional Glauber theory and nuclear physics was also applied to αp and $\alpha\alpha$ collisions. The effects of including the quark degrees of freedom were to make the diffraction pattern narrower and, in the $\alpha\alpha$ case, to increase the scattered intensities at the maxima. The quark model calculations yielded no improvement over the conventional nuclear physics calculations. Indeed, the latter were, on the whole, more satisfactory, although discrepancies between different sets of measurements make the comparisons difficult. On the other hand, we can say that our simple quark model, with essentially no free parameters, yielded differential cross sections for αp and $\alpha\alpha$ elastic scattering which were in rough qualitative agreement with experiment and may be taken as a starting point for more elaborate and sophisticated calculations.

There are, of course, a number of improvements that can, in principle, be made in our quark model. They involve, for example, more sophisticated wave functions for the quarks in the nuclei which take into account clustering aspects of the quarks, and more sophisticated quark-quark interactions which lead to fits of NN scattering out to larger momentum transfers. In the fully microscopic approach that we use, such modifications are nontrivial. Nevertheless, from our model one cannot make any definitive conclusions regarding the necessity (or lack thereof) of incorporating the quark degrees of freedom in nucleus-nucleus collisions unless the wave functions used for the quark distribution are more realistic.

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