# Theory of compound-nucleus reactions: Gaussian versus non-Gaussian statistics of its parameters

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(Received 8 August 1986)

A theory of compound-nucleus reactions is formulated which is valid for most of the physical situations—from the domain of isolated to the domain of overlapping resonances. We allow for a more general than Gaussian statistics of the resonance decay amplitudes. The energy spectrum is parametrized in terms of a variable $\sigma_p$ that measures its stiffness. The following results are obtained: We formulate a condition under which Hauser-Feshbach expressions emerge. They include an elastic enhancement factor $W$. This factor essentially depends on the fourth moment of the decay amplitudes and the stiffness parameter $\sigma_p$. Within our model, $\sigma_p$ is given by requiring Wigner's level repulsion. If, in addition, one specializes to the Gaussian statistics of the decay amplitudes, the present results yield the analytical solution to the earlier Monte Carlo simulations. In the same limit, we find agreement with experimental results on $W$ that are available for the regimes of well-isolated and of strongly overlapping resonances.

## I. INTRODUCTION

In recent years, there was a discussion[1–4] in the published literature on the question whether the decay amplitudes of compound-nucleus resonances—as they are observed, e.g., in low energy nucleon scattering—have a Gaussian probability distribution. This discussion prompted the investigation presented here.

In 1975 Agassi *et al.*[5] formulated the theory of statistical nuclear reactions under the assumptions that the resonance decay amplitudes are Gaussian variables and that the energy spectrum of the resonances forms a "picket fence," i.e., the energies have a constant distance $D$ without fluctuations. This model takes care of the experimentally observed "stiffness" of nuclear spectra, but its simplicity allows one to handle the regime of overlapping resonances only. Reformulating the diagrammatic expansion technique of the authors of Ref. 5, we generalize the older results in two respects:

(i) A "disordered picket fence" permits us to introduce fluctuations of the energy levels; the importance of the fluctuations is controlled by a parameter $\sigma_p$ that can be considered as characterizing the stiffness of the spectrum.

(ii) The joint probability distribution of the resonance decay amplitudes is not restricted to be Gaussian.

The first generalization provides a unified theory that essentially covers the whole range from the regime of isolated resonances to the regime of strongly overlapping resonances. The second point allows us to work out the observables—for the case of overlapping resonances—that contain information on the statistics of the decay amplitudes.

Currently, much progress is being achieved in relating the theory of statistical reactions to the theory of statistical spectra. This is done[6,7] by treating that part of the Hamiltonian that generates the compound states as a random matrix[8] taken from the Gaussian orthogonal ensemble (GOE). Progress was also achieved in deducing a probability distribution for statistical $S$ matrices essentially from a requirement of minimum information.[9] Both approaches are intimately related.[10] In the present work we try to specify as little as possible about the statistics of the parameters that characterize the compound nuclear levels. This helps to find the observables that are sensitive to the Gaussian assumption of the above approaches.

The material of the present paper is organized in the following way: In Sec. II we state our statistical assumptions. In Sec. III the expansion technique is explained and the average $S$ matrix calculated. Correlation functions between two $S$-matrix elements, especially average compound-nucleus cross sections, are obtained in Sec. IV. The remaining sections are devoted to a thorough discussion and to various specializations of the results of Secs. III and IV. In Sec. V the case of isolated resonances implying a small number $N$ of open channels is considered. In Sec. VI we state the condition under which Hauser-Feshbach expressions are obtained. This is the case, if $N$ is sufficiently large. For compound-elastic scattering the Hauser-Feshbach expressions include an elastic enhancement factor $W$. This quantity is essentially the only one to be sensitive to the statistics of the decay amplitudes and the spectral fluctuations. It is discussed in Sec. VII. We then specialize, in Sec. VIII, the general results to the case of Gaussian statistics and equivalent channels. The parameter $\sigma_p$ of the spectral stiffness is determined in Sec. IX by requiring level repulsion. This means that the probability of finding any two levels at the same energy is zero. Except for the transmission coefficients no free parameter is then left in the theory. We compare the resulting enhancement factor $W$ with Monte Carlo calculations by Hofmann *et al.*[11] and with experiments. The results are summarized in Sec. X.

 

## II. SPECIFICATION OF THE STATISTICAL ASSUMPTIONS

We start out from the $K$-matrix representation of the scattering matrix; see, e.g., Sec. 8.8 of Ref. 12, which is

$$S=e^{i\delta}(1+iK)(1-iK)^{-1}e^{i\delta} . \tag{2.1}$$

Here, $\exp(i\delta)$ is a diagonal and unitary matrix of potential scattering phases. It will be omitted in the formalism that follows. The quantity $K$ is a matrix in the open channels, whose elements are

$$K_{ab}=\pi\langle a \mid V(H-E)^{-1}V \mid b\rangle . \tag{2.2}$$

Here, $H$ is the Hamiltonian that generates the bound states of the system. We designate the bound states $|\lambda\rangle$, $\lambda=1,\ldots,\Lambda$, by Greek letters. The eigenvalues of $H$ are $E_\lambda$. The $E_\lambda$ may very well be above the first threshold for particle emission, i.e., the spectrum of $H$ contains "bound states embedded in the continuum" (BSEC). The details are described in Ref. 12. The residual interaction $V$ couples the bound states to the channel wave functions $|a\rangle, |b\rangle,\ldots,$ which we denote by Latin letters. There shall be $N$ open channels at the energy $E$ of the system. The coupling between bound and continuum states gives rise to resonances, i.e., to the formation of a long lived compound system out of the channels [note that the ansatz of Eqs. (2.1) and (2.2) implies that a reaction leading from channel $a$ to a different channel $b$ can take place only via intermediate resonance states: direct channel-channel coupling has been neglected; see Sec. 4.2 of Ref. 12]. The $K$ matrix depends on the energy $E$ explicitly and implicitly through the channel wave functions that have to be taken at $E$. Except close to thresholds, this implicit energy dependence should, however, be weak (cf. Secs. 9.3 and 9.15 of Ref. 12) and will be neglected in the sequel.

The matrix $K$ is Hermitian, since $H$ and $V$ are Hermitian. This entails unitarity of $S$. If $H$ and $V$ can be chosen real, i.e., if time reversal invariance holds, $K$ is symmetric, which entails symmetry of $S$. This will be assumed throughout the present work.

Let us introduce the rectangular coupling matrix $\gamma$ with elements

$$\gamma_{\lambda e}=\sqrt{\pi}\langle\lambda V \mid e\rangle,\quad \lambda=1,\ldots,\Lambda,\quad e=1,\ldots,N \tag{2.3}$$

referring to an arbitrary basis of the bound state space and the physical space of open channels. One has

$$K=\gamma^T(H-E)^{-1}\gamma , \tag{2.4}$$

and one easily shows that $S$ can be rewritten

$$\begin{aligned} S(E)&=1+2iK(1-iK)^{-1}\\ &=1-2i\gamma^T(E-H+i\gamma\gamma^T)^{-1}\gamma . \end{aligned} \tag{2.5}$$

It is now tempting to consider the matrix $H$—given in the arbitrary but fixed basis $|\lambda\rangle$—as the statistical variable of the problem. In the absence of any information other than that $H$ should be real and symmetric, one would most naturally assume it to be an element of the GOE; cf. Ref. 8. Average $S$-matrix elements and correlation functions would then be calculated by performing ensemble averages over the GOE. This has indeed been done recently.[7]

As pointed out in the Introduction, the route taken here is different. We want to keep the statistical assumptions as flexible as possible and allow for non-Gaussian statistical variables. The price that we have to pay in comparison with Ref. 7 is twofold: (i) We shall be able to solve the problem only to first order in $1/N$, the inverse number of open channels, or to first order in the absorption in all channels. This covers, however, almost all of the experimentally occurring cases. (ii) We shall have to resort to a hybrid model in the sense described now. Let the basis $|\lambda\rangle$ henceforth be the one that diagonalizes $H$. Then, $K$ reads

$$K_{ab}(E)=\sum_\lambda \frac{\gamma_{\lambda a}\gamma_{\lambda b}}{E_\lambda-E} . \tag{2.6}$$

The quantities $\gamma_{\lambda c}$ and $E_\lambda$ will be treated as the statistical parameters of the present formalism. Average $S$-matrix elements and correlation functions will be calculated by averaging over the statistical ensemble of the multidimensional parameter $(\gamma_{\lambda c},E_\lambda)$. This ensemble is subject to the following assumptions. (i) The set of energies $(E_\lambda,$ $\lambda=1,\ldots,\Lambda)$ is statistically independent of the set of decay amplitudes $(\gamma_{\lambda e},\ \lambda=1,\ldots,\Lambda,\ e=1,\ldots,N)$, i.e., the joint probability distribution factorizes. (ii) The set $(\gamma_{\lambda e},$ $e=1,\ldots,N)$ of decay amplitudes of a given level $\lambda$ is statistically independent of the amplitudes $(\gamma_{\mu e},$ $e=1,\ldots,N)$ of any other level $\mu\neq\lambda$. Moreover, the distribution $W_\lambda$ of the amplitudes $(\gamma_{\lambda e},\ e=1,\ldots,N)$ does not depend on $\lambda$,

$$W_\lambda(\gamma_{\lambda 1},\ldots,\gamma_{\lambda N})\equiv w(\gamma_{\lambda 1},\ldots,\gamma_{\lambda N}),\quad \lambda=1,\ldots,\Lambda . \tag{2.7}$$

Equation (2.7) guarantees statistical stationarity with respect to the energy $E$; see the discussion in Ref. 5. Since the average $\overline{\gamma_{\lambda e}^n}$ is then independent of $\lambda$, the notation

$$\overline{\gamma_{\lambda e}^n}=\overline{\gamma_e^n}\equiv\langle\gamma_e^n\rangle \tag{2.8}$$

is used.

It has become customary to use horizontal bars for ensemble averages and angular brackets for energy averages. For the convenience of printing, we henceforth use both symbols without any distinction for the ensemble averages. This is the only type of average considered in the present context.

A third assumption is put in for convenience; it suppresses direct reactions, however, without any loss of generality, cf. Sec. III: (iii) The amplitudes $\gamma_{\lambda a}$ and $\gamma_{\lambda b}$ are statistically independent for any pair of channels $a\neq b$, i.e.,

$$w(\gamma_{\lambda 1},\ldots,\gamma_{\lambda N})=\prod_{e=1}^{N} w_e(\gamma_{\lambda e}) . \tag{2.9}$$

The problem will be tractable only if one requires that (iv) the distributions $w_e$ are even functions so that all odd moments vanish,

$$\overline{\gamma_{\lambda e}^{2n+1}}=0,\quad n\geq 0 . \tag{2.10}$$

Guided by the experimentally observed stiffness of nu-

clear spectra, we introduce a disordered picket fence model for the eigenvalues $E_\lambda$ as follows: (v) Any two $E_\lambda, E_\mu$ are statistically independent. The probability distribution $p_\lambda$ of $E_\lambda$ is supposed to be given in terms of a distribution $p$, which is independent of $\lambda$, as

$$p_\lambda(E_\lambda)=p(\epsilon_\lambda-E_\lambda) . \tag{2.11}$$

The function $p$ shall be even and have its maximum at zero. The average of $E_\lambda$ is then equal to $\epsilon_\lambda$, and the $\epsilon_\lambda$ shall form a regular lattice,

$$\epsilon_\lambda-\epsilon_\mu=(\lambda-\mu)D . \tag{2.12}$$

The quantity $D$ is the mean compound nuclear level distance. The energy $E$ shall, of course, be within the interval $[\epsilon_1,\epsilon_\Lambda]$ sufficiently far from the endpoints so that edge effects are irrelevant.

A comment about the stiffness of nuclear spectra seems appropriate. It can be characterized by the distribution of the distance between an arbitrary level and its $n$th neighbor. At any rate, the mean value has to be $nD$. How about the variance $\sigma(n)$? It will grow as the square root of $n$, precisely $\sigma(n)=\sqrt{n}D$, if $p_\lambda(E_\lambda)$ is a constant independent of $\lambda$. The statistics introduced above will yield this behavior, if the variance $\sigma_p$ of the distribution $p$ is taken to be sufficiently large compared to $D$. If, on the contrary, one takes $\sigma_p \ll D$ so that the ordering of the levels is with a high probability identical to the sequence of their average values $\epsilon_\lambda$, the present statistics yields $\sigma(n)=\sqrt{2}\sigma_p$, independent of $n$. Hence, the parameter $D/\sigma_p$ measures the spectral stiffness. Now in Fig. 1, experimental values of $\sigma(n)$ are reproduced. They were determined[13] from a set of $\frac{1}{2}^+$ resonances in the system $^{56}\mathrm{Fe}+\mathrm{p}$ at proton energies between 3 and 4 MeV. The $\sigma(n)$ start with $\sigma(1)\approx 0.5D$ and slowly grow with increasing $n$, but fall far below $\sqrt{n}D$, indicated by the curve in Fig. 1. This behavior is what is meant by calling the spectrum "stiff." Figure 1 would roughly require $\sigma_p \approx D$. In the spirit of the present approach, we shall specify neither the distribution $p$ nor the stiffness parameter $\sigma_p$, but rather try to find the dependence of the results on them.

The statistical properties of the $E_\lambda$ enter into the following formalism via two different functions: (a) The probability of finding a level at energy $E$,

$$\sum_\lambda p(\epsilon_\lambda-E)\approx\frac{1}{D}\int p(\epsilon-E)d\epsilon=1/D , \tag{2.13}$$

which is taken to be independent of $E$. Technically, this means that the width of the distribution $p$ should be large enough to allow replacement of the sum by the integral in Eq. (2.13). (b) The correlation function

$$\sum_\lambda p(\epsilon_\lambda-E_1)p(\epsilon_\lambda-E_2)\approx\frac{1}{D}\int p(\epsilon-E_1)p(\epsilon-E_2)d\epsilon \equiv g(E_1-E_2) . \tag{2.14}$$

This expression depends on the difference of the arguments $E_1, E_2$ only, since the integral is left unchanged under a common translation of $E_1$ and $E_2$. One easily shows that the distribution $g$ is even and has its maximum at zero. Its variance is $\sqrt{2}\sigma_p$.

It is assumed that the number $\Lambda$ of levels is very large compared to the number $N$ of channels. The results are then independent of $\Lambda$. Effects that arise from a limited number $\Lambda$ of states are known in the experimental literature as "finite range of data errors."[14] They are neglected here.

On the basis of the model defined above, quantities of physical interest will be calculated in the next sections. We start with the average $S$-matrix element. Its evaluation is easy and serves as illustration of the mathematical technique. The more complicated calculation of the correlation between two $S$-matrix elements $S_{ab}$ and $S^*_{cd}$, which includes the evaluation of average cross sections, is performed in Sec. IV.

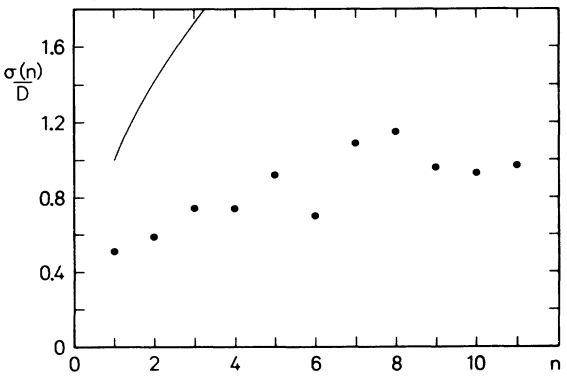


FIG. 1. Measured variances $\sigma(n)$ of the distance between a given resonance level and its $n$th neighbor. The variance is given in units of the mean level distance $D$. The data are from Ref. 13. The curve is the function $\sqrt{n}$; see text.

## III. CALCULATION OF THE AVERAGE $S$ MATRIX

The general idea of all the formal developments of the present paper is to expand the $S$ matrix into a geometrical series which is averaged term by term and subsequently resummed. This reminds one of the method used in Ref. 5; the details are, however, different, in order to allow for the more general statistical assumptions introduced in Sec. II.

Thus one writes

$$\langle S(E)\rangle=\langle(1+iK(E^+))(1-iK(E^+))^{-1}\rangle =1+2\sum_{n=1}^{\infty}\langle(iK(E^+))^n\rangle . \tag{3.1}$$

In order to avoid the singularities of $K$, the complex energy

$$E^+=E+i\eta \tag{3.2}$$

has been introduced. It is implied that the limit of $\eta\to 0$ is taken after performing the averages and the sum in Eq. (3.1).

The sum over the powers of $K$ need not exist. If, e.g., $K$ was equal to a real variable $x$ with Gaussian distribution, one would have $\langle x^{2k+1}=0\rangle$ and $\langle x^{2k}\rangle=(2k-1)!!\langle x^2\rangle^k$; hence, the sum in Eq. (3.1) would diverge for every $\langle x^2\rangle\neq 0$, although $\langle(1+ix)(1-ix)^{-1}\rangle$

exists. We assume that the series of the second part of Eq. (3.1) is equal to the desired average, wherever it converges. By analytic continuation, one then has the general result. In the case at hand, this procedure works, since we now show that $\langle K^n\rangle=\langle K\rangle^n$.

Introducing the diagonal level matrix $b$ with elements

$$b_{\lambda\mu}(E)=b_\lambda(E)\delta_{\lambda\mu}=\frac{i}{E_\lambda-E^+}\delta_{\lambda\mu}\ , \tag{3.3}$$

one can rewrite Eq. (3.1) by virtue of Eq. (2.6) as

$$\bar{S}=1+2\sum_{n=1}^{\infty}\overline{(\gamma^T b\gamma)^n}\ . \tag{3.4}$$

Let us calculate $\overline{\gamma^T b\gamma}$:

$$\overline{(\gamma^T b\gamma)_{ab}}=\sum_\lambda\overline{\gamma_{\lambda a}\gamma_{\lambda b}\,b_\lambda}=\delta_{ab}\overline{\gamma_a^2}\sum_\lambda\overline{b_\lambda}\ . \tag{3.5}$$

Using Eq. (2.13), the sum over $\overline{b_\lambda}$ becomes

$$\sum_\lambda\overline{b_\lambda}=\sum_\lambda\int\frac{i}{E'-E^+}p(\epsilon_\lambda-E')dE'$$
$$=\frac{1}{D}\int\frac{i}{E'-E^+}dE'$$

and, if the edges $\epsilon_1$ and $\epsilon_N$ of the energy levels are taken to be sufficiently far from the energy $E$, one obtains

$$\sum_\lambda\overline{b_\lambda}=-\pi/D\ , \tag{3.6}$$

whence

$$\overline{(\gamma^T b\gamma)_{ab}}=-\pi\overline{\gamma_a^2}/D\delta_{ab}\ . \tag{3.7}$$

We introduce the diagonal matrix

$$x=-\overline{\gamma^T b\gamma}\ \text{ with elements }\ x_a=\pi\overline{\gamma_a^2}/D\ . \tag{3.8}$$

Consider now the term with $n=2$ of Eq. (3.4),

$$\langle(\gamma^T b\gamma)^2_{ab}\rangle=\langle\gamma_{\lambda a}b_\lambda\gamma_{\lambda e}\gamma_{\mu e}b_\mu\gamma_{\mu b}\rangle$$
$$=\langle\gamma_{\lambda a}b_\lambda\gamma_{\lambda e}\rangle\langle\gamma_{\mu e}b_\mu\gamma_{\mu b}\rangle$$
$$+\langle\gamma_{\lambda a}b_\lambda\gamma_{\lambda e}\gamma_{\lambda e}b_\lambda\gamma_{\lambda b}\rangle$$
$$-\langle\gamma_{\lambda a}b_\lambda\gamma_{\lambda e}\rangle\langle\gamma_{\lambda e}b_\lambda\gamma_{\lambda b}\rangle\ . \tag{3.9}$$

Here, repeated indices have to be summed over. In the second part of this equation, we have factorized the average with respect to the indices $\lambda$ and $\mu$. By the statistical assumptions of Sec. II, this is justified for the terms with $\lambda\neq\mu$. For the terms with $\lambda=\mu$, the appropriate correction has been added. Since this procedure is repeatedly used in the following, we introduce a special notation for the correction term,

$$\langle\gamma_{\lambda a}b_\lambda\gamma_{\lambda e}\gamma_{\lambda e}b_\lambda\gamma_{\lambda b}\rangle-\langle\gamma_{\lambda a}b_\lambda\gamma_{\lambda e}\rangle\langle\gamma_{\lambda e}b_\lambda\gamma_{\lambda b}\rangle$$
$$=(\gamma^T\underline{b\gamma\gamma^T}b\gamma)_{ab}\ , \tag{3.10}$$

and call it a *contraction*. The contraction line implies that the connected $b$'s are summed with their indices equal and that there are two terms: the average of the whole expression reduced by the average that is factorized with respect to the connected $b$'s. The factorization shall be done such that each $b$ is taken together with the adjacent $\gamma$'s.

The value of the contraction (3.10) is

$$(\gamma^T\underline{b\gamma\gamma^T}b\gamma)_{ab}=\left\langle\gamma_a^2\sum_e\gamma_e^2\right\rangle\delta_{ab}\sum_\lambda\langle b_\lambda^2\rangle$$
$$-\langle\gamma_a^2\rangle\langle\gamma_b^2\rangle\sum_\lambda\langle b_\lambda\rangle^2=0\ . \tag{3.11}$$

In order to prove that $\sum\overline{b_\lambda^2}=0$, note that

$$b_\lambda^{n+1}=\frac{i^n}{n!}\frac{d^n}{dE^n}b_\lambda\ , \tag{3.12}$$

whence

$$\sum_\lambda\overline{b_\lambda^{n+1}}=\frac{i^n}{n!}\frac{d^n}{dE^n}\sum_\lambda\overline{b_\lambda}=0\ \text{ for } n>0\ , \tag{3.13}$$

because the result of Eq. (3.6) is independent of $E$.

One easily sees that $\sum\overline{b_\lambda^2}$ vanishes also:

$$\sum_\lambda\overline{b_\lambda^2}=\sum_\lambda\int\int\frac{i}{E'-E^+}\frac{i}{E''-E^+}$$
$$\times p(\epsilon_\lambda-E')p(\epsilon_\lambda-E'')dE'dE''$$
$$=\int\int\frac{i}{E'-E^+}\frac{i}{E'+x-E^+}g(x)dE'dx$$
$$=0\ , \tag{3.14}$$

since the integral over $E'$ vanishes. This entails, with the help of Eq. (3.12),

$$\sum_\lambda\overline{b_\lambda^n}\,\overline{b_\lambda^m}=0\ \text{ for } n,m\geq 1\ . \tag{3.15}$$

Using Eqs. (3.13) and (3.14), it is not difficult to show by induction that

$$\langle(\gamma^T b\gamma)^k\rangle=\langle\gamma^T b\gamma\rangle^k\ , \tag{3.16}$$

which yields the average $S$ matrix

$$\bar{S}=(1-x)(1+x)^{-1} \tag{3.17}$$

and the transmission coefficients

$$\tau_a=1-|\overline{S_{aa}}|^2$$
$$=4x_a(1+x_a)^{-2}\ . \tag{3.18}$$

The average $S$ matrix is found to be diagonal. This means that the statistical assumptions set up in Sec. II preclude direct reactions. Inclusion of direct reactions is possible by use of the transformation introduced in Ref. 15.

We now turn to correlation functions and average cross sections.

## IV. CORRELATION FUNCTIONS AND AVERAGE CROSS SECTIONS

### A. Exposition of the scheme of calculation: Cross contractions

Let us define the fluctuating part of the $S$ matrix by

$$S=\bar{S}+S^{\mathrm{fl}} . \tag{4.1}$$

The quantities to be calculated in the present section are the correlation functions

$$\begin{aligned} C_{ab,cd}(\epsilon)&=\langle S^{\mathrm{fl}}_{ab}(E)S^{\mathrm{fl}*}_{cd}(E+\epsilon)\rangle \\ &=\langle S_{ab}(E)S^{*}_{cd}(E+\epsilon)\rangle-\langle S_{ab}\rangle\langle S^{*}_{cd}\rangle \\ &=\langle S_{ab}(E)S^{\dagger}_{dc}(E+\epsilon)\rangle-\langle S_{ab}\rangle\langle S^{\dagger}_{dc}\rangle ; \end{aligned} \tag{4.2}$$

for $a=c$, $b=d$, and $\epsilon=0$, they yield the average compound nuclear cross sections $\sigma^{\mathrm{CN}}_{ab}$.

We start from the expansion of the $S$ matrix, which is implied by Eq. (3.4). This gives

$$C_{ab,cd}(\epsilon)=4\left\langle\left[\sum_{n=1}^{\infty}(\gamma^{T}b(E)\gamma)^{n}\right]_{ab}:\left[\sum_{m=1}^{\infty}(\gamma^{T}b(E+\epsilon)^{*}\gamma)^{m}\right]_{dc}\right\rangle-4\left\langle\left[\sum_{n=1}^{\infty}(\gamma^{T}b\gamma)^{n}\right]_{ab}\right\rangle:\left\langle\left[\sum_{m=1}^{\infty}(\gamma^{T}b^{*}\gamma)^{m}\right]_{dc}\right\rangle . \tag{4.3}$$

The two factors on the right-hand side (rhs), namely the matrix elements $(ab)$ and $(dc)$, have been separated by a bold colon, which we shall also call the "center line." It does not denote a mathematical operation but merely emphasizes the place where the average is factorized in the second term.

Consider the first term on the rhs; it is a multiple sum over channels, levels, and powers. We shall average it term by term and subsequently resum the leading terms. Imagine the center line in every term of the multiple sum. Note that each term in which the parameters to the left of the center line are statistically independent of the parameters to the right is cancelled by the appropriate term within the second expression of Eq. (4.3). This independence is given whenever all level indices that occur to the left of the center line are different from the ones occurring to the right. If, on the contrary, at least one level index to the left coincides with one level index to the right, the two parts of Eq. (4.3) do not cancel each other and we are left with a contraction as defined in Sec. III, which connects two $b$'s on either side of the center line. We call this a "cross contraction" and shall have to sum over all possible ways to construct them.

As an example, consider the simplest cross contraction that arises in Eq. (4.3). In symbolic and explicit form it is

$$\underline{(\gamma^{T}b\gamma)_{ab}}:\underline{(\gamma^{T}b^{*}\gamma)}_{dc}=\sum_{\lambda}\langle\gamma_{\lambda a}b_{\lambda}\gamma_{\lambda b}:\gamma_{\lambda d}b^{*}_{\lambda}\gamma_{\lambda c}\rangle-\sum_{\lambda}\langle\gamma_{\lambda a}b_{\lambda}\gamma_{\lambda b}\rangle:\langle\gamma_{\lambda d}b^{*}_{\lambda}\gamma_{\lambda c}\rangle . \tag{4.4}$$

Note that, for brevity, the energy arguments are suppressed; here and in the following, $b$ has to be taken at $E$ and $b^{*}$ at $E+\epsilon$. (The "propagators" $b$ should not be confused with the channel $b$, the latter one appearing only as an index.) It is unnecessary to evaluate Eq. (4.4); the next subsection will show that one can proceed quite far without being more explicit.

We shall now construct and sum the relevant cross contractions in three steps. This will be successful within the order $1/N$, where $N$ was defined as the number of open decay channels: terms of second and higher order in $1/N$ are neglected. One shall find, however, that the neglected terms are of second or higher order in the parameters $x_a$ defined in Eq. (3.8), so that our approximation is also valid for arbitrary $N$ in the limit of weak absorption [i.e., $\tau_e \ll 1$; see Eq. (3.18)] in all channels $e$.

### B. Endfactors of contractions

Consider again the simple contraction of Eq. (4.4). It occurs together with terms that are cross-contracted in the same way but carry factors with independent indices at the ends on either side of the center line. One obtains, e.g., a sum over structures

$$\begin{aligned} \sum_{q}[\langle(\gamma^{T}b\gamma)^{q}\rangle\gamma^{T}\underline{b\gamma}]_{ab}:\underline{(\gamma^{T}b^{*}\gamma)}_{dc}&=\sum_{q}\left[-\frac{\pi}{D}\overline{\gamma_a^2}\right]^{q}\underline{(\gamma^{T}b\gamma)_{ab}}:\underline{(\gamma^{T}b^{*}\gamma)}_{dc} \\ &=\left[1+\frac{\pi}{D}\overline{\gamma_a^2}\right]^{-1}\underline{(\gamma^{T}b\gamma)_{ab}}:\underline{(\gamma^{T}b^{*}\gamma)}_{dc} , \end{aligned} \tag{4.5}$$

which we have evaluated with the results of Sec. III. Defining the renormalized $\gamma$ matrix

$$\hat{\gamma}=\gamma(1+x)^{-1}\ , \tag{4.6}$$

we find the sums over all possible endfactors,

$$\sum_{qrst}[\overline{\gamma^T b\gamma}^{\,q}\gamma^T b\gamma\overline{\gamma^T b\gamma}^{\,r}]_{ab}:[\overline{\gamma^T b^*\gamma}^{\,s}\gamma^T b^*\gamma\overline{\gamma^T b^*\gamma}^{\,t}]_{dc}=(\hat{\gamma}^T b\hat{\gamma})_{ab}:(\hat{\gamma}^T b^*\hat{\gamma})_{dc}\ . \tag{4.7}$$

All contraction patterns will carry these endfactors.

### C. Contractions with branchings

A cross contraction may have branchings, such as, e.g.,

$$(\hat{\gamma}^T b\gamma\gamma^T b\hat{\gamma})_{ab}:(\hat{\gamma}^T b^*\hat{\gamma})_{dc}\ ;$$

this means that there may be additional $b$'s carrying the same index as the cross contracted ones. We shall now sum all possible branched contractions. Before doing so, note that in the same way as there are independent endfactors, there may be factors $\gamma^T b\gamma$ bracketed by the branch but summed independently. Hence, one has to evaluate

$$\sum_q(\hat{\gamma}^T b\gamma\overline{(\gamma^T b\gamma)^q}\gamma^T b\hat{\gamma})_{ab}:(\hat{\gamma}^T b^*\hat{\gamma})_{dc}=(\hat{\gamma}^T b\gamma(1+x)^{-1}\gamma^T b\hat{\gamma})_{ab}:(\hat{\gamma}^T b^*\hat{\gamma})_{dc}$$
$$=(\hat{\gamma}^T b\tilde{\gamma}\,\tilde{\gamma}^T b\hat{\gamma})_{ab}:(\hat{\gamma}^T b^*\hat{\gamma})_{dc}\ . \tag{4.8}$$

Here, yet another renormalized $\gamma$ matrix,

$$\tilde{\gamma}=\gamma(1+x)^{-1/2}\ , \tag{4.9}$$

has been introduced. Let us rewrite Eq. (4.8) by help of the diagonal matrix $\Gamma$ with elements

$$\Gamma_\lambda=2(\tilde{\gamma}\,\tilde{\gamma}^T)_{\lambda\lambda}=2\sum_e\tilde{\gamma}_{\lambda e}^2\ , \tag{4.10}$$

which gives

$$(\hat{\gamma}^T b\tilde{\gamma}\,\tilde{\gamma}^T b\hat{\gamma})_{ab}:(\hat{\gamma}^T b^*\hat{\gamma})_{dc}=(\hat{\gamma}^T\tfrac{1}{2}\Gamma b^2\hat{\gamma})_{ab}:(\hat{\gamma}^T b^*\hat{\gamma})_{dc}\ . \tag{4.11}$$

In general, contractions with $q$ branchings may be expressed and summed as

$$\sum_q(\hat{\gamma}^T b\tilde{\gamma}\tilde{\gamma}^T b\tilde{\gamma}\cdots{}_{q\text{ branches}}\cdots b\hat{\gamma})_{ab}:(\hat{\gamma}^T b^*\hat{\gamma})_{dc}=\sum_q[\hat{\gamma}^T(\tfrac{1}{2}\Gamma)^q b^{q+1}\hat{\gamma}]_{ab}:(\hat{\gamma}^T b^*\hat{\gamma})_{dc}$$
$$=(\hat{\gamma}^T\tilde{b}\hat{\gamma})_{ab}:(\hat{\gamma}^T b^*\hat{\gamma})_{dc}\ , \tag{4.12}$$

where the convention

$$\tilde{b}=b(1-\tfrac{1}{2}\Gamma b)^{-1}$$

has been used. The elements of this diagonal matrix are

$$\tilde{b}_\lambda=i(E_\lambda-E^+-i\Gamma_\lambda/2)^{-1}\ . \tag{4.13}$$

These quantities have no poles on the real axis in the limit of $\eta\to 0$, see Eq. (3.2), and one can therefore take this limit without any problem. One should note, however, that the decay amplitudes $\gamma_{\lambda e}$ and the new "propagators" $\tilde{b}_\lambda$ are no longer statistically independent.

Evidently, the structures with a single cross contraction and arbitrary numbers of branchings to the left and to the right of the center line sum up to

$$(\hat{\gamma}^T\tilde{b}\hat{\gamma})_{ab}:(\hat{\gamma}^T\tilde{b}^*\hat{\gamma})_{dc}=\langle\hat{\gamma}_{a\lambda}^T\tilde{b}_\lambda\hat{\gamma}_{\lambda b}:\hat{\gamma}_{d\lambda}^T\tilde{b}_\lambda^*\hat{\gamma}_{\lambda c}\rangle$$
$$-\langle\hat{\gamma}_{a\lambda}^T\tilde{b}_\lambda\hat{\gamma}_{\lambda b}\rangle:\langle\hat{\gamma}_{d\lambda}^T\tilde{b}_\lambda^*\hat{\gamma}_{\lambda c}\rangle\ . \tag{4.14}$$

The contraction of Eq. (4.14) obviously vanishes unless the indices $a,b,c,d$ coincide pairwise. For simplicity, we specialize the following discussion to the case of $a=c$, $b=d$. This means that we calculate the correlation function $C_{ab,ab}(\epsilon)=\langle S_{ab}^{\rm fl}(E)S_{ab}^{\rm fl*}(E+\epsilon)\rangle$. Equation (4.14) then gives

$$(\hat{\gamma}^T\tilde{b}\hat{\gamma})_{ab}:(\hat{\gamma}^T\tilde{b}^*\hat{\gamma})_{ba}=(1+x_a)^{-1/2}(1+x_b)^{-1/2}\sum_\lambda\{\langle\tilde{\gamma}_{\lambda a}^2\tilde{b}_\lambda\tilde{b}_\lambda^*\tilde{\gamma}_{\lambda b}^2\rangle-\delta_{ab}\langle\tilde{\gamma}_{\lambda a}^2\tilde{b}_\gamma\rangle\langle\tilde{\gamma}_{\lambda b}^2\tilde{b}_\lambda^*\rangle\}$$
$$=(1+x_a)^{-1}(1+x_b)^{-1}\{A_{ab}-\delta_{ab}B_{ab}\}\ . \tag{4.15}$$

The final result will be expressed in terms of the matrices $A$ and $B$ defined by the last equation. It is not difficult to evaluate the elements of $A$. Remember that $b$ and $b^*$ are taken at different energies:

$$A_{ab}(\epsilon)=\sum_{\lambda}\int\langle\tilde{\gamma}^2_{\lambda a}\tilde{\gamma}^2_{\lambda b}(E'-E-i\Gamma_\lambda/2)^{-1}\times(E'-E-\epsilon+i\Gamma_\lambda/2)^{-1}\rangle p(\epsilon_\lambda-E')dE' . \tag{4.16}$$

Here, the overbar denotes the average over the $\gamma$'s only, since the average over the $E_\lambda$ has been written explicitly. The probability distribution of the $\gamma_{\lambda e}$ is independent of $\lambda$; therefore, the sum over $\lambda$ can be performed by help of Eq. (2.13) and we drop the level index at $\tilde{\gamma}^2_{\lambda e}$, $\Gamma_\lambda$. The result is

$$A_{ab}(\epsilon)=\frac{2\pi}{D}\langle\tilde{\gamma}^2_a\tilde{\gamma}^2_b(\Gamma+i\epsilon)^{-1}\rangle . \tag{4.17}$$

The elements of $B$ are

$$B_{ab}(\epsilon)=\int\langle\tilde{\gamma}^2_a(E'-E-i\Gamma/2)^{-1}\rangle\times\langle\tilde{\gamma}^2_b(E''-E-\epsilon+i\Gamma/2)^{-1}\rangle\times g(E'-E'')dE'dE'' , \tag{4.18}$$

where again the overbars denote averages over the $\gamma$'s only and Eq. (2.14) has been used. One finds

$$B_{ab}(\epsilon)=2\pi i\int\langle\tilde{\gamma}^2_a\tilde{\gamma}'^2_b(x-\epsilon+i(\Gamma+\Gamma')/2)^{-1}\rangle g(x)dx . \tag{4.19}$$

In this equation, independent averages over $\gamma$ and $\gamma'$ are implied.

The quantity $A_{ab}(\epsilon)$ has the upper bound $A_{ab}(0)$ which is of the order $1/N$ and of the order of the absorption coefficients defined in Eq. (3.8), i.e.,

$$|A_{ab}(\epsilon)|\le A_{ab}(0)=O(1/N)O(x) , \tag{4.20}$$

where here $x$ stands for a "typical value" of the $x_e$. The $B_{ab}(\epsilon)$ obey the inequalities

$$|B_{ab}(\epsilon)|\le B_{ab}(0)\le[A_{aa}(0)+A_{bb}(0)]/2 , \tag{4.21}$$

where $B_{ab}(0)$ is real and non-negative. The second of the last two relations can be proved by help of the inequality

$$\frac{ab}{a+b+c+d}\le\left[\frac{a^2}{a+c}+\frac{b^2}{c+d}\right]\Big/4 \quad \text{for } a,b,c,d\ge 0 . \tag{4.22}$$

### D. Multiple cross contractions

The last step of the summation procedure is to add up structures that contain more than one cross contraction. Consider the simplest one, with two independent pairs of $\tilde{b},\tilde{b}^*$ (we omit the tildes that appear on all symbols),

$$(\gamma^T b\gamma\gamma^T b\gamma)_{ab}:(\gamma^T b^*\gamma\gamma^T b^*\gamma)_{ba}=\sum_{\lambda\mu e}\langle\gamma^2_{\lambda a}b_\lambda b^*_\lambda\gamma^2_{\lambda e}\rangle\langle\gamma^2_{\lambda e}b_\mu b^*_\mu\gamma^2_{\lambda b}\rangle-\delta_{ab}\left[\sum_\lambda\langle\gamma^2_{\lambda a}b_\lambda\rangle\langle\gamma^2_{\lambda a}b^*_\lambda\rangle\right]^2$$
$$=(A^2)_{ab}-\delta_{ab}(B_{aa})^2 . \tag{4.23}$$

In this contraction, there are two independent level summations and two channel summations from the products $\gamma\gamma^T$ on either side of the center line. After averaging, only one sum over channels remains in the first term and none in the second term, which allows one to express the result by the squares of $A$ and of the diagonal elements of $B$. Note that $(A^2)_{ab}$ is of the order of $1/N$ but of second order in $x$; more generally,

$$(A^q)_{ab}=O(1/N)O(x^q) . \tag{4.24}$$

We observe that there is another possible contraction of the string of $\gamma^T b\gamma$ factors in Eq. (4.23), namely

$$(\gamma^T b\gamma\gamma^T b\gamma)_{ab}:(\gamma^T b^*\gamma\gamma^T b^*\gamma)_{ba}=\left[\sum_\lambda\langle\gamma^2_{\lambda a}b_\lambda b^*_\lambda\gamma^2_{\lambda b}\rangle\right]^2=(A_{ab})^2 , \tag{4.25}$$

if channels $a$ and $b$ are different. In this "interlaced contraction," no sum over channels remains and, therefore, the result $(A_{ab})^2$ is of the order of $1/N^2$. We neglect it, working out only the terms of order $1/N$. The quantity $(A_{ab})^2$ is of second order in $x$, so that the approximation is valid also for arbitrary $N$ in the case of weak absorption in all channels.

The case $a=b$ is different. One can then convert the interlaced cross contraction into a structure similar to that of Eq. (4.23) by observing that one can reverse the order of the factors $\gamma^T b\gamma$ on one side of the center line without changing the value of the contraction:

$$(\gamma^T b\gamma\gamma^T b\gamma)_{aa}:(\gamma^T b^*\gamma\gamma^T b^*\gamma)_{aa}=\sum_{\lambda\mu e}\langle\gamma^2_{\lambda a}b_\lambda b^*_\lambda\gamma^2_{\lambda e}\rangle\langle\gamma^2_{\mu e}b_\mu b^*_\mu\gamma^2_{\mu a}\rangle-\left[\sum_\lambda\langle\gamma^2_{\lambda a}b_\lambda\rangle\langle\gamma^2_{\lambda a}b_\lambda\rangle\langle\gamma^2_{\lambda a}b^*_\lambda\rangle\right]^2$$
$$=(A^2)_{aa}-(B_{aa})^2$$
$$=(\gamma^T b\gamma\gamma^T b\gamma)_{aa}:(\gamma^T b^*\gamma\gamma^T b^*\gamma)_{aa} , \tag{4.26}$$

see Eq. (4.23).

Equations (4.15), (4.23), and (4.26) give the values of simple and twofold cross contractions. We state the rules for $q$-fold cross contractions. The proof is done by induction, but is lengthy and therefore suppressed here.

(i) The nested contraction, as in Eq. (4.23), is of the order of $1/N$ and has the value $(A^q)_{ab}-\delta_{ab}(B_{aa})^q$.

(ii) If $a\neq b$, all contractions with interlacings, as in Eq. (4.25), are of second or higher order in $1/N$ and in $x$. They are neglected.

(iii) If $a=b$ and $q\geq 2$, there is one interlaced contraction of order $1/N$. It can be converted into a nested one by reversing the order of the factors on one side of the center line, as in Eq. (4.26). Its value is $(A^q)_{aa}-(B_{aa})^q$.

With these rules, one obtains the sum over all cross contractions to the order of $1/N$:

$$\sum_{q\geq 1}(\gamma^T b\gamma\cdots\underbrace{\gamma^T b\gamma)_{ab}:(\gamma^T b^*\gamma\cdots}_{q\text{ contractions}}\gamma^T b^*\gamma)_{ba}+\delta_{ab}\sum_{q\geq 2}(\gamma^T b\gamma\cdots\underbrace{\gamma^T b\gamma)_{aa}:(\gamma^T b^*\gamma\cdots}_{q\text{ contractions}}\gamma^T b^*\gamma)_{aa}$$

$$=\sum_{q\geq 1}(A^q)_{ab}-\delta_{ab}B_{aa}+\delta_{ab}\sum_{q\geq 2}(A^q)_{aa}=(1+\delta_{ab})[(1-A)^{-1}]_{ab}-\delta_{ab}(2+A_{aa}+B_{aa})\ . \tag{4.27}$$

Here, higher powers of $B_{aa}$ have been suppressed, since they are of second or higher order in $1/N$; see the inequalities (4.20) and (4.21).

Collecting all results we find the following: (i) The correlation functions $C_{ab,cd}(\epsilon)$ of Eq. (4.2) vanish unless the indices $a,b,c,d$ coincide pairwise.

(ii) The cases $a=c$, $b=d$ and $a=d$, $b=c$ yield the same result due to the symmetry of the $S$ matrix, namely

$$C_{ab,ab}(\epsilon)=C_{ab,ba}(\epsilon)=4(1+x_a)^{-1}(1+x_b)^{-1}\{(1+\delta_{ab})[(1-A)^{-1}]_{ab}-\delta_{ab}(2+A_{aa}+B_{aa})\}\ , \tag{4.28}$$

where $x$, $A$, and $B$ are defined in Eqs. (3.8), (4.17), and (4.19), respectively.

(iii) The technique of the present section can be used to calculate the remaining nontrivial case with $a=b, c=d$. One finds

$$C_{aa,bb}(\epsilon)=4(1+x_a)^{-1}(1+x_b)^{-1}(A-B)_{ab} \tag{4.29}$$

for $a\neq b$.

The following sections of the present paper are devoted to the specialization and discussion of these results.

## V. ISOLATED RESONANCES

Let us assume that the transmission coefficients (3.18) in all channels are small compared to unity so that the formalism of Secs. III and IV can be reduced to the first order in the coefficients $x$ of Eq. (3.8). In this limit, the formulae (4.28) and (4.29) are applicable to any number $N$ of channels, since the neglected terms of higher order in $1/N$ are of higher order in $x$ anyway. The limit shall be understood such that the compound-nucleus resonances are well isolated, i.e.,

$$\overline{\Gamma}_\lambda\approx 2\sum_e\overline{\gamma_{\lambda e}^2}\ll D\ . \tag{5.1}$$

The superscripts on the $\gamma$'s can be omitted, since the denominators in Eqs. (4.6) and (4.9) contribute only higher orders of $x$.

Under these conditions the matrix $B$—see Eq. (4.19)—becomes

$$B_{ab}(\epsilon)\approx 2\pi i\left\langle\gamma_a^2\gamma_b'^2\int g(x)/(x-\epsilon+i(\Gamma+\Gamma')/2)dx\right\rangle$$

$$\approx 2\pi\overline{\gamma_a^2}\,\overline{\gamma_b^2}\pi g(\epsilon)\leq 2\pi^2\overline{\gamma_a^2}\,\overline{\gamma_b^2}g(0)=O(x^2)\ , \tag{5.2}$$

because the width $\sqrt{2}\sigma_p$ of the function $g$ is approximately equal to $D$; see Sec. II. Together with the normalization

$$\int g(x)dx=1/D\ , \tag{5.3}$$

this implies

$$g(0)\propto(\sigma_p D)^{-1}\propto D^{-2}\ , \tag{5.4}$$

which ensures that expression (5.2) is of the order of $x^2$. Hence, $B$ is negligible.

Because of the estimate (4.24), higher powers of $A$ are neglected and one obtains the correlation functions

$$\begin{aligned}C_{ab,ab}(\epsilon)&=C_{ab,ba}(\epsilon)\\&=C_{aa,bb}(\epsilon)\\&=4A_{ab}(\epsilon)\\&=\frac{8\pi}{D}\langle\gamma_a^2\gamma_b^2(\Gamma+i\epsilon)^{-1}\rangle\ .\end{aligned} \tag{5.5}$$

We note that the compound nuclear cross section [see Eq. (4.2)] is

$$\sigma_{ab}^{\mathrm{CN}}\equiv\langle\,|S_{ab}^{\mathrm{fl}}|^2\rangle=\frac{4\pi}{D}\left\langle\gamma_a^2\gamma_b^2\Big/\sum_{e=1}^N\gamma_e^2\right\rangle\ , \tag{5.6}$$

which yields just the transmission coefficient in the one-channel case,

$$\sigma_{aa}^{\mathrm{CN}}=4\pi\overline{\gamma_a^2}/D=\tau_a\quad\text{for } N=1\ , \tag{5.7}$$

as it should.

## VI. HAUSER-FESHBACH EXPRESSIONS

If there are so many open channels that in the elements of the matrices $A$ and $B$ of Eqs. (4.17) and (4.19) one can independently average numerator and denominator, then for $a \neq b$ the cross sections factorize into transmission coefficients as stated by the well known Hauser-Feshbach[16] formula.

How large must $N$ be to allow independent averaging of numerator and denominator in Eqs. (4.17) and (4.19)? From a Taylor expansion of $\gamma_a^2\gamma_b^2/\sum_e \gamma_e^2$ at the point $\gamma_f^2=\overline{\gamma^2}$ for all channels $f$ (the same idea is used in the first one of Refs. 3), one finds

$$\left\langle \gamma_a^2\gamma_b^2 \left[\sum_e \gamma_e^2\right]^{-1}\right\rangle = \overline{\gamma^2}N^{-1}[1-N^{-1}(\overline{\gamma^4}/\overline{\gamma^2}^2-1)+N^{-2}(\overline{\gamma^6}/\overline{\gamma^2}^3-\overline{\gamma^4}/\overline{\gamma^2}^2-(\overline{\gamma^4}/\overline{\gamma^2}^2)^2+1)], \quad a\neq b \ . \tag{6.1}$$

Hence, the approximation of the present section requires the correction terms in the square brackets to be small compared to unity. For Gaussian statistics, this condition is met if $N \gg 2$, as one can verify by help of the statistical moments given within the discussion that precedes Eq. (3.3). See also the explicit result in Sec. VIII. Generally, the number $N$ required for the present approximation is a function of the higher moments of the $\gamma$'s.

Let us assume now the matrices $A$ and $B$ to be

$$A_{ab}=\frac{2\pi}{D}\overline{\tilde\gamma}_a^2\overline{\tilde\gamma}_b^2(\overline{\Gamma}+i\epsilon)^{-1} \tag{6.2}$$

and

$$B_{ab}=2\pi i\overline{\tilde\gamma}_a^2\overline{\tilde\gamma}_b^2\int (x-\epsilon+i\overline{\Gamma})^{-1}g(x)dx \ . \tag{6.3}$$

We define a column vector $u^T$ with elements

$$u_e=\left[\frac{2\pi}{D(\overline{\Gamma}+i\epsilon)}\right]^{1/2}\overline{\tilde\gamma}_e^2, \quad e=1,\ldots,N \tag{6.4}$$

and split $A$ into the separable matrix $u^Tu$ and an extra diagonal matrix $F$,

$$A=u^Tu+F \ , \tag{6.5}$$

with

$$F_{ab}=\delta_{ab}\frac{2\pi}{D}(\overline{\tilde\gamma}_a^4-\overline{\tilde\gamma}_a^2{}^2)(\overline{\Gamma}+i\epsilon)^{-1} \ . \tag{6.6}$$

The resolvent of $A$ is

$$\begin{aligned}[(1-A)^{-1}]_{ab}&=[(u^Tu)(1-uu^T)^{-1}+1+F]_{ab}\\ &=\overline{\tilde\gamma}_a^2\overline{\gamma}_b^2\left[\sum_e \tau_e+2\pi i\epsilon/D\right]^{-1}\\ &\quad+\delta_{ab}(1+F_{aa}) \ ,\end{aligned} \tag{6.7}$$

and one sees that within the present approximation all results can be expressed by the second and fourth moments of the $\gamma$'s.

The correlation function $C_{ab,ab}(\epsilon)$ of Eq. (4.28) becomes

$$C_{ab,ab}(\epsilon)=\tau_a\tau_b\left[\sum_e \tau_e+2\pi i\epsilon/D\right]^{-1} \quad \text{for } a\neq b \ . \tag{6.8}$$

This Lorentzian form—with the correlation length independent of the reaction channels $a,b$—is familiar from Ericson's theory of cross section fluctuations.[17] The correlation length is

$$\Gamma_{\rm cor}=\left[\sum_e \tau_e\right]D/2\pi \ , \tag{6.9}$$

a result first proved in Ref. 5 under the assumptions of Gaussian statistics and overlapping resonances.

A more complicated expression is found, if $a=b$; namely

$$\begin{aligned}C_{aa,aa}(\epsilon)&=2\tau_a^2\left[\sum_e \tau_e+2\pi i\epsilon/D\right]^{-1}\\ &\quad+(K-2)\tau_a^2(\overline{\Gamma}+i\epsilon)^{-1}D/2\pi\\ &\quad-\tau_a^2 i\int g(x)(x-\epsilon+i\overline{\Gamma})^{-1}dx\,D^2/2\pi \ ,\end{aligned} \tag{6.10}$$

where $K$ is the ratio of statistical moments,

$$K=\overline{\gamma_a^4}/\overline{\gamma_a^2}^2 \ , \tag{6.11}$$

assumed to be independent of the channel $a$. (It should not be confused with the $K$ matrix, introduced in Sec. II in a completely different context.) Hence, the autocorrelation function of elastic scattering cannot be expressed in terms of the transmission coefficients: it carries information on the details of the statistics of the $\gamma$'s (via $K$) and the level positions (via $g$). To have an easier discussion of Eq. (6.10), let us specialize it to the case of strongly overlapping resonances. This means $\overline{\Gamma}\gg D$ and, therefore, $\overline{\Gamma}$ is much larger than the width $\sqrt{2}\sigma_p$ of the function $g$, so that the integral can be approximated by

$$i\int g(x)(x-\epsilon+i\overline{\Gamma})^{-1}dx\approx[D(\overline{\Gamma}+i\epsilon)]^{-1} \tag{6.12}$$

using Eq. (5.3). We then have

$$\begin{aligned}C_{aa,aa}(\epsilon)&=2\tau_a^2(\Gamma_{\rm cor}+i\epsilon)^{-1}D/2\pi\\ &\quad+(K-3)\tau_a^2(\overline{\Gamma}+i\epsilon)^{-1}D/2\pi \ .\end{aligned} \tag{6.13}$$

In the limit of strongly overlapping resonances, one may roughly estimate to have strong absorption in all relevant channels, i.e., $\tau_e=1$ and $x_e=1$ for all $e$, and therefore $\overline{\Gamma}=2\Gamma_{\rm cor}$; see Eqs. (3.8), (3.18), (4.9), (4.10), and (6.9). In this approximation, the positions of the poles in Eq. (6.13) are different by a factor of 2 and the ratio $K$ of the statistical moments could, in principle, be measured by studying

autocorrelation functions of elastic scattering cross sections. If the statistics of the $\gamma$'s is Gaussian, then $K=3$ and the second term in Eq. (6.13) vanishes.

The correlation function of Eq. (4.29) is in the limit of the Hauser-Feshbach approximation, introduced at the top of the present section,

$$C_{aa,bb}(\epsilon)=\tau_a\tau_b(\overline{\Gamma}+i\epsilon)^{-1}D/2\pi - \tau_a\tau_b i\int g(x)(x-\epsilon+i\overline{\Gamma})^{-1}dx\, D^2/2\pi\ , \tag{6.14}$$

for $a\neq b$. This function vanishes in the regime of strongly overlapping resonances when the integral tends towards the approximation (6.12). This behavior is confirmed by the Monte Carlo simulation in Ref. 11.

The correlation function $C_{ab,ab}(\epsilon)$ equals the compound nuclear cross section $\sigma_{ab}^{\rm CN}$ at $\epsilon=0$; one obtains

$$\sigma_{ab}^{\rm CN}=\tau_a\tau_b\left[\sum_e \tau_e\right]^{-1}[1+\delta_{ab}(W-1)]\ . \tag{6.15}$$

For nonelastic reactions, $a\neq b$, this is the well known Hauser-Feshbach formula. Except for the condition formulated at the top of the present section, it holds within all generality of our statistical assumptions. The elastic cross sections $\sigma_{aa}^{\rm CN}$ are enhanced over the Hauser-Feshbach result by the factor

$$W=2+\sum_e \tau_e\frac{D}{2\pi}\left\{(K-2)/\overline{\Gamma}-iD\int g(x)(x+i\overline{\Gamma})^{-1}dx\right\}. \tag{6.16}$$

The existence of an elastic enhancement factor is experimentally[18,19] established. The implications of Eq. (6.16) are discussed in the next section.

## VII. THE ELASTIC ENHANCEMENT FACTOR

We continue to work under the assumption formulated at the beginning of Sec. VI. It was shown that inelastic cross sections $\sigma_{ab}^{\rm CN}$ with $a\neq b$ can then be expressed by the transmission coefficients which are functions of the second moments $\overline{\gamma_e^2}$. Any more details of the statistics, such as, e.g., higher moments of the $\gamma$'s and the function $g$ characterizing the distribution of the energy eigenvalues, enter only into the expression that describes elastic scattering, namely the elastic enhancement factor $W$ defined by Eq. (6.15) and given explicitly in Eq. (6.16).

Let us discuss $W$ in the limiting cases of overlapping and isolated resonances. (i) In the case of strongly overlapping resonances, characterized by $\overline{\Gamma}\gg D$, the integral in Eq. (6.16) is approximated by Eq. (6.12) and the enhancement factor becomes

$$W(\text{overl. res.})=2+(K-3)\sum_e \tau_e D/(2\pi\overline{\Gamma})\ . \tag{7.1}$$

Here, $K$ measures the fourth moments of the decay amplitudes; cf. Eq. (6.11). If their distribution is Gaussian, then $K=3$ and we expect $W(\text{overl. res.})=2$, in agreement with other theoretical work[5,20] and with an experiment by Kretschmer and Wangler.[19] These authors find from elastic and inelastic proton scattering on $^{28}$Si, where the compound nucleus $^{29}$P was excited to $E_x=16.8$ MeV in the domain of overlapping resonances,

$$W(\text{overl. res.})=2.09\pm0.14\ . \tag{7.2}$$

If we assume strong absorption, i.e., $\tau_e=x_e=1$, for all relevant channels $e$, then Eq. (7.1) yields

$$W(\text{overl. res.})=2+(K-3)/2\ ; \tag{7.3}$$

cf. the discussion following Eq. (6.13). Hence, the experimental result (7.2) yields the fourth moment of the decay amplitudes, namely

$$K=\overline{\gamma_a^4}/\overline{\gamma_a^2}^2=3.18\pm0.28\ , \tag{7.4}$$

and thus confirms Gaussian statistics.

(ii) In the limit of isolated resonances characterized by $D\gg\overline{\Gamma}$, the integral in Eq. (6.16) is approximately $\pi g(0)$; see Eq. (5.2) and the discussion there. Note that $D\gg\overline{\Gamma}$ implies weak absorption in all channels which yields

$$\sum\tau_e\approx 4\sum x_e\approx 2\pi\overline{\Gamma}/D$$

by Eqs. (3.8), (3.18), (4.9), and (4.10). The elastic enhancement factor reduces to

$$W\approx K-\left[\sum_e\tau_e\right]g(0)D^2/2 \tag{7.5}$$

to first order in the absorption coefficients and neglecting terms of order $1/N$. With $\tau_e\to 0$ we have

$$W(\text{isol. res.})\approx K=\overline{\gamma_a^4}/\overline{\gamma_a^2}^2\ . \tag{7.6}$$

Assuming Gaussian statistics, this gives $W(\text{isol. res.})=3$.

We note, however, that this treatment of the limit of isolated resonances is not entirely realistic, since—in actual compound nuclei—it implies low excitation energy, and, hence, only a few channels will be open. Therefore the assumption $N\gg 1$ is unrealistic in this limit and we should use the results of Sec. V rather than those of Sec. VI. Here, one can define an elastic enhancement factor only if all channels are equivalent, i.e., $\overline{\gamma_e^2}=\overline{\gamma^2}$ is independent of the channel $e$. Then all the inelastic cross sections of Eq. (5.6) are equal and the elastic ones are enhanced by the factor

$$W(\text{isol. res.})=\left\langle\gamma_a^4\Big/\sum_{e=1}^N\gamma_e^2\right\rangle \times\left[\left\langle\gamma_a^2\gamma_b^2\Big/\sum_{e=1}^N\gamma_e^2\right\rangle\right]^{-1},\quad a\neq b\ . \tag{7.7}$$

For Gaussian statistics, this again gives $W(\text{isol. res.})=3$ for any number $N$ of open channels. The proof will be given in Sec. VIII.

Experimental studies of low energy neutron[21] and proton[22] scattering find the decay widths $\gamma_\lambda^2$ to follow Porter-Thomas[23] distributions. This distribution is implied by a Gaussian statistics of the decay amplitudes $\gamma_{\lambda e}$. Hence, the experimental results imply $W(\text{isol. res.})=3$.

In summary, the elastic enhancement factor $W$ is a function of the statistics of both the decay amplitudes $\gamma_{\lambda e}$ and the energy spectrum $E_\lambda$. Assuming Gaussian distri-

bution of the $\gamma$'s, $W$ decreases from 3 to 2 when the physical situation changes from well isolated to strongly overlapping resonances. Both limiting cases have been investigated experimentally and were found to be in agreement with Gaussian statistics. No experimental result is available for the intermediate situation.

The essential property of the energy spectrum showing up in the enhancement factor $W$ is its stiffness. As discussed in Sec. II, the variance $\sqrt{2}\sigma_p$ of the distribution $g$ is inversely proportional to the spectral rigidity, $D/\sigma_p$. Based on experimental evidence, we have so far assumed $\sigma_p \approx D$. If, however, we consider $\sigma_p$ to be a free parameter, we obtain, from Eqs. (5.4) and (7.5),

$$\left.\frac{\partial W}{\partial \tau_a}\right|_{\text{all } \tau_e=0} = -g(0)D^2/2 \propto D/\sigma_p \ . \tag{7.8}$$

This roughly means that the stiffer the spectrum, the steeper the slope of $W$ with increasing absorption. If one assumes $g$ to be Gaussian, i.e.,

$$g(x)=D^{-1}(4\pi\sigma_p^2)^{-1/2}\exp(-x^2/4\sigma_p^2) \ , \tag{7.9}$$

Eq. (7.8) gives

$$\left.\frac{\partial W}{\partial \tau_a}\right|_{\text{all } \tau_e=0} = -\tfrac{1}{4}(\pi)^{-1/2}D/\sigma_p \ . \tag{7.10}$$

The specialization of our results to Gaussian distributions is treated in detail in the next section.

## VIII. THE CASE OF GAUSSIAN STATISTICS AND EQUIVALENT CHANNELS

In this section we specialize the results of Sec. IV to the case in which the distributions $w_e(\gamma_{\lambda e})$ and $p(\epsilon_\lambda - E_\lambda)$—cf. Eqs. (2.9) and (2.11)—are normal. In order to keep this discussion within reasonable limits, we set the parameter $\epsilon$ of Eq. (4.2) equal to zero and require equivalence of all channels, i.e.,

$$w_e(\gamma_{\lambda e})=w(\gamma_{\lambda e})=(2\pi\overline{\gamma^2})^{-1/2}\exp(-\gamma_{\lambda e}^2/2\overline{\gamma^2}) \ . \tag{8.1}$$

This means that the absorption coefficients $x_e$ of Eq. (3.8) and the transmission coefficients $\tau_e$ of Eq. (3.18) are independent of the channel $e$,

$$x_e = x \quad \text{and} \quad \tau_e = \tau \ . \tag{8.2}$$

If $p$ is normal with variance $\sigma_p$, then Eq. (2.14) yields Eq. (7.9).

The matrices $A$ and $B$ of Eqs. (4.17) and (4.19) must be evaluated. This is best done using the generating function

$$f(z)=\int_{-\infty}^{\infty} e^{-\gamma^2 z} w(\gamma)d\gamma \ . \tag{8.3}$$

One easily verifies the relations

$$\left\langle \gamma_a^2\gamma_b^2\left[\sum_{e=1}^{N}\gamma_e^2\right]^{-1}\right\rangle = \int_0^\infty [f(z)]^{N-2}[f'(z)]^2 dz \qquad \text{for } a\neq b \tag{8.4}$$

and

$$\left\langle \gamma_a^4\left[\sum_{e=1}^{N}\gamma_e^2\right]^{-1}\right\rangle = \int_0^\infty [f(z)]^{N-1}f''(z)dz \ , \tag{8.5}$$

that are valid for any distribution $w$. If $w(\gamma)$ is normal, one obtains

$$f(z)=(1+2\overline{\gamma^2}z)^{-1/2} \ , \tag{8.6}$$

and this immediately gives

$$\left\langle \gamma_a^2\gamma_b^2\left[\sum_{e=1}^{N}\gamma_e^2\right]^{-1}\right\rangle = \overline{\gamma^2}/(N+2) \quad \text{for } a\neq b \tag{8.7}$$

and

$$\left\langle \gamma_a^4\left[\sum_{e=1}^{N}\gamma_e^2\right]^{-1}\right\rangle = 3\overline{\gamma^2}/(N+2) \ . \tag{8.8}$$

One sees that the approximation of Sec. VI which requires independent averages of numerator and denominator in these equations is obtained if $N \gg 2$. This conclusion had already been drawn from Eq. (6.1). The elements of the matrix $A$ are

$$A_{ab}=(1+\delta_{ab})\frac{\pi}{D}\overline{\tilde{\gamma}^2}/(N+2) \ , \tag{8.9}$$

where $\overline{\tilde{\gamma}_a^2}=\overline{\tilde{\gamma}^2}$ is defined in Eq. (4.9). To evaluate the matrix $B$, one uses the identity

$$\int_{-\infty}^{\infty}\left\langle \gamma_a^2\gamma_b'^2\left[-iy+\sum_{e=1}^{N}(\gamma_e^2+\gamma_e'^2)\right]^{-1}\right\rangle g(y)dy = \int_0^\infty dz\,[f(z)]^{2N-2}[f'(z)]^2\int_{-\infty}^{\infty}dy\,e^{-iyz}g(y) \ . \tag{8.10}$$

The Fourier transform of $g(y)$ is $\exp(-z^2\sigma_p^2)/D$ and one therefore arrives at

$$B_{ab}=\tfrac{1}{4}\tau^2(1+x)^2 I(x,N,D/\sigma_p) \ , \tag{8.11}$$

where a special notation has been introduced for the integral,

$$I(x,N,D/\sigma_p)=D(2\pi\sigma_p)^{-1}\times\int_0^\infty [1+z\tau(1+x)D(2\pi\sigma_p)^{-1}]^{-N-2}\times\exp(-z^2)dz \ . \tag{8.12}$$

The resolvent $(1-A)^{-1}$ can be calculated with the method used in Sec. VI, and Eq. (4.28) yields the compound nuclear cross sections

$$\sigma_{ab}^{\text{CN}}=\tau^2[N\tau+2\tau(1+x)]^{-1}[1+\delta_{ab}(W-1)] \ , \tag{8.13}$$

with the elastic enhancement factor

$$W=2+\frac{N+2(1+x)}{(N+2)(1+x)}-\tau[N+2(1+x)]I(x,N,D/\sigma_p) \ . \tag{8.14}$$

In Fig. 2 we reproduce the enhancement factor $W$ obtained by Hofmann *et al.*[11] via Monte Carlo calculations for $N=5$ channels. Compared with their results is the function (8.14), with the choice

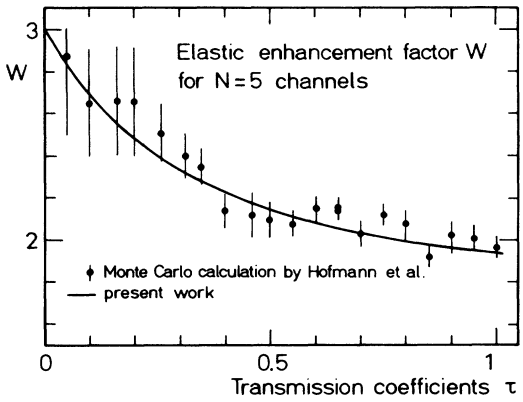


FIG. 2. Elastic enhancement factor $W$ for $N=5$ open channels as a function of the transmission coefficients $\tau$. The points with error bars are from Ref. 11, the curve is given by Eq. (8.14); see text.

$$D/\sigma_p = 2\sqrt{\pi} \ . \tag{8.15}$$

If one requires level repulsion, this choice is imposed—as we shall show in Sec. IX. The agreement between the Monte Carlo calculations and the present analytical result seems to be perfect within the errors of the former. This is so despite the fact that the calculations of Ref. 11 have not been done for equivalent channels but rather for various five channel cases with the transmission coefficients different in different channels. These results are given as a function of the coefficient $\tau_a$ in that channel $a$ for which the elastic enhancement was determined.

In Fig. 3 the enhancement factor of Eq. (8.14) is displayed for $N=10$ channels and in Fig. 4 for the limit of $N\to\infty$. This limit is taken such that the sum $N\tau$ of the transmission coefficients remains finite. Equation (8.14) then yields

$$W(N\to\infty)=3-N\tau I_\infty(N\tau(2\sqrt{\pi})^{-1}) \ , \tag{8.16}$$

with

$$I_\infty(\xi)=\pi^{-1/2}\exp(\xi^2)\int_\xi^\infty \exp(-z^2)dz \ . \tag{8.17}$$

The function of Eq. (8.16) is given in Fig. 4.

We now turn to the justification of Eq. (8.15).

## IX. LEVEL REPULSION

So far we have specified the quantity $\sigma_p$ that characterizes the statistics of the spectrum only qualitatively: The experimentally observed stiffness of statistical spectra tells us that it should be of the order of the mean level distance $D$; see Sec. II. One can go a step farther and relate $\sigma_p$ to the probability of finding degenerate levels. This can be done in such a way that an unsatisfactory feature of the joint probability distribution

$$\tilde{P}(E_1,\ldots,E_\Lambda)=\prod_{\lambda=1}^{\Lambda} p(\epsilon_\lambda - E_\lambda) \ , \tag{9.1}$$

introduced in Sec. II, is removed at the same time. According to Eq. (9.1), a level $\lambda$ is distinguished from another level $\mu$, since they are found in the neighborhood of different mean values $\epsilon_\lambda$ and $\epsilon_\mu$. There is no physical reason for this; the levels should all be equivalent. We therefore symmetrize $\tilde{P}$ and consider the distribution

$$P_\Lambda(E_1,\ldots,E_\Lambda)=(\Lambda!)^{-1}\sum\prod_{\lambda=1}^{\Lambda} p(\epsilon_{\tau_\lambda}-E_\lambda) \ , \tag{9.2}$$

where the sum goes over all permutations of

$$\begin{pmatrix}\tau_1,\ldots,\tau_\Lambda\\ 1,\ldots,\Lambda\end{pmatrix} .$$

Note that this modification of the energy level statistics does not affect any result derived so far: the results are independent of the $\epsilon_\lambda$ and therefore symmetrization with respect to $\epsilon_\lambda$ does not change them.

Let us now construct the two-point function $P_2$ by integrating $P$ over all variables but two, which gives

$$P_2(E_1,E_2)=\sum_{\substack{\mu,\lambda\\ \mu\neq\lambda}} p(\epsilon_\lambda-E_1)p(\epsilon_\mu-E_2)[\Lambda(\Lambda-1)]^{-1} \ . \tag{9.3}$$

Using the approximations (2.13) and (2.14), one obtains a

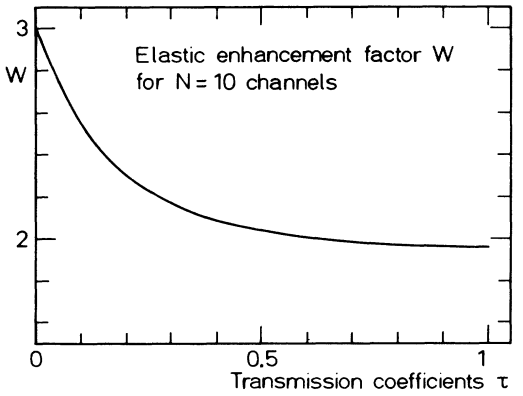


FIG. 3. Elastic enhancement factor $W$ for $N=10$ open channels as a function of the transmission coefficients $\tau$. The stiffness of the spectrum is given by Eq. (8.15); see text.

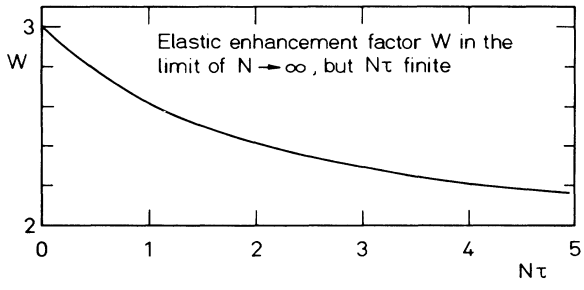


FIG. 4. Elastic enhancement factor $W$ in the limit of $N\to\infty$, but $N\tau$ finite. The stiffness of the spectrum is given by Eq. (8.15); see text.

relation between $g$ and the two-point function $P_2$; namely

$$P_2(E_1,E_2)=[D^{-2}-g(E_1-E_2)][\Lambda(\Lambda-1)]^{-1}\,. \quad (9.4)$$

The value of $P_2$ for $E_1=E_2$ is the probability of finding two levels degenerate. If we inject this information, then the ansatz (7.9) yields the stiffness parameter. Requiring $P_2(E,E)=0$, one obtains

$$g(0)=D^{-2}\,, \quad (9.5)$$

and, with Eq. (7.9), the parameter given in Eq. (8.15).

Equation (9.5) entails—by virtue of Eq. (7.10)—the derivative

$$\left.\frac{\partial W}{\partial \tau_a}\right|_{\text{all } \tau_e=0}=-\tfrac{1}{2}\,. \quad (9.6)$$

It is remarkable that this result is in exact agreement with the theory[7] which treats the matrix $H$ of Eq. (2.4) as an element of the GOE; see Sec. 3 of Ref. 10. We disagree with Ref. 24, where the derivative of Eq. (9.6) was obtained to be $-\frac{1}{4}$.

An unsatisfactory feature of the present approach should be mentioned: Although Eq. (9.5) is very plausible and although it correctly leads via Eq. (9.6) to the limit of the GOE theory, it also leads to a ratio $\sigma_p/D$ which is somewhat too small—see Eq. (8.15)—for the approximations (2.13) and (2.14) to be very good. These approximations are, however, needed if the ensemble averages shall be stationary with respect to the total energy $E$. There does not seem to be a simple and physically reasonable ansatz for the level statistics that would improve this situation, and with the present section we have reached the limit of the model presented here.

## X. SUMMARY AND CONCLUSION

The main achievements of the present work are (ordered by their degree of specialization):

(i) A method for calculating first and second moments of statistical scattering operators $S$ is presented. The ensemble of the statistical variables is not restricted to be Gaussian. It takes care of fluctuations of the energy levels and decay amplitudes. We calculate term by term the ensemble averages of a power series expansion of $S$ and subsequently resum the series. The nonvanishing elements of the mean scattering matrix $\bar{S}$ are given by the well known[12] expression (3.17). Thus $\bar{S}$ is given by an energy-scale independent *ratio* of model parameters; this ratio does *not* depend on the resonance level distribution. All further results are also stationary with respect to the energy.

The compound nucleus reaction cross section $\overline{|S_{ab}^{fe}|^2}$ is given in Eq. (4.28) in terms of the probability integrals (4.16) and (4.18). Equation (4.28) contains an elastic enhancement factor. Our calculation reveals the symmetry of the $S$ matrix as one of the origins of the elastic enhancement factor; cf. Ref. 25. We emphasize that the derivation of Eq. (4.28) contains an essential approximation; namely that the number $N$ of open decay channels is large compared to unity. One sees, however, that the neglected terms are of higher order in the strength function $x_a$ of Eq. (3.8). Therefore the final result is valid for two complementary limiting conditions which cover all the physics of absorption into the compound nucleus: The present formalism applies for any absorption if $N$ is large and it applies for any $N$ if the absorption is small. Note that small $N$ occur mainly below neutron emission threshold where absorption is small.

(ii) We have singled out the observable that is sensitive to the properties of the statistical ensemble. This observable is the factor $W$ of enhancement over the Hauser-Feshbach formula occurring for elastic scattering; see Eq. (6.16). It turns out to depend on the ratio $\overline{\gamma^4}/\overline{\gamma^2}^2$ of statistical moments of the decay amplitudes and on the autocorrelation function $g$ [see Eq. (2.14)] of the resonance level distribution. Introduction of Gaussian decay amplitudes yields experimentally known results: $W=2$ for overlapping resonances,[19] $W=3$ for isolated resonances.[21,22] We note that $W$ does not depend on the resonance level distribution in both limiting cases: very weakly and very strongly overlapping resonances. This means that the experimental results—in connection with the present formalism—confirm the Gaussian statistics of resonance decay amplitudes. Considering $W$ as a function of the transmission coefficient, its slope is—for weak absorption—inversely proportional to the stiffness parameter of the spectrum. The stiffer the spectrum, the steeper the slope of $W$; see end of Sec. VII. For intermediate absorption, $W$ is given by a functional in the autocorrelation function $g$. Since $W$ essentially measures the correlation between the in- and out-going channels, one may say "the autocorrelation of the resonance level distribution weakens the correlation of entrance and exit channel."

(iii) If Gaussian statistics is accepted for the decay amplitudes, then the results of the present formalism depend only on the stiffness of the energy spectrum (and, of course, on the transmission coefficients). This parameter can be determined by requiring Wigner's level repulsion; cf. Eq. (6.5). We incorporate it into our model by constructing a joint probability distribution for the resonance levels which is symmetrized in the energy arguments [Eq. (9.2)]. This removes at the same time an artificial ordering of the resonance levels.

(iv) The present result for the enhancement factor $W$ agrees with the Monte Carlo calculations of Ref. 11 within the errors of the latter and for the full range of absorption; see Fig. 2.

In conclusion, we hope that the present work, in connection with sufficiently precise experiments, will help to decide whether Gaussian statistics and the fundamental ideas connected with it (cf. Refs. 7–9) are realized in nature.

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[1]B. H. Chou, G. E. Mitchell, E. G. Bilpuch, and C. R. Westerfeldt, Phys. Rev. Lett. **45**, 1235 (1980); Z. Phys. A **300**, 157 (1981); J. F. Shriner, Jr., E. G. Bilpuch, C. R. Westerfeldt, and G. E. Mitchell, *ibid.* **305**, 307 (1982); J. F. Shriner, Jr., K. M. Whatley, E. G. Bilpuch, C. R. Westerfeldt, and G. E. Mitchell, *ibid.* **313**, 51 (1983); P. Ramakrishnan, B. H. Chou, G. E. Mitchell, E. G. Bilpuch, R. O. Nelson, J. F. Shriner, Jr., and C. R. Westerfeldt, *ibid.* **311**, 157 (1983); G. E. Mitchell, E. G. Bilpuch, J. F. Shriner, Jr., and A. M. Lane, Phys. Rep. **117**, 1 (1985).

[2]H. M. Hofmann, T. Mertelmeier, and H. A. Weidenmüller, Z. Phys. A **311**, 289 (1983).

[3]H. L. Harney, Z. Phys. A **316**, 177 (1984); H. L. Harney, Phys. Rev. Lett. **53**, 537 (1984).

[4]J. F. Shriner, Jr., Phys. Rev. C **32**, 694 (1985).

[5]D. Agassi, H. A. Weidenmüller, and G. Mantzouranis, Phys. Rep. **22c**, 145 (1975).

[6]H. A. Weidenmüller, Ann. Phys. (N.Y.) **158**, 120 (1984).

[7]J. J. M. Verbaarschot, H. A. Weidenmüller, and M. R. Zirnbauer, Phys. Lett. **149B**, 263 (1984); J. J. M. Verbaarschot, H. A. Weidenmüller, and M. R. Zirnbauer, Phys. Rep. **129**, 367 (1985).

[8]T. A. Brody, J. Flores, J. B. French, P. A. Mello, A. Pandey, and S. S. W. Wong, Rev. Mod. Phys. **53**, 385 (1981).

[9]P. A. Mello, P. Pereyra, and T. H. Seligman, Ann. Phys. (N.Y.) **161**, 254 (1985).

[10]J. J. M. Verbaarschot, Ann. Phys. (N.Y.) **168**, 368 (1986).

[11]H. M. Hofmann, J. Richert, J. W. Tepel, and H. A. Weidenmüller, Ann. Phys. (N.Y.) **90**, 403 (1975).

[12]C. Mahaux, and H. A. Weidenmüller, *Shell Model Approach to Nuclear Reactions* (North-Holland, Amsterdam, 1969).

[13]W. A. Watson III, E. G. Bilpuch, and G. E. Mitchell, Z. Phys. A **300**, 89 (1981).

[14]A. Richter, in *Nuclear Spectroscopy and Reactions,* edited by J. Cerny (Academic, New York, 1974), Pt. B, p. 343.

[15]C. A. Engelbrecht and H. A. Weidenmüller, Phys. Rev. C **8**, 859 (1973).

[16]W. Hauser and H. Feshbach, Phys. Rev. **87**, 366 (1952).

[17]T. Ericson and T. Mayer-Kuckuk, Annu. Rev. Nucl. Sci. **16**, 183 (1966).

[18]J. Ernst, H. L. Harney, and K. Kotajima, Nucl. Phys. **A136**, 87 (1969).

[19]W. Kretschmer and M. Wangler, Phys. Rev. Lett. **41**, 1224 (1978).

[20]C. Mahaux and H. A. Weidenmüller, Annu. Rev. Nucl. Part. Sci. **29**, 1 (1979).

[21]A. Bohr and B. R. Mottelson, *Nuclear Structure* (Benjamin, New York, 1969), Vol. I, p. 183; A. Michaudon, in *Statistical Properties of Nuclei,* proceedings of the International Conference, Albany, 1971, edited by J. B. Garg (Plenum, New York, 1972), p. 149.

[22]W. K. Wells, E. G. Bilpuch, and G. E. Mitchell, Z. Phys. A **297**, 215 (1980); E. Arai, M. Futakuchi, H. Kamada, J. Komaki, T. Matsuzaki, M. Ogawa, and Y. Oguri, Nucl. Phys. **A378**, 259 (1982); E. Arai, M. Futakuchi, J. Komaki, M. Ogawa, and Y. Oguri, J. Phys. Soc. Jpn. **52**, 802 (1983); Y. Ozawa, Y. Oguri, and E. Arai, Nucl. Phys. **A440**, 13 (1985).

[23]C. E. Porter and R. G. Thomas, Phys. Rev. **104**, 483 (1956).

[24]T. H. Seligman, Z. Phys. A **305**, 69 (1982).

[25]Z. Vager, Phys. Lett. **36B**, 269 (1971).