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Interpretation of relativistic dynamical effects in proton-nucleus scattering

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A plausibility basis for the relativistic (Dirac) approach to proton-nucleus elastic scattering is outlined. It discriminates between implicit assumptions which appear to be under physical control and those which are not. Recent conjectures concerning the essence of the approach are shown to be either misleading or problematical. The essential physics is dynamical in nature; no pathologies, "unphysical" characteristics, or internal inconsistencies are evident. Remaining areas of concern are identified.

Recently, improved descriptions of proton-nucleus elastic scattering have resulted from use of the Dirac equation.[1-5] That the antinucleon degree of freedom actually plays the crucial role, as is inferred from use of a covariant wave equation, was established in Ref. 2, where the *projectile Z*-graph mechanism (ZGM), virtual $N\bar{N}$ pair production and annihilation, was shown to be the essential new feature. This has resulted in arguments claiming that an alternative interpretation is necessary due to perceived physical inconsistencies of the ZGM, including quenching via anticorrelations,[6] free propagator properties,[6,7] form factor suppression of ZGM,[8] and neglected elementary processes.[9] We analyze these issues via a physically plausible extension of nonrelativistic (NR) multiple scattering theory (MST) and a set of numerical "experiments."

In NR MST the elastic nucleon-nucleus transition operator PTP can be obtained from an auxiliary operator $T'=[(A-1)/A]T$ which satisfies[10-12]

$$T'=(A-1)t+(A-1)tG_0T' \ . \tag{1}$$

After definition of an optical potential operator U' we have[10,11] the elastic scattering equation $PT'P=PU'P+PU'PG_0PT'P$ with

$$U'=(A-1)t+(A-1)^2tQG_0t+\cdots \ . \tag{2}$$

Here, A is the number of target nucleons, P projects onto the nuclear ground state $P=|\phi_0\rangle\langle\phi_0|$, $Q=1-P$, and G_0 is the free projectile-target propagator. The two-nucleon transition operator embedded in the many-body system[10-12] t is replaced in impulse approximation by the free NN t matrix. Two relativistic additions are customary:[2,13] Inclusion of the Møller factor in frame transformations of t, and use of the relativistic energy-momentum relation in G_0. These extensions have become standard in NR analyses,[13] and we regard them so here. The A-dependent factors arise from the rule that the projectile does not interact with the same nucleon consecutively; such repeated interactions are summed by t.

The physical rationale for Eqs. (1) and (2) is the "coherence" property of elastic matrix elements.[11] Elastic matrix elements of t are favored by a statistical factor A relative to inelastic matrix elements since all A nucleons contribute "coherently." Thus, the second-order contribution to $PT'P$ from the first term in Eq. (2) is enhanced by A^2 relative to the first-order contribution of the second term of Eq. (2) for a given single-particle Q-space state. In ^{40}Ca, e.g., the enhancement factor in T' is 1600 [$(1600)^2$ in cross section]. Elastic matrix elements are also enhanced at low momentum transfer (q) because of orthogonality of ground and excited states. Finally, the number of important Q-space states is drastically reduced by the restricted range of relevant momenta and by the need for strong ground state coupling. For these reasons, $\langle\phi_0|U'|\phi_0\rangle$ is almost invariably taken as just the first term of Eq. (2), the familiar $t\rho$ or impulse approximation.

The above picture is spoiled in the event of coupling to very collective intermediate states whose "suppression factor" is naturally $(n/A)^2$ rather than $(1/A)^2$, where n is the number of "active" nucleons. Another such possibility is short range (SR) NN correlations, which can first enter in the second term[14] of Eq. (2). The relevant construct is then the correlation function

$$C_2(\mathbf{r}_1,\mathbf{r}_2)=\rho_{(2)}(\mathbf{r}_1,\mathbf{r}_2)-\rho_{(1)}(\mathbf{r}_1)\rho_{(1)}(\mathbf{r}_2) \ ,$$

with $\rho_{(n)}$ an n-body density. If correlations are important, truncation of Eq. (2) at low order will not be physically consistent.

In adapting this framework to the Dirac approach one first replaces G_0 by the free Feynman propagator $S_F=(\not p-m+i\varepsilon)^{-1}$ and reinterprets the remaining quantities as operators in Dirac, rather than Pauli, spinor space. S_F is currently treated in a static limit. The final step is to identify t as a Dirac-space operator whose positive-energy ($+E$) Dirac spinor matrix elements reproduce NN scattering.[15] An analogous treatment of the target states is also natural, but, being of secondary importance,[2] we ignore it. From the expansion of S_F in free spinors

$$S_F=\int d^3p\,|\mathbf{p}\rangle\left[\frac{u(\mathbf{p},+)\bar{u}(\mathbf{p},+)}{E-E_p+i\varepsilon}+\frac{u(\mathbf{p},-)\bar{u}(\mathbf{p},-)}{E+E_p-i\varepsilon}\right]\langle\mathbf{p}| , \tag{3}$$

i.e., $S_F=G_0^+ +G_0^-$, we see that if G_0^- is neglected then, *by construction,* we recover the NR formulation.[2] Otherwise, there are two coupled (spinor) subspaces: one, characterized by G_0^+, provides the familiar NR dynamics; the other, characterized by G_0^-, adds the antinucleon degree of freedom. Internal consistency of this relativistic extension requires that it be faithful to the original principles of the MST. Clearly maintained are both the crucial coherence property [the target can remain in P space as the projectile scatters into a $(-E)$ state] and the meaning of the statistical factors. The coherence feature is essential since the experimental data require a low q effect. Further consistency questions are addressed below.

It has recently been argued[6] that (SR) nuclear anticorrelations preclude $N\bar{N}$ pair effects because two target nucleons must be closer than the range R of G_0^- [which is $R \sim 1/m$ due, e.g., to the branch point of $E_p = (p^2+m^2)^{1/2}$]. This ignores the range β of the interaction.[7] In general, double scattering terms are fully suppressed if the sum (Σ) of R and 2β is $\Sigma \ll d$. Here, a SR NN "anticorrelation hole" is characterized by $\rho_{(2)}=0$, for $r=|\mathbf{r}_1-\mathbf{r}_2| \leq d$ (typically $d \sim 0.5$ fm), so that $C_2 = -\rho_{(1)}(\mathbf{r}_1)\rho_{(1)}(\mathbf{r}_2)$, for $r \leq d$. In that situation the projectile just cannot reach the second scatterer. Consider in detail three cases: (a) $R \sim 0.2$ fm *and* $\beta = 0$. Then $\Sigma \lesssim d$, and the ZGM *would be* completely suppressed. ZGM effects from iterations of a first-order U' are misleading since they are canceled in (neglected) higher order: The strong second-order optical potential exactly cancels the quadratic ZGM contribution to T', that is $PtPG_0^- tP + PtQG_0^- tP = 0$. Actually, such strong effects must be included to all orders and, if we reexpress Eq. (1) as (i) $T' = \hat{T} + \hat{T}G_0^+ T'$ plus (ii) $\hat{T} = (A-1)t + (A-1) \times tG_0^- \hat{T}$, where propagator components rather than the subspaces P and Q are separated, it is clear that ZGM effects cancel to all orders of MST: The second term of (ii) involves $\rho_{(n)}$ $(n \geq 2)$ and $\rho_{(n)}=0$ when two or more nucleons satisfy $r<d$. Thus in this case, the NR and relativistic results are physically identical. (b) $R \gtrsim 1$ fm *and* $\beta \sim 1$ fm, as in the $(+E)$ Dirac sector or in the NR case. Here, $\Sigma \gg d$ and correlation effects are negligible, as we have already assumed. (c) The actual circumstance for the ZGM is $R \sim 0.2$ fm, but $\beta \gtrsim 1$ fm. Here, $\beta > d$ is of the order of the equilibrium NN separation in nuclei, so that the projectile can interact successively with distinct nucleons without propagation. One expects correlation effects to be small, although not as small as in (b). The suppression is easily estimated for a double scattering term with $R \sim 1/m$ and is found to be a 20% effect.[7] Thus, the ZGM is not removed by SR anticorrelations and the ZGM is consistent with the NR theory.

It is also suggested in Ref. 6 that the empirical merit of the ZGM is due to its accidental removal of perceived "pathologies" in the NR theory. The argument[6] uses a NR reduction of the Dirac equation with vector (V) and scalar (S) potentials, $[\not{p} - \gamma_0 V - (m+S)]\Psi = 0$, to "upper-component" form

$$\left[E - \frac{p^2}{2m} - U_c - \boldsymbol{\sigma}\cdot\mathbf{p}\frac{W_{LS}}{2m}\boldsymbol{\sigma}\cdot\mathbf{p}\right]u = 0 \ , \tag{4}$$

with $W_{LS} = U_{LS}/(1-U_{LS})$, $U_{LS} = (V-S)/2m$, and $U_c = V+S$. The argument in Ref. 6 has three steps: (1) Since the nonlinear part of W_{LS} arises essentially from the ZGM, Eq. (4) yields the NR limit when $W_{LS} \to U_{LS}$. (2) In that NR limit, contributions to the T matrix iterative in U_{LS} contain an effective propagator $p^2 G_0$ by virtue of the explicit momentum factors that accompany U_{LS}, as in Eq. (4). This effective propagator does not fall off at high momenta and one can identify an r-space δ-function (contact) term. (3) When U_{LS} is replaced by W_{LS} and iterative contributions of U_{LS} are again examined, no such contact terms are present. The conclusion in Ref. 6 is that this is the source of the Dirac success. This analysis contains a number of serious flaws.

A well-behaved potential $V(p',p)$ can always be rewritten as $V(p',p) = p'\hat{V}(p',p)p$ to obtain an intermediate $p^2 G_0$ upon iteration, and create the illusion of this type of SR problem without changing the physics. The free propagator is dynamically linked to the potentials: The effective propagation is determined by the detailed interplay between them. Consider Eq. (4) reexpressed as

$$\left[E - (1+W_{LS})\frac{p^2}{2m} - V_{\text{Rest}}\right]u = 0 \ , \tag{5}$$

which defines V_{Rest} as consisting of a velocity-dependent central potential plus a standard NR spin-orbit potential. In realistic NR treatments of nuclear scattering, high momentum components of V_{Rest} are strongly damped by the form factor of the nucleus [e.g., by $\rho(q)$ in $t\rho$]. Generally, nonlocalities[2] in t also damp the potentials in the variable orthogonal to q. However, even for a local approximation, the nuclear form factor, combined with restricted (on-shell) initial and final momenta, ensures proper damping of high intermediate momenta. There is no pathology in V_{Rest}. In fact, we have numerically verified that only nearly-on-shell scatterings are significant. These remarks also apply to the $W_{LS}p^2/2m$ term of Eq. (5), which need not be treated specially.[2] The apparent kinetic-energy-like behavior is suppressed by the nuclear form factor contained in W_{LS} and no contact terms arise upon iteration.[16] Thus, realistic NR spin-orbit potentials do not produce badly behaved phenomena at high momenta, and the success of relativistic approaches *cannot* be attributed to cancellation of (nonexistent) contact terms.

The suggestion[6] of anomalous SR behavior of free propagators carries over to Ref. 7, where it is argued that the origin of the Dirac success is the canceling by G_0^- of "undesirable" SR effects in G_0^+. There, the discussion is in terms of $S_F = G_0^+ + G_0^-$. It is noted that elimination of square root branch cut contributions in the contour integral for G_0^+ (which is connected to SR propagation), yields the full S_F from G_0^+. On this basis three conclusions are drawn: (1) G_0^- cancels (rather than adds) SR structure (found in G_0^+). (2) Mathematically, this is the primary role of antiparticles in the Dirac approach. (3) Physically, the elimination of SR propagation in G_0^+ results in the Dirac success, presumably as opposed to spin-dependent ZG dynamics.

Although the analysis in Ref. 7 is convoluted, (1) and (2) are simply truisms if S_F is adopted as correct (as in Ref. 7), as is (3) in part. For $S_F = G_0^+ + G_0^-$ is just $S_F = (S_F - G_0^-) + G_0^-$. Since we know G_0^- has a very short range, it is clear that removing SR contributions from G_0^+, i.e., $(S_F - G_0^-)$, yields S_F again. Conclusions (1)–(3) then follow. Moreover, the Dirac success is not disconnected from the interactions[2] and (3) seems not to clarify the issue. We have, however, numerically examined the question of SR detail by removing branch cuts using $E_p \simeq m + p^2/2m$ in propagator denominators and with E_p fixed at its asymptotic value E_a in the free spinors. The propagator G_0^- is then of much shorter range, while G_0^+ has no special SR structure at all. The calculation, which includes target recoil effects[2] and maintains the residue at the pole of G_0^+ at its relativistic value, yields spin observables for 500 MeV $p + {}^{40}\mathrm{Ca}$ elastic scattering unchanged to six significant figures. Similarly, replacing E_p by E_a everywhere in G_0^- causes no change. Thus, SR details are insignificant and do not affect the role of G_0^-. This could have been anticipated by consideration of the second-order term $\int d^3p' V(\mathbf{p},\mathbf{p}')G_0^+(p')V(\mathbf{p}',\mathbf{p})$ as a contour integral in the p' plane. Here, p is on shell and, apart from the poles of G_0^+ and V, there is a branch cut contribution along the line $p' = [im, i\infty]$. Along this cut $q = |\mathbf{p}' - \mathbf{p}| > m$ and $V(\mathbf{p}',\mathbf{p})$ is dominated by the nuclear shape through the density $\rho(q)$. Branch cut effects are suppressed by the enormous cost in q due to the distance to the on-shell momentum point.

It has also been suggested[8] that the Dirac approach is inappropriate due to suppression of pair effects by the nucleon form factor. This originates from general kinematic considerations at the vertex coupling a field quanta to a physical $N\bar{N}$ pair, where the form factor must support four-momentum transfer $q^2 > 4m^2$. A specific argument in the language of quantum chromodynamics (QCD) is that it is difficult to turn three quarks around (into antiquarks) since nucleons are extended "floppy" objects. When an interaction with one quark occurs, many internal interactions must occur to influence the other quarks, yielding suppression. This argument is appropriate to the perturbative domain of QCD where form factors must support large q^2. However, the relativistic mechanism under discussion here occurs at low q^2, the strong coupling domain. Consider the limit wherein a microscopic, covariant theory such as QCD may go over to a quantum electrodynamics (QED)-like framework for nucleons with form factors to describe the "quasiparticles." The residual covariance then dictates the same form factors $f(q^2)$ for NN and $N\bar{N}$ vertices. Pair effects near production threshold are drastically suppressed by form factors since $q^2 \sim 4m^2$, while the $(-E)$ propagator acquires a pole and causes no suppression. This circumstance is reversed in the Dirac approach under present discussion for which $q = (0,\mathbf{q})$. At $q^2 \sim 0$ suppression comes not from form factors but from the propagator which is $\gtrsim 2m$ away from the physical antiparticle unitarity cut. Physically, in the $q^2 \sim 0$ region of QCD the quarks are stiffly connected and move in unison so that a quasiparticle approach is reasonable. The nucleon is an extended object, but it is not floppy for small q^2, and a covariant treatment with form factors leads naturally to the Dirac approach with the form factors absorbed into the interactions. Although the low q^2 limit of a fundamental theory like QCD is certainly problematical, the Dirac approach seems more natural for such investigations than an *ad hoc* NR presumption.

Finally, one must ask whether the Dirac successes may be due to simulation of neglected degrees of freedom (e.g., quark manifestations). Such possibilities are drastically reduced in our formulation since the target must remain in the P space as the projectile couples to the new degree of freedom, or else face the enormous suppression factors described earlier.[17] In general, candidate processes must possess the coherence property, must support low q effects, and must show definite spin character.[1-5] This question merits further investigation, but, even if alternatives are found, it is still necessary to explain why ZG effects should be absent.

Remaining concerns span topics ranging from NR MST to QCD. In the NR domain, use of the free NN t and neglect of higher-order terms of Eq. (2) are of concern, and current relativistic extensions of NN amplitudes are uncertain: How are the $(-E)$ spinor matrix elements constrained by experiment? From field theory, under what circumstances are leading corrections to a NR limit described by the ZGM and form factors? Are nonstatic effects important? What other implications of relativistic dynamics can be confirmed? Finally, consistency with a microscopic theory remains uncertain: What is the role of covariance in the description of composite objects? These questions cannot be ignored. Pending their resolution the Dirac approach must be regarded with a certain skepticism. However, specific criticisms have been shown to be either readily dismissed or themselves problematical: None vitiates the approach or seems to tip the balance in that direction.

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[14]An infinite series of such terms, involving scattering from correlated pairs, sum to the second-order spectator term. See Ref. 12.

[15]This is not enough to specify t; model-dependent extensions fix negative energy spinor matrix elements.

[16]Moreover, if one looks upon this term of Eq. (5) as defining a position-dependent effective mass m^*, then whether the full W_{LS} or just U_{LS} is used, $m^*/m \sim 2/3$ at the peak of U_{LS} (which is ~ 0.4). This term, when absorbed into the free propagator, thus causes suppression rather than enhancement of large intermediate momenta (it costs more energy to give momentum to lighter particles) in either case.

[17]A nice example is intermediate Δ excitation. For an isospin zero target, isospin conservation precludes the coherent process and thus damps intermediate Δ excitation.