Comment on "Method for observation of neutrinos and antineutrinos"

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I discuss a derivation of the total cross section for neutrino (and antineutrino) scattering by a crystal, with emphasis on the problem of coherence or incoherence over the volume of the crystal in the process.

In two papers,[1,2] Weber has presented a new approach to the derivation of the cross section for neutrino (antineutrino) scattering by a crystal. The interesting feature is that his total cross section exhibits coherent behavior in a kinematic region where the maximum momentum transfer from the neutrinos is large compared to the inverse of the bulk size of the scatterer. Standard discussions of coherent scattering require that the maximum momentum transfer be *smaller* than the inverse bulk size.[3] I discuss inconsistencies in his derivation and possible corrections to it. Ordinary incoherent scattering total cross sections result.

We can see from simple physical arguments given in Ref. 3 that large *total* cross sections are not possible for the kinematic regime under consideration (that given in the preceding paragraph for incoherent scattering). The differential cross section for N scattering sites in three dimensions is $\mathscr{O}(N^2)$ *only* within a forward diffraction peaks subtending a solid angle $\mathscr{O}(N^{-2/3})$, and is $\mathscr{O}(N)$ for angles greater than that. This gives a *total* cross section $\mathscr{O}(N^{4/3})$. However, the total force (proportional to the momentum-transfer-weighted total cross section) will be $\mathscr{O}(N)$, since the "typical" momentum transfer is $\mathscr{O}(N^{-1/3})$ (roughly proportional to the square root of the solid angle). Scattering at larger solid angles occurs, but it does not contribute significantly to the total cross section for scattering (the height of the diffraction peaks here being smaller).

Weber argued that since neutrinos are weakly interacting and essentially massless, they were "special" in that they could scatter through any angle, including 180°. We have just shown that argument to be wrong. *Any* particle can scatter through *any* angle. This being true, then there is really no restriction on the type of scattering process that we could apply Weber's results to. A recent comment by Ho has ruled out large cross sections for neutron scattering,[4] and there is no evidence anywhere for such in x-ray scattering. This means that there must be something fundamentally wrong with Weber's derivation.

I will use Weber's notation for my discussion. He is unfortunately not completely clear or consistent in its use. However, to make my point I need to follow his derivations as closely as possible, and this includes the notation.

The basic problem lies in Weber's construction of his final state crystal excitation operator for the scattering process [this is derived in his equations (26a)-(26c)]

$$\Psi_{SF}^{\dagger}=\sum_{n=1}^{N}\psi_{SOn}^{*}e^{-i\Delta P_{\mu}x^{\mu}}a_{On}^{\dagger}\ , \tag{1}$$

where $\mathbf{\Delta P}$ is the total momentum transfer to the scatterer [see Ref. 1 for the definitions of other symbols in Eq. (1)]. We can compare this to the operator given in Eq. (26) of Weber's paper,

$$\Psi_{SF}^{\dagger}=\sum_{n=1}^{N}\psi_{SOn}^{*}e^{-i(\Delta p_{\mu})_n x^{\mu}}a_{On}^{\dagger}\ , \tag{2}$$

where $\sum_{n=1}^{N}\Delta\mathbf{p}_n=\Delta\mathbf{P}$. Weber uses this in the case where all $\Delta\mathbf{p}_n=\Delta\mathbf{P}/N$ to obtain the "standard" incoherent total scattering cross section proportional to N. Clearly, if momentum is to be conserved, the two results are inconsistent. Equation (2) is certainly valid. The problem with Eq. (1), as I will show, lies in the normalization of the operator.

Let us reconsider Weber's derivation of Eq. (1). If we start with his Eq. (26a), we see that he *sums* only the first term over all possible scattering sites (since he assumes that the actual site is unknown in an experiment). However, the correct treatment is to *average* over all possible scattering sites. This is needed to not only avoid the problem of momentum nonconservation mentioned earlier, but also in order to fulfill the normalization condition given by Weber in his Eq. (24). It is not at all reasonable to ignore the second term in his Eq. (26a). If it is kept so that

$$\Psi_{SF}^{\dagger}=\frac{1}{N}\sum_{n=1}^{N}\psi_{SOn}^{*}(e^{-i\Delta P_{\mu}x^{\mu}}+N-1)a_{On}^{\dagger}\ , \tag{3}$$

we see that it contributes a term $\mathscr{O}(N)$. If we now average over the number of particles in the crystal, the excitation operator behaves as

$$\Psi_{SF}^{\dagger}=\sum_{n=1}^{N}\psi_{SOn}^{*}a_{On}^{\dagger}+\mathscr{O}(N^{-1})\ , \tag{4}$$

from which we get an incoherent total cross section. This is now consistent with our earlier physical arguments.

In conclusion, we have shown that Weber's derivation of large total cross sections is wrong on the basis of elementary physical arguments and that it is a result of an incorrect mathematical derivation.

Note: Bertsch and Austin have published a Comment[5] which uses similar physical arguments to ours to disallow Weber's result.

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[1]J. Weber, Phys. Rev. C **31**, 1468 (1985).

[2]J. Weber, Found. Phys. B **14**, 1185 (1984).

[3]See, for example, R. Feynman, *Feynman Lectures in Physics* (Addison-Wesley, Reading, Mass., 1963), Vol. I, Sec. 32-5; L. D. Landau and E. N. Lifshitz, *Electrodynamics of Continuous Media,* 4th ed. (Pergamon, Oxford, 1979), Chap. 9; J. D. Jackson, *Classical Electrodynamics,* 2nd ed. (Wiley, New York, 1975), Sec. 14.8.

[4]T. H. Ho, Phys. Lett. **168B**, 295 (1986).

[5]G. F. Bertsch and S. M. Austin, Phys. Rev. C **34**, 361 (1986).