Configuration mixing among $11/2^-$ states in ^{19}F

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The apparent contradiction between alpha strengths of $11/2^-$ states in ^{19}F and their $E2$ decays to $7/2^-$ states is removed by considering configuration mixing in both $7/2^-$ and $11/2^-$ states.

Configuration mixing[1] among the $11/2^-$ states[2–10] in ^{19}F is a complicated[1,3,5,6,9] and still somewhat controversial[10–12] topic. Near 8.5 MeV excitation in ^{19}F there should exist one $13/2^-$ and one $11/2^-$ state that arise from coupling a $1p_{1/2}$ proton hole to the 6^+ level at 8.78 MeV in ^{20}Ne. These two states should have three basic properties:

(1) Strong $L=6$ α-particle transfer in ^{15}N(^{6}Li,d) or ^{15}N(^{7}Li,t). The α spectroscopic factors, S_α, for the two states should be equal and equal to S_α for the ^{20}Ne (6^+) level.

(2) Strong $E2$ decays from the $13/2^-$ and $11/2^-$ to the $9/2^-$ and $7/2^-$ levels, respectively, at $E_x=4.03$ and 4.00 MeV. By comparison with ^{20}Ne and with lower-lying members of the $1/2^-$ band in ^{19}F, the $B(E2)$ values should be about 20 W.u.

(3) The $13/2^-$ state should lie slightly below the $11/2^-$. If we take the splitting from the $5/2^-,3/2^-$ doublet at 1.35,1.46 MeV and assume a $(2L+1)$ dependence, the $13/2^-$ state should be about 294 keV below the $11/2^-$. (This ordering is not exhibited by the $9/2^-,7/2^-$ pair of levels at 4.03,4.00 MeV, but we return to this point below.)

The actual physical situation is quite different. There is only one $13/2^-$ state [at 8.288 MeV (Ref. 2)] in the appropriate energy region, but three $11/2^-$ states[2,5,6,9]—at 7.166, 8.953, and 9.873 MeV. (These excitation energies are from the latest compilation, but they differ only very slightly from those in the most recent gamma-decay study.[11]) The $13/2^-$ state has the expected angular-distribution shape and cross section magnitude in ^{15}N(^{6}Li,d) (Ref. 10), but the $11/2^-$ α strength is split—with most of it in the middle $11/2^-$ level at 8.95 MeV. Furthermore, either the total $11/2^-$ α strength is much larger than that observed for the $13/2^-$ state, or an unresolved state is contributing to the 8.95-MeV cross section in (^{6}Li,d). If the total $11/2^-$ α strength is comparable to that for the $13/2^-$ state, the splitting of S_α among the three $11/2^-$ states is listed (as case I) in Table I. If, on the other hand, all the 8.95-MeV cross section belongs to the $11/2^-$ state, the splitting is as listed for case II.

In the ^{15}N(^{13}C,^{9}Be) reaction,[1] which favors high L transfer much more dramatically than does (^{6}Li,d), the 8.95- and 8.29-MeV cross sections are roughly equal. In fact, in Ref. 1, the $11/2^-$ yield is about 10% larger than one would expect from the observed yield for the $13/2^-$ state, if the two states had equal S_α. In that reference, the $L=6$ to $L=2$ ratio (for equal spectroscopic factors) is about 20 to 1, whereas in (^{6}Li,d) (Ref. 10), that ratio is roughly 2 to 1. These considerations might suggest that the "extra" cross section observed in (^{6}Li,d) for the 8.95-MeV state indeed arises from an unresolved low J state—as suggested in Ref. 10.

The gamma decay of the $13/2^-$ is as expected—an $E2$ of 19 ± 2 (Ref. 2) or 22 ± 3 (Ref. 13) W.u. to the lowest $9/2^-$ state. However, the $E2$ decays[2,5,6,9] of the $11/2^-$ states are quite complicated. It is the lowest $11/2^-$ level, at 7.17, that has the strongest $E2$ decay (12.6 ± 2.4 W.u. in Ref. 11, 15 ± 5 W.u. in Ref. 9) to the lowest $7/2^-$ state. The 8.95-MeV $11/2^-$ state decays[2,6] to *two* $7/2^-$ states with comparable strengths. These numbers are given in Table II.

In both α transfer[1,3,10] and $E2$ decays,[2,5,6] the third $11/2^-$ level possesses little strength. In three-nucleon transfer,[7,8] the situation is complex—all three states have some strength. In Ref. 8, the 9.87-MeV state is about twice as strong as the 8.95-MeV level, but the resolution is poor—about 300 keV, so that the presence of additional states cannot be ruled out. In Ref. 7, the ratio of integrated cross sections is 28:19, while at forward angles they are about equal. In both reactions, the $13/2^-$ at 8.28 MeV is quite weak.

It was in Ref. 1 that the solution to the $11/2^-$ puzzle was first attempted. In the present paper, we suggest what appears to be the most reasonable explanation and attempt to reconcile the available data in a consistent model. The presence of three $11/2^-$ states in this energy region is not, of itself, surprising. In addition to the $^{20}\mathrm{Ne}\otimes(1p_{1/2})^{-1}$ state discussed above, an $11/2^-$ level of $(sd)^2(fp)$ character should exist nearby.[1] The most likely configuration[1] for a third $11/2^-$ state is $(sd)^4\,(1p)^{-1}$, but with the four particles coupled to $T=1$, i.e.,

TABLE I. Relative alpha-particle spectroscopic factors of the $11/2^-$ states.

State	$S_\alpha(11/2^-)/S_\alpha(13/2^-)$ Case I	Case II
7.17 MeV	0.216	0.20
8.95 MeV	0.784	1.80

TABLE II. $B(E2)$ values for the $11/2^- \to 7/2^-$ decays in ^{19}F.

E_i (MeV)	E_f (MeV)	$B(E2)$ (W.u.)	Ref.
7.167	3.999	12.6±2.4	12
8.953	3.999	5.1±1.1	6
	5.420	5.3±1.2	6
9.872	3.999	1.8±0.4	6

$^{20}\mathrm{F}\otimes(1p_{1/2})^{-1}$. The three observed $11/2^-$ levels would then be linear combinations of these three basis states.

We look first at the gamma decays. The existence[2] of a strong $E2$ from 8.95 to 5.42, $7/2^-$ suggests the possibility of configuration mixing between two (at least) $7/2^-$ states. This conjecture is supported by the presence[2] of a strong $E2$ from the 5.42-MeV state to the lowest $3/2^-$. In fact, the numerical values are very similar—5.7 ± 1.2 W.u. for $5.42\to1.46$ and 5.3 ± 1.1 W.u. for $8.95\to5.42$. Hence, the $E2$'s cannot be understood without considering mixing of the $7/2^-$ levels.

Mixing of the $7/2^-$ states is also suggested by the "wrong" order of the lowest $9/2^-,7/2^-$ pair. With no mixing, the $7/2^-$ should be about 203 keV above the $9/2^-$, whereas it is 338 keV below it. In a two-state model, this energy shift corresponds to an "impurity" of 16.7% in the 4.00-MeV, $7/2^-$ state. If the admixed $7/2^-$ state has no direct $E2$ to the $^{20}\mathrm{Ne}\otimes(1p_{1/2})^{-1}$ $3/2^-$ level [a reasonable assumption for either of the two expected configurations—$(sd)^2fp$ and $^{20}\mathrm{F}\otimes(1p_{1/2})^{-1}$], then the measured $B(E2)$ for $5.42\to1.46$ can be used to infer an independent value of the mixing amplitude. For this calculation, we need the total $E2$ strength in the unmixed limit, for which we use the weighted average in ^{20}Ne and ^{19}F—20.6 ± 0.8 W.u. (See Table III.) The 5.7 ± 1.2 W.u. $E2$ for $5.42\to1.46$ then gives $(27.7\pm5.9)\%$ for the admixed probability. This latter number is probably consistent with the value arrived at above from the observed $7/2^-$ energy, considering the uncertainties. Thus, it appears that the two $7/2^-$ states are of the form

$$\Psi(4.00)=\alpha\,^{20}\mathrm{Ne}\otimes(1p_{1/2})^{-1}+\beta \text{ other },$$

$$\Psi(5.42)=-\beta\,^{20}\mathrm{Ne}\otimes(1p_{1/2})^{-1}+\alpha \text{ other },$$

with β in the range 0.409—0.526.

We now return to the α strengths for the $11/2^-$ states and ask if they can be understood with only two states and not three. This restriction might seem reasonable, as the third $11/2^-$ level is weak in α transfer and has a small $B(E2)$ to the lowest $7/2^-$ state. We consider cases I and II separately.

In case I we assume the total $11/2^-$ strength is equal to the strength observed for the one $13/2^-$ state, i.e., we assume that the summed S_α for the $11/2^-$ states is equal to S_α for the $13/2^-$ state at 8.29 MeV—the summed $11/2^-$ peak differential cross section would be 12/14 of $\sigma(13/2^-)$. In that case, part of the 8.95-MeV cross section arises from a nearby unresolved state, as suggested in Ref. 10. The $11/2^-$ strength is then split between the two lowest $11/2^-$ states in the ratio 0.216:0.784. Then

$$\Psi_{\mathrm{I}}(7.17)=A\,^{20}\mathrm{Ne}\otimes(1p_{1/2})^{-1}+B \text{ other },$$

$$\Psi_{\mathrm{I}}(8.95)=-B\,^{20}\mathrm{Ne}\otimes(1p_{1/2})^{-1}+A \text{ other },$$

and the α strength splitting immediately gives $|A|=0.465$, $|B|=0.885$. This value of A is (perhaps coincidentally) right in the middle of the range derived above for the $7/2^-$ mixing amplitude. But, of course, the ordering is opposite—it is the second $11/2^-$ state that is dominantly $^{20}\mathrm{Ne}\otimes 1p^{-1}$.

The unperturbed energies of the basis states then turn out to be 8.567 MeV for the $^{20}\mathrm{Ne}\otimes 1p^{-1}$ state and 7.552 MeV for the other $11/2^-$ level. The $13/2^-$, $11/2^-$ splitting is then 279 keV, which agrees quite well with the 294 keV expected from the $5/2^-,3/2^-$ splitting.

These wave functions can account for the $E2$'s (if at all) only if the states labeled "other" also have a nonzero $E2$ matrix element, which we take to be $u\sqrt{\mathrm{W.u.}}$ We still assume no cross terms in the $E2$. It turns out to be possible to fit within error bars the $B(E2)$'s from both $11/2^-$ states to the lowest $7/2^-$ state with a reasonable value of $u=3$–$5\sqrt{\mathrm{W.u.}}$ But the predicted $11/2_2^-\to7/2_2^-$ $B(E2)$ always turns out to be too large by about a factor of 2. [Because B/A $(11/2^-)$ is roughly equal to α/β $(7/2^-)$, the $11/2_2^-\to7/2_2^-$ $B(E2)$ will always be comparable to that for $11/2_1^-\to7/2_1^-$.] Of course, we have neglected a cross term in the $B(E2)$ and a small value of such a term, because it would be destruc-

TABLE III. Pertinent $B(E2)$ values (in W.u.) in ^{20}Ne and ^{19}F.

	^{20}Ne			^{19}F	
Initial	Final	$B(E2)$	Initial	Final	$B(E2)$
2^+,1.63	0^+,g.s.	20.3±1.0	$5/2^-$,1.35	$1/2^-$,0.11	20±3
			$3/2^-$,1.46	$1/2^-$,0.11	25±11
4^+,4.25	2^+,1.63	22±2	$9/2^-$,4.03	$5/2^-$,1.35	28±6
			$7/2^-$,4.00	$3/2^-$,1.46	16±10
			$7/2^-$,5.42	$3/2^-$,1.46	5.7±1.2
6^+,8.78	4^+,4.25	20±3	$13/2^-$,8.29	$9/2^-$,4.03	19±2
					22±3
			$11/2^-$,7.17	$7/2^-$,4.00	12.6±2.4
			$11/2^-$,8.95	$7/2^-$,4.00	5.1±1.2
			$11/2^-$,8.95	$7/2^-$,5.42	5.3±1.1

tive in one case and constructive in the other, could fix the discrepancy. The cross term needed is only about 0.2—0.4 W.u.

We turn now to case II, but still in a two-state model. For case II to be correct, both $11/2^-$ basis states must have appreciable α strength. In fact, the total $11/2^-$ cross section in case II is 2.12 times that expected from the observed $13/2^-$ yield and a $2J+1$ ratio. The "other" $11/2^-$ basis state must then have $S_\alpha = 1.12$ times S_α for $^{20}\mathrm{Ne}\otimes 1p^{-1}$. To make the arithmetic easy, we assume equal α strengths for the two basis states.

Then the observed ratio of cross sections for the two $11/2^-$ states provides the constraint

$$[(A+B)/(A-B)]^2 = 0.11$$

giving $A/B = -1.99$ or -0.50. The minus sign is necessary for the lower $11/2^-$ to be weaker in α transfer. If we pick the second choice, $A/B = -0.50$, the results $A = 0.447, B = -0.894$ are remarkably similar (up to a sign) to the situation in case I. All remarks about the $E2$ mixing problem will still hold, but of course with opposite signs for the interfering terms.

To summarize the two-state mixing results: Case I and case II give (coincidentally) very similar wave functions for the $11/2^-$ states. In both cases, it is necessary for the "other" $11/2^-$ basis state to have a strong $E2$ to the "other" $7/2^-$ basis state. Furthermore, the fit to the $11/2^- \rightarrow 7/2^-$ $B(E2)$'s is only semiquantitative.

In Ref. 1, it is pointed out that the $(sd)^2fp$ states would have appreciable α-transfer strength, but that the $^{20}\mathrm{F}\otimes 1p^{-1}$ states would lie significantly lower in energy than the $(sd)^2fp$. It is thus very likely that the extra $E2$ strength comes from mixtures of $^{20}\mathrm{F}\otimes 1p^{-1}$, whereas the extra α strength in case II arises from $(sd)^2fp$ admixtures. That is, we must use a three-state basis space, at least for the $11/2^-$ levels. We let ϕ_1, ϕ_2, and ϕ_3 be the basis states and Ψ_A, Ψ_B, and Ψ_C be the physical states at 7.17, 8.95, and 9.87 MeV, respectively. We assume basis states 1 and 2 have the α strength, whereas it is state 3 that has the $E2$ to the "other" $7/2^-$ state.

For any given value of the $7/2^-$ mixing amplitude, this model has four unknown parameters—three independent wave-function amplitudes for the $11/2^-$ states and the $E2$ matrix element connecting ϕ_3 with the "other" $7/2^-$ basis state. The four pieces of experimental information we use to determine these four quantities are the three $B(E2)$'s from the lowest two $11/2^-$ states and the S_α ratio for these two states.

It turns out that these four experimental quantities *can* be fitted simultaneously for a range of values of the $7/2^-$ mixing amplitude. We plot in Fig. 1 the $11/2^-$ wave-function amplitudes versus this $7/2^-$ mixing amplitude. These $11/2^-$ wave functions fit exactly the four quantities mentioned above—i.e., the calculated value goes through the center of the error bar. The fitted value of the $B(E2)$ connecting ϕ_3 and $7/2^-$ (other) is also plotted

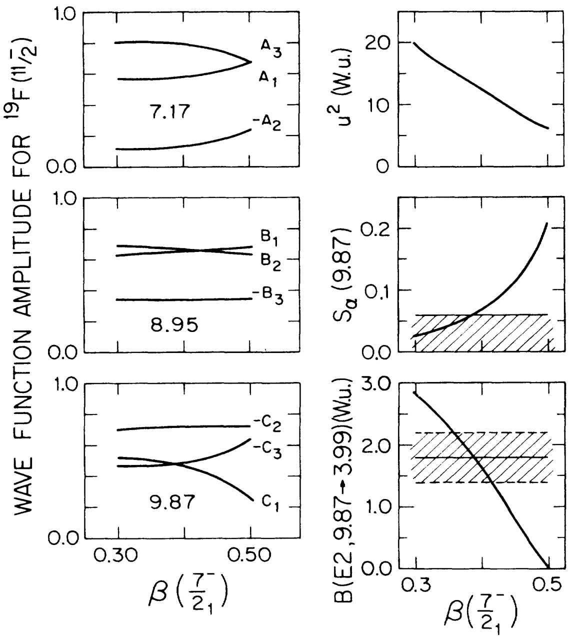

FIG. 1. Left: Wave function amplitudes for $11/2^-$ states of ^{19}F, plotted versus mixing amplitude in the lowest $7/2^-$ state. Right: Top—the $B(E2)$ connecting the $11/2^-$ ϕ_3 basis state with the "other" $7/2^-$ basis state; middle—α-particle spectroscopic factor of the 9.87-MeV state (relative to that of the $13/2^-$ level); bottom—$B(E2)$ for 9.87, $11/2^-$ to 3.99, $7/2^-$.

vs $\beta\,(7/2^-)$ in Fig. 1.

The S_α and $E2$ properties of the third $11/2^-$ state are now fully determined within the model, but do they fit the data? The predicted S_α (9.87) and $B(E2;9.87\text{-}3.99)$ are plotted in the figure for various values of β $(7/2^-)$. (Remember the wave functions for all three $11/2^-$ states depend somewhat on the value of this amplitude.) The experimental limit for S_α is ≤ 0.06, and the $B(E2)$ is 1.8 ± 0.4 W.u. We note agreement with these two experimental values for β in the allowed range.

If the basis states are as expected, neither ϕ_1 nor ϕ_3 has any three-nucleon transfer strength, so that the three-nucleon strengths should simply be proportional to the square of the coefficient of ϕ_2. For the full range of β displayed in Fig. 1, the ratio $(B_2/C_2)^2$ is approximately consistent with the ratio observed in three-particle transfer.

It thus appears that the long-standing puzzle of the $11/2^-$ states in ^{19}F can be understood. The basic ideas concerning the identities of the basis states were already present in Ref. 1. It only remained to derive the wave functions from experimental quantities. These $11/2^-$ wave functions should now be used to calculate other observables not yet measured, to see how well (or badly) they do in predicting other quantities.

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[1] A. A. Pilt *et al.*, Nucl. Phys. **A273**, 189 (1976).
[2] F. Ajzenberg-Selove, Nucl. Phys. **A392**, 1 (1983).
[3] R. Middleton, in *Nuclear Reactions Induced by Heavy Ions, Heidelberg* (North-Holland, Amsterdam, 1970), p. 263.
[4] B. Y. Underwood *et al.*, Nucl. Phys. **A225**, 253 (1974).
[5] L. K. Fifield *et al.*, Phys. Lett. **68B**, 125 (1977).

[6]T. J. M. Symons *et al.,* J. Phys. G **4,** 411 (1978).
[7]K. Van der Borg, R. J. de Meijer, and A. Van der Woude, Nucl. Phys. **A273,** 172 (1976).
[8]M. Hamm *et al.,* Phys. Rev. Lett. **36,** 846 (1976).
[9]D. W. O. Rogers, W. R. Dixon, and R. S. Storey, Can. J. Phys. **51,** 1 (1973).
[10]S. Mordechai and H. T. Fortune, Phys. Rev. C **29,** 1765 (1984).
[11]S. Mordechai and H. T. Fortune, Phys. Rev. C **32,** 2207 (1985).
[12]W. R. Dixon, D. W. O. Rogers, R. S. Storey, and A. A. Pilt, Phys. Rev. C **32,** 2205 (1985).
[13]B. Y. Underwood *et al.,* Nucl. Phys. **A225,** 253 (1974).