

Effect of deuteron structure on hyperfine structure of deuterium

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The effect of deuteron structure on the hyperfine structure of deuterium is reexamined by using several realistic nucleon-nucleon (NN) potentials. We find that the value of the Bohr-Low correction is almost the same for all the potentials considered, and is larger than the value obtained by Low and by Low and Salpeter by using much simpler potentials. Among other corrections to the hyperfine structure, the most important one is due to angular momentum (L) dependent terms in the NN potential. This correction is sensitive to the choice of the potential. When these other corrections are included, theory and experiment become compatible for some of the potentials, while the situation is worsened for others. However, there are remaining ambiguities related to mesonic and isobaric effects, the implications of which are discussed.

I. INTRODUCTION

The hyperfine structure (hfs) of deuterium has been a subject of special interest due to its relatively large anomaly as compared to that of hydrogen. According to Fermi's model,¹ in which the nucleus is treated as a point particle, the hfs of the hydrogen-like atom is given by

$$h\nu = \frac{8\pi}{3} \left[\frac{2I+1}{I} \right] \mu_0 \mu \psi^2(0), \quad (1.1)$$

where I is the nuclear spin, μ the magnetic moment of the nucleus, μ_0 the Bohr magneton, and $\psi(0)$ the value of the Schrödinger wave function of the electron evaluated at the origin. There are two types of corrections to Fermi's formula; the first one is of quantum electrodynamic (QED) origin while the other is hadronic in nature. The hfs predicted by Eq. (1.1), together with these corrections, agrees with experiment for hydrogen;² however, there is a significant discrepancy in the case of deuterium.³ This discrepancy is large in view of the degree of accuracy of QED.

In comparing theory and experiment it is convenient to take the ratio ν_D/ν_H , where D and H refer to deuterium and hydrogen, respectively. Fermi's formula (1.1) gives

$$\left[\frac{\nu_D}{\nu_H} \right]_{\text{Fermi}} = \frac{3}{4} \frac{\mu_D}{\mu_H} \left[\frac{m_D}{m_H} \right]^3, \quad (1.2)$$

where m_D and m_H are the electron reduced masses for the deuterium and hydrogen atoms, respectively. The experimental value for the ratio deviates from Fermi's value; hence we write

$$\frac{\nu_D}{\nu_H} = \left[\frac{\nu_D}{\nu_H} \right]_{\text{Fermi}} (1 + \Delta). \quad (1.3)$$

An important point to note is that all QED corrections cancel in this ratio so that Δ is entirely due to hadronic corrections. The experimental value for Δ is given by³

$$\Delta_{\text{expt}} = (170.3 \pm 0.5) \times 10^{-6}. \quad (1.4)$$

Immediately after the first measurement of ν_D/ν_H , Bohr⁴ pointed out that the bulk of Δ_{expt} can be explained as being due to the loose structure of the deuteron. At small electron-deuteron separations the electron wave function is centered on the proton rather than the center of the deuteron; this results in a reduction of the effect of the neutron magnetic moment and hence an enhancement of the hfs. Subsequently, Low⁵ reexamined Bohr's mechanism in detail and obtained

$$\Delta_{\text{Low}} = (183 \pm 22) \times 10^{-6}, \quad (1.5)$$

which is consistent with the experimental value, Eq. (1.4). The uncertainty in Δ_{Low} of Eq. (1.5) is mainly due to the spread of its values for different potentials considered. Low used very simple models for the nucleon-nucleon (NN) interaction, i.e., the square well, the exponential, and the Hulthén potentials. A few years later, with improved knowledge of the deuteron wave function, Low and Salpeter⁶ recalculated Δ_{Low} and obtained⁶

$$\Delta_{\text{Low}} = (198 \pm 10) \times 10^{-6}. \quad (1.6)$$

This exceeds Δ_{expt} by about 20 ppm (parts per million) or more.

Later Greenberg and Foley⁷ used the more realistic potential of Signell and Marshak,⁸ which contains a spin-orbit interaction. They also included the nucleon structure effect on the hfs and obtained

$$\Delta = \begin{cases} (195 \pm 41) \times 10^{-6} - 0.004\mu_{LS} & \text{with } P_D = 7\%, \\ (218 \pm 46) \times 10^{-6} - 0.004\mu_{LS} & \text{with } P_D = 4\%, \end{cases} \quad (1.7)$$

where P_D is the D -state probability, and μ_{LS} ($= -0.024$) is the deuteron magnetic moment, in nuclear magnetons, produced by the spin-orbit force. With the large contribution of the spin-orbit interaction (96 ppm), the discrepancy between theory and experiment becomes much greater. This is the situation in which the deuterium hfs problem has been left. Since 1960, however, our understanding of nuclear forces has considerably improved. A number of phenomenological NN potentials have been constructed

which reproduce the properties of the deuteron and scattering data very well. The purpose of this paper is to reexamine the problem by using a number of modern realistic NN potentials.⁹⁻¹⁴ For the method of calculation we closely follow Low.⁵ We estimate the Bohr-Low correction Δ_{Low} with these potentials. We find it remarkable that Δ_{Low} varies little from one potential to the other; its values¹⁵ range between 213 and 222 ppm.

At this point we should note that, in addition to Δ_{Low} , there are other corrections which are not directly related to the NN interaction. They are those due to the finite electromagnetic size of the nucleon and those due to nuclear recoil. We denote the part of the Δ that is due to these corrections by Δ_{other} . Its value has been estimated as^{7,16-21}

$$\Delta_{\text{other}} = (-2 \pm 25) \times 10^{-6}. \quad (1.8)$$

The large uncertainty in Δ_{other} stems from the recoil correction.

In addition to the standard spin-orbit term, the modern NN potentials, except the potential of Glendenning and Kramer, contain terms which are linear or quadratic in angular momentum (L). The potentials used by Low⁵ do not have such terms. The magnetic moment arising from such a force contributes an amount Δ_L to the hfs anomaly of deuterium. We show that when the correction Δ_L is included in Δ , agreement with experiment is obtained for some of the potentials, while the situation is worsened for the others. The implications of the effect of the L -dependent force on the hfs of deuterium will be discussed. We will also briefly discuss the mesonic and isobaric effects on the hfs.

In Sec. II we summarize the main points of the calculations. Numerical results and discussions are presented in Sec. III, and some details of the calculations are given in the Appendices.

II. HYPERFINE STRUCTURE OF DEUTERIUM

In this section we summarize Low's calculations.⁵

A. Hamiltonian

When the electron is close to the deuteron it centers around the proton instead of the center of the deuteron. Therefore it is convenient to take the coordinate system in which the proton, rather than the center of mass of the deuteron, is at rest. The Hamiltonian then takes the form

$$H = H_e + H_c + H_D + H'_p + H'_n + H'_L + H' + H'', \quad (2.1)$$

where

$$H_e = c\boldsymbol{\alpha} \cdot \mathbf{p} + \beta mc^2, \quad (2.2)$$

$$H_c = -e^2/r, \quad (2.3)$$

$$H_D = \frac{p^2}{M} + V, \quad (2.4)$$

$$H'_p = e\boldsymbol{\alpha} \cdot \nabla_r \left[\frac{1}{r} \right] \times \boldsymbol{\mu}_p, \quad (2.5)$$

$$H'_n = e\boldsymbol{\alpha} \cdot \nabla_r \left[\frac{1}{|\mathbf{r} - \mathbf{R}|} \right] \times \boldsymbol{\mu}_n, \quad (2.6)$$

$$H'_L = -\frac{e^2}{c} (\mathbf{v} \cdot \boldsymbol{\alpha}/r), \quad (2.7)$$

$$H' = \mathbf{p} \cdot \mathbf{v}, \quad (2.8)$$

$$H'' = \frac{p^2}{4M} - \left[\frac{1}{2\hbar^2} \right] V_{\text{ex}} (\mathbf{p} \cdot \mathbf{R})^2, \quad (2.9)$$

where $\mathbf{r}$ is the position of the electron relative to the deuteron center, $\mathbf{p}$ the momentum conjugate to $\mathbf{r}$, $\mathbf{R}$ the position of the neutron relative to proton, $\mathbf{P}$ the momentum conjugate to $\mathbf{R}$, $\mathbf{v} = \dot{\mathbf{R}}/2$, m the mass of the electron, M the mass of the proton (or neutron), and $\boldsymbol{\alpha}, \beta$ the usual Dirac matrices. The different parts of the Hamiltonian given by Eqs. (2.2)–(2.4) are the Dirac Hamiltonian for a free electron, the electron-proton Coulomb interaction, and the deuteron Hamiltonian, respectively. H'_p , H'_n , and H'_L are the proton spin, neutron spin, and orbital hfs interactions, respectively. When the origin of the coordinates is transformed from the deuteron center of mass to the proton, the term H_e remains the same, whereas other terms are affected. The terms H' and H'' emerge in this transformation.

In Low's calculation⁵ the nuclear potential V was taken to be

$$V = [(1-t) + tP_{\text{ex}}]V(\mathbf{R}), \quad (2.10)$$

where P_{ex} is the position exchange operator and t is the fraction of exchange force. In the case of realistic NN potentials V contains L -dependent terms, as well, which will be discussed separately in Sec. IID. The realistic nuclear potentials are not given in the above form. But they can be put into this form by determining an effective value of t ; see Appendix A. Our estimate shows that $t \approx 0.5$. Moreover, the hfs of deuterium is not sensitive to t . Therefore, we can safely assume that $t = 0.5$.

B. First-order effects

The proton-spin hyperfine interaction is due to H'_p . In first-order perturbation theory, we obtain

$$\begin{aligned} \langle H'_p \rangle &= \left\langle e\boldsymbol{\alpha} \cdot \nabla_r \left[\frac{1}{r} \right] \times \boldsymbol{\mu}_p \right\rangle \\ &= \frac{8\pi}{3} \psi^2(0) \mu_0 \mu_p (1 - \frac{3}{2} \sin^2 \omega), \end{aligned} \quad (2.11)$$

where $\sin^2 \omega$ is the D -state probability of the deuteron. This $\langle H'_p \rangle$ is already contained in Fermi's formula and hence does not contribute to Δ .

The neutron-spin contribution in first-order perturbation theory is

$$\langle H'_n \rangle = \left\langle e\boldsymbol{\alpha} \cdot \nabla_r \left[\frac{1}{|\mathbf{r} - \mathbf{R}|} \right] \times \boldsymbol{\mu}_n \right\rangle. \quad (2.12)$$

Because the deuteron wave function consists of S and D components, there are three terms in $\langle H'_n \rangle$; the S term, D

term, and SD cross term. The S term is

$$\langle H'_n \rangle_S = \frac{8\pi}{3} \cos^2 \omega \psi^2(0) \mu_0 \mu_n \left[1 - \frac{2}{a_0} \int_0^\infty R \phi_S^2(R) dR \right], \quad (2.13)$$

where the second term in the large square brackets is the correction obtained by Bohr.⁴ From the formula

$$\frac{1}{|\mathbf{r} - \mathbf{R}|} = \begin{cases} \sum_{l=0}^{\infty} \frac{R^l}{r^{l+1}} P_l(\cos \gamma), & r > R \\ \sum_{l=0}^{\infty} \frac{r^l}{R^{l+1}} P_l(\cos \gamma), & r < R \end{cases} \quad (2.14)$$

where $P_l(\cos \gamma)$ is the Legendre polynomial of order l , the D term can be obtained in the form

$$\langle H'_n \rangle_D = \langle H'_n \rangle_D^{l=0} + \langle H'_n \rangle_D^{l=2}, \quad (2.15)$$

where l refers to the l appearing in Eq. (2.14), and

$$\begin{aligned} \langle H'_n \rangle_D^{l=0} &= \frac{8\pi}{3} \sin^2 \omega \psi^2(0) \mu_0 \mu_n \left(-\frac{1}{2} \right) \\ &\times \left[1 - \frac{2}{a_0} \int_0^\infty R \phi_D^2(R) dR \right], \end{aligned} \quad (2.16)$$

$$\langle H'_n \rangle_D^{l=2} = \frac{8\pi}{3} \sin^2 \omega \psi^2(0) \mu_0 \mu_n \int_0^\infty \left[\frac{R}{4a_0} \right] \phi_D^2(R) dR. \quad (2.17)$$

The SD cross term is

$$\langle H'_n \rangle_{SD} = \frac{8\pi}{3} \cos \omega \sin \omega \psi^2(0) \mu_0 \mu_n \int_0^\infty \left[\frac{\sqrt{2}R}{4a_0} \right] \phi_D \phi_S dR, \quad (2.18)$$

where a_0 is the Bohr radius. The deuteron S and D radial wave functions are normalized as

$$\int_0^\infty \phi_S^2 dR = \int_0^\infty \phi_D^2 dR = 1. \quad (2.19)$$

The first order neutron-spin contribution to Δ is

$$E_n = \frac{-32i}{9} \psi^2(0) \frac{e^3}{\hbar c^2} \mu_n \left\{ \left[\ln \left[\frac{mc^2}{|W_0|} \frac{\hbar/mc}{\gamma d/2} \right] + \frac{4}{3} \right] \langle \mathbf{R} \cdot \mathbf{v} \rangle - \langle \ln(2\alpha R) \mathbf{R} \cdot \mathbf{v} \rangle - \sum_m (\mathbf{R}_{0m} \cdot \mathbf{v}_{m0}) \ln \left[\frac{W_m - W_0}{|W_0|} \right] \right\}, \quad (2.22)$$

where $\gamma = 1.78$ is Euler's constant. The terms in the large curly braces can be calculated by using the results

$$\langle \mathbf{R} \cdot \mathbf{v} \rangle = \frac{3i\hbar}{2M} (1 + \eta t), \quad (2.23)$$

$$\langle \ln(2\alpha R) \mathbf{R} \cdot \mathbf{v} \rangle = \frac{3\hbar i}{2M} \left(\frac{1}{3} + A + Bt \right), \quad (2.24)$$

$$\begin{aligned} \Delta_1 &= \frac{-\mu_n d}{\mu_D a_0} \left[\cos^2 \omega \int_0^\infty 2(\alpha R) \phi_S^2 dR \right. \\ &\quad - \frac{5}{4} \sin^2 \omega \int_0^\infty (\alpha R) \phi_D^2 dR \\ &\quad \left. - \frac{\sqrt{2}}{4} \sin \omega \cos \omega \int_0^\infty (\alpha R) \phi_S \phi_D dR \right], \end{aligned} \quad (2.20)$$

where $\alpha = (MW_0/\hbar^2)^{1/2}$, $d (=1/\alpha)$ is the size of deuteron, and W_0 is the deuteron binding energy. The major contribution to Δ comes from Δ_1 and, in particular, from its first term. The orbital hyperfine interaction H'_L does not contribute in first order.

C. Second-order effects

For second-order effects we may combine the terms H'_p , H'_n , H'_L , H' , and H'' with one another.

1. Combination of H' and H'_p

The term H' gives a nonzero contribution between two nuclear states of opposite parity, whereas H'_p is nonzero between states of the same parity. Hence the combination of H' and H'_p does not contribute to the hfs of deuterium.

2. Combination of H' and H'_n

The energy shift arising from this combination is

$$\begin{aligned} E_n &= -2 \sum_m \left\{ \sum_{n_+} \left[\frac{(H'_n)_{0n,0m} (H')_{n0,m0}}{E_n - E_0 + W_m - W_0} \right] \right. \\ &\quad \left. - \sum_{n_-} \left[\frac{(H'_n)_{n0,0m} (H')_{0n,m0}}{E_0 - E_n + W_m - W_0} \right] \right\}. \end{aligned} \quad (2.21)$$

the sums $\sum_{n_{\pm}}$ are over the positive and negative energy states of the electron, and $\sum_m$ is over the energy states of the nucleus. Because E_n is a small correction, the continuum states of the electron in the Coulomb field can safely be replaced by plane waves. Furthermore, it is sufficient to take the deuteron to be in the S state. Then, as shown in Ref. 5, E_n reduces to

where

$$\eta = \frac{-2M}{3\hbar^2} \int_0^\infty R^2 V(R) \phi_S^2 dR. \quad (2.25)$$

Furthermore, we have

$$A = \int_0^\infty \phi_S^2 \ln(2\alpha R) dR, \quad (2.26)$$

$$B = \frac{-2M}{3\hbar^2} \int_0^\infty \phi_S^2 [\ln(2\alpha R) R^2 V(R)] dR, \quad (2.27)$$

$$\kappa = \frac{1}{ic} \sum_m \mathbf{R}_{0m} \cdot \mathbf{v}_{m0} \ln \left[\frac{W_m - W_0}{|W_0|} \right] \quad (2.28)$$

$$= \frac{|W_0|}{\pi \hbar c} \int_0^\infty dk \left[k^2 \left[\frac{W_k + |W_0|}{|W_0|} \right] \ln \left[\frac{W_k + |W_0|}{|W_0|} \right] \right. \\ \left. \times \left[\int_0^\infty dR [R^2 j_1(kR) \phi_S] \right]^2 \right]. \quad (2.29)$$

In deriving Eq. (2.29), the nuclear intermediate state has been taken as a free state with kinetic energy $W_k = \hbar^2 k^2 / M$. The contribution to Δ_{expt} due to E_n becomes

$$\Delta_2 = \frac{4}{\pi} \frac{e^2}{\hbar c} \frac{m}{M} \frac{\mu_n}{\mu_D} \left\{ (1 + \eta t) \left[\ln \left[\frac{2\hbar c}{\gamma} \frac{\alpha}{|W_0|} \right] + \frac{4}{3} \right] \right. \\ \left. - \left(\frac{1}{3} + A + Bt \right) - \frac{2Mc^2}{3\hbar c} \kappa \right\}. \quad (2.30)$$

3. Combination of H' and H'_L

The orbital hfs produced by this combination is given⁵ by

$$L_2 = \frac{1}{\hbar} \sum_m \hat{\mathbf{z}} \cdot (M \mathbf{v}_{0m} \times \mathbf{R}_{0m}) \left[\frac{|W_0|}{W_m + |W_0|} \ln \left[\frac{W_m + |W_0|}{|W_0|} \right] \right] \quad (2.35)$$

$$= \frac{3|W_0|Mc^2}{10\pi\hbar^2c^2} \left[\int_0^\infty k^2 dk \ln \left[\frac{W_k + |W_0|}{|W_0|} \right] \right] \left\{ \left[\int_0^\infty R^2 dR j_1(kR) \phi_D \right]^2 - \left[\int_0^\infty R^2 dR j_3(kR) \phi_D \right]^2 \right\} \sin^2 \omega. \quad (2.36)$$

Some details for L_2 are presented in Appendix C.

There are other combinations of the terms of Eq. (2.1), but their net contribution to the hfs is negligible.⁵ For Δ_{Low} it is sufficient to calculate Δ_1 , Δ_2 , and Δ_3 ; i.e.,

$$\Delta_{\text{Low}} = \Delta_1 + \Delta_2 + \Delta_3.$$

D. Contribution of L-dependent force to the hfs of deuterium

In Low's calculation very simple models for the NN interaction were used which did not include any L-dependent terms. On the other hand, all realistic potentials⁹⁻¹⁴ contain terms which are linear or quadratic in L, such as $\mathbf{LS}$, L^2 , Q_{12} , L_{12} , and H_{12} . They all contribute to the magnetic moment of the deuteron and also to the anomaly in the hfs of deuterium. Sessler and Foley²² estimated the correction Δ_{LS} to the anomaly in the hfs of

$$E_L = -2 \sum_m \left\{ \sum_{n+} \left[\frac{(H')_{0n,0m} (H'_L)_{n0,m0}}{E_n - E_0 + W_m - W_0} \right] \right. \\ \left. - \sum_{n-} \left[\frac{(H')_{n0,0m} (H'_L)_{0n,m0}}{E_0 - E_n + W_m - W_0} \right] \right\}. \quad (2.31)$$

On the other hand, the normal orbital hfs is⁵

$$E_F = -2 \sum_m \left\{ \sum_{n+} \left[\frac{(H')_{0n,0m} (H'_L)_{n0,m0}}{W_m - W_0} \right] \right. \\ \left. - \sum_{n-} \left[\frac{(H')_{n0,0m} (H'_L)_{0n,m0}}{W_m - W_0} \right] \right\}. \quad (2.32)$$

The difference $E_L - E_F$ results in a contribution to the hfs of deuterium,

$$\Delta_3 = \left[\left[\frac{4}{\pi} \right] \left[\frac{\mu_L}{\mu_D} \right] \left[\frac{e^2}{\hbar c} \right] \left[\frac{mc^2}{|W_0|} \right] \right] L_2, \quad (2.33)$$

where

$$\mu_L = \frac{e\hbar}{2Mc} \quad (2.34)$$

and

deuterium arising from the spin-orbit force. They obtained

$$\Delta_{LS} = \frac{-\mu_{LS}}{\mu_D} \left[\frac{2D}{a_0} \right], \quad (2.37)$$

where μ_{LS} is the magnetic moment of the deuteron due to the same spin-orbit force. The value of D depends upon the range of the force. For a short-range force, like the spin-orbit force in the NN interaction, D is given by

$$D = \frac{mc^2}{|W_0|} \alpha a_0, \quad (2.38)$$

where α is the fine structure constant. With this value of D , Eq. (2.37) can be written as

$$\frac{\Delta_{LS}}{\mu_{LS}} = -39 \times 10^{-4}. \quad (2.39)$$

As mentioned above, D depends on the range of the

TABLE I. Potentials and corrections used in our calculations.

Potential	Correction	Δ_1 (ppm)	Δ_2 (ppm)	Δ_3 (ppm)	Δ_{Low} (ppm) ± 10 ppm	$\Delta_{\text{Low}} + \Delta_{\text{other}}$ (ppm) ± 35 ppm
Reid soft core (RSC)		246	-21	-10	215	213
Reid soft core alternative (RSCA)		247	-22	-9	216	214
Reid hard core (RHC)		247	-21	-9	217	215
Tourelil and Sprung (TS)		254	-24	-9	221	219
Tourelil, Rouben, and Sprung (TRS)		250	-23	-9	218	216
Paris (PAR)		250	-19	-9	222	220
Glendenning and Kramer (GK9)		243	-20	-10	213	211
Hamada and Johnston (HJ)		247	-20	-10	217	215

force, and the range of the quadratic $\mathbf{L}$ -dependent interaction is about the same as that of the spin-orbit force. It therefore seems reasonable to assume that the same ratio, (2.39), applies to all other components of this force. That is to say,

$$\frac{\Delta_L}{\mu_{LL}} = -39 \times 10^{-4}, \quad (2.40)$$

where Δ_L is the correction to Δ and μ_{LL} is the magnetic moment arising from the $\mathbf{L}$ -dependent force²³ in the NN interaction. We find that the correction Δ_L to the hfs of deuterium is quite substantial, as can be seen from Table II.

III. RESULTS AND DISCUSSION

The hadronic correction to the hfs of deuterium is

$$\Delta = \Delta_{\text{Low}} + \Delta_L + \Delta_{\text{other}}, \quad (3.1)$$

which is to be compared with Δ_{expt} of Eq. (1.4). As we discussed in Sec. II, Δ_{Low} consists of three terms: $\Delta_{\text{Low}} = \Delta_1 + \Delta_2 + \Delta_3$. The Δ_L is due to the $\mathbf{L}$ -dependent force in the NN interaction. The Δ_{other} is

$$\Delta_{\text{other}} = \Delta_{p-D} + \Delta_{n-D} + \Delta_{\text{recoil}} - \Delta_{p-H}, \quad (3.2)$$

where Δ_{p-H} is the finite-size correction (including the recoil effects) of the proton to the hfs of hydrogen.^{16,17} The corrections Δ_{p-D} and Δ_{n-D} are the finite-size corrections of the proton and the neutron to the hfs of deuterium, respectively. These terms have been estimated to be^{7,16-21} $\Delta_{p-H} = (-38 \pm 2)$ ppm, $\Delta_{p-D} = (-137 \pm 5)$ ppm, $\Delta_{n-D} = (21 \pm 2)$ ppm, and $\Delta_{\text{recoil}} = (76 \pm 16)$ ppm. Combining all of these we obtain Eq. (1.8), i.e., $\Delta_{\text{other}} = (-2 \pm 25)$ ppm. It may be useful to point out here that in estimating the nucleon size corrections we have used the rms charge radius of proton $r_{cp} = 0.862 \pm 0.012$ fm, its magnetic radius $r_{mp} = 0.84 \pm 0.03$ fm, and the neutron magnetic radius $r_{mn} = r_{mp}$.^{19,20}

Table I contains Δ_{Low} as obtained with the NN potentials.⁹⁻¹⁴ These potentials differ from each other in a number of ways and their D -state probability ranges from 5.45% to 7.42%. It is, however, remarkable that Δ_{Low} varies only between 213 and 222 ppm. Therefore, Δ_{Low} is

insensitive to the D -state probability as it is insensitive to the potential. In all the cases considered, $\Delta_{\text{Low}} + \Delta_{\text{other}}$ exceeds Δ_{expt} .

In their work Greenberg and Foley⁷ obtained $\Delta_{\text{Low}} + \Delta_{\text{other}} = (195 \pm 41)$ ppm for 7% D -state probability, which is substantially lower than our values. We therefore reexamined the result of Greenberg and Foley.⁷ They calculated Δ_1 ($\epsilon_{\text{Low}}^{\text{SM}}$ in their notation) and found $\Delta_1 = (224 \pm 5)$ ppm, with 7% D -state probability. Our estimate shows that $\Delta_1 = 245$ ppm with 6.57% D -state probability. If this value of Δ_1 is used in the result of Greenberg and Foley,⁷ one obtains $\Delta_{\text{Low}} + \Delta_{\text{other}} = (216 \pm 41)$ ppm. This result is compatible with ours, as can be seen from Table I.

From our results we find that $\Delta_{\text{Low}} + \Delta_{\text{other}}$ is not quite enough to reproduce the experimental result. To remove this discrepancy we have to look for some other source of correction in the NN interaction. We know that all realistic potentials contain an $\mathbf{L}$ -dependent force which contributes to the deuteron magnetic moment and hence to the hfs anomaly of deuterium.

We estimate this correction Δ_L for a few potentials and present the results in Table II. We find that Δ_L is negative in the case of the Paris potential, the potential of Glendenning and Kramer (GK9), the Tourelil-Sprung potential, and the Tourelil-Rouben-Sprung potential; thus, agreement between theory and experiment is attained. The situation is, however, worsened in the case of the Reid soft-core (RSC) potential and the Hamada-Johnston (HJ) potential. The Δ_L of the HJ potential is twice as large as that of the RSC potential. For the potentials con-

TABLE II. Potentials and corrections used in our calculations.

Potential	Correction	Δ_L (ppm)	Δ (ppm) ± 35 ppm
RSC		12	225
TS		-17	202
TRS		-16	200
PAR		-28	192
GK9		-26	185
HJ		29	244

TABLE III. Contribution to the deuteron magnetic moment from the L -dependent components of a few realistic NN potentials and the total magnetic moment (Ref. 23) $\mu_{LL} (= \sum_i \mu_i)$ (μ_i in nuclear magnetons).

Potential $\backslash i$	LS (10^{-3})	L^2 (10^{-3})	Q_{12} (10^{-3})	L_{12} (10^{-3})	H_{12} (10^{-3})	P^2 (10^{-3})	μ_{LL} (10^{-3})
RSC	2.14	2.71	-7.83				-2.98
TS	7.63	2.36	-5.73				4.26
TRS	9.87	3.13		-8.88			4.12
PAR	6.05			-6.19		7.40	7.26
GK9	6.78						6.78
HJ	2.40				-9.82		-7.42

by us, Δ_L varies between -28 and 29 ppm, and the effect is far less in magnitude than that obtained with the Signell-Marshak potential.

From Table III we observe that the magnetic moments produced by the LS , L^2 , and P^2 terms are positive and reduce the discrepancy, whereas the Q_{12} , L_{12} , and H_{12} terms have the opposite effects. This suggests that the agreement between theory and experiment can be improved if the parameters of a given potential can be readjusted in such a way that the contributions of the terms like LS , L^2 , and P^2 are enhanced while the contributions of other terms like Q_{12} , L_{12} , and H_{12} are reduced. We note, however, that some ambiguities remain. It is known that nuclear forces are, at least partly, mediated by the exchange of mesons. The magnetic moment arising from meson exchange currents contribute to the anomaly in the hfs of deuterium. Furthermore, the deuteron may have a small admixture of some isobaric component which also contributes to Δ . For example, the presence of Δ^{++} and Δ^- in the deuterium nucleus can affect the hfs. However, it is difficult to estimate these effects in a reliable way. Due to these ambiguities in Δ , it is difficult, at this stage, to make any categorical statement about the superiority of any one potential over the others.

In conclusion, we mention the quark effect on the anomaly in the hfs of deuterium. The quark degrees of freedom have already been speculated²⁴ on the deuteron structure. In fact, we were motivated by this idea to look into the hfs of deuterium. We soon realized, however, that the correction to Fermi's formula, $\approx \int_0^{R_c} R \phi_S^2 dR$, is negligible for small values of R_c . The quark effect is expected to be of a short range and hence its impact on the hfs anomaly will be insignificant. Some simple model calculations which we carried out substantiated our belief.

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APPENDIX A

As mentioned in Sec. II, the NN potential V in Low's model was taken to be

$$V = [(1-t) + tP_{ex}]V(\mathbf{R}), \quad (A1)$$

where t is the fraction of exchange force in the NN potential. To put the realistic NN potential in the above form, we will develop an expression for t along the following lines.

Let us write the NN potentials²⁵ as follows:

$$V = -[V_W(R) + V_M(R)P_x + V_B(R)P_\sigma + V_H(R)P_\sigma P_x], \quad (A2)$$

where P_x and P_σ are the space exchange and spin exchange operators, respectively, and V_W , V_M , V_B , and V_H are the coefficients giving relative contributions of the various potential terms. For a spin triplet and isospin singlet system, which is the case of interest, Eq. (A2) is reduced to

$$V = -[(V_W + V_B) + (V_M + V_H)P_x]. \quad (A3)$$

By comparing Eqs. (A1) and (A3) we get

$$\frac{t}{1-t} = \frac{V_M + V_H}{V_W + V_B}. \quad (A4)$$

To evaluate t from Eq. (A4) we use the following relations deduced from Eq. (A2):

$$V(^3\text{Even}) = -[(V_W + V_B) + (V_M + V_H)], \quad (A5)$$

$$V(^3\text{Odd}) = -[(V_W + V_B) - (V_M + V_H)], \quad (A6)$$

$$V(^3\text{Even}) + V(^3\text{Odd}) = -2(V_W + V_B), \quad (A7)$$

$$V(^3\text{Even}) - V(^3\text{Odd}) = -2(V_M + V_H). \quad (A8)$$

Therefore,

$$\frac{t}{1-t} = \frac{V(^3\text{Even}) - V(^3\text{Odd})}{V(^3\text{Even}) + V(^3\text{Odd})}. \quad (A9)$$

Using the values of $V(^3\text{Odd})$ and $V(^3\text{Even})$ from the realistic NN potentials, for example, the Reid hard core potential, t can easily be estimated from Eq. (A9).

APPENDIX B

The quantity κ was introduced in Eq. (2.28) and was presented in a computable form in Eq. (2.29). Equation (2.29) is derived below. The deuteron is considered to be in the S state and its excited state is assumed to be a free state. One can write

$$\langle \mathbf{R} | 0 \rangle = \frac{1}{\sqrt{4\pi}} \frac{\phi_S(R)}{R} \chi_1^1, \quad (\text{B1})$$

$$\begin{aligned} \langle \mathbf{R} | n \rangle &= \frac{1}{\sqrt{V}} e^{i\mathbf{k} \cdot \mathbf{R}} \chi_1^{m_s} \\ &= \frac{4\pi}{\sqrt{V}} \sum_{l,m} i^l j_l(kR) Y_{lm}^*(\hat{\mathbf{k}}) Y_{lm}(\hat{\mathbf{R}}) \chi_1^{m_s}, \end{aligned} \quad (\text{B2})$$

where $\langle \mathbf{R} | 0 \rangle$ and $\langle \mathbf{R} | n \rangle$ denote the wave functions for the ground and the excited states of the deuteron, respectively. Using $\mathbf{R} = \sum_q R_q \hat{\xi}_q^*$, $R_q = (4\pi/3)^{1/2} R Y_{1q}(\hat{\mathbf{R}})$, and the wave functions given in Eqs. (B1) and (B2), we obtain

$$\begin{aligned} \langle n | R_q | 0 \rangle &= \frac{4\pi}{\sqrt{V}} \sum_{l,m} i^l j_l(kR) Y_{lm}^*(\hat{\mathbf{k}}) Y_{lm}(\hat{\mathbf{R}}) \chi_1^{m_s} \left| \left[\frac{4\pi}{3} \right]^{1/2} R Y_{1q}(\hat{\mathbf{R}}) \left[\frac{\phi_S}{R} \chi_1^1 \right] \left[\frac{1}{4\pi} \right]^{1/2} \right| \\ &= \left[\frac{16\pi^2}{3V} \right]^{1/2} \delta_{m_s,1} \left\{ (-i) Y_{1q}(\hat{\mathbf{k}}) \int_0^\infty dR [R^2 j_1(kR) \phi_S] \right\}. \end{aligned} \quad (\text{B3})$$

With the help of Eq. (B3) we can write

$$\begin{aligned} \sum_n \langle 0 | \mathbf{R} | n \rangle \cdot \langle n | \mathbf{R} | 0 \rangle &= \left[\frac{16\pi^2}{3V} \right]^{1/2} \left[\frac{V}{(2\pi)^3} \sum_{m_s} \int_0^\infty k^2 dk \int d\Omega_{\hat{\mathbf{k}}} \left\{ \delta_{m_s,1} \sum_q \xi_q \left\{ i Y_{1q}^*(\hat{\mathbf{k}}) \int_0^\infty dR [R^2 j_1(kR) \phi_S] \right\} \right\} \right. \\ &\quad \times \left. \left\{ \delta_{m_s,1} \sum_{q'} \xi_{q'}^* \left\{ -i Y_{1q'}(\hat{\mathbf{k}}) \int_0^\infty dR [R^2 j_1(kR) \phi_S] \right\} \right\} \right] \\ &= \frac{2}{3\pi} \int_0^\infty k^2 dk \left[\left\{ \int d\Omega_{\hat{\mathbf{k}}} \left[\sum_q Y_{1q}^*(\hat{\mathbf{k}}) Y_{1q}(\hat{\mathbf{k}}) \right] \right\} \left\{ \int_0^\infty dR [R^2 j_1(kR) \phi_S] \right\}^2 \right] \\ &= \frac{2}{\pi} \int_0^\infty k^2 dk \left\{ \int_0^\infty dR [R^2 j_1(kR) \phi_S] \right\}^2. \end{aligned} \quad (\text{B4})$$

We have replaced the summation over intermediate states by integration in Eq. (B4), using the prescription

$$\sum_n \rightarrow \frac{V}{(2\pi)^3} \int_0^\infty k^2 dk \int d\Omega_{\hat{\mathbf{k}}} \sum_{m_s}.$$

Next let us define

$$\begin{aligned} \kappa &= \frac{1}{ic} \sum_n \left[\mathbf{R}_{0n} \cdot \mathbf{v}_{n0} \ln \left[\frac{W_n - W_0}{|W_0|} \right] \right] \\ &= \frac{|W_0|}{2\hbar c} \sum_n \left[\mathbf{R}_{0n} \cdot \mathbf{R}_{n0} \left[\frac{W_n + |W_0|}{|W_0|} \right] \right. \\ &\quad \times \ln \left[\frac{W_n + |W_0|}{|W_0|} \right] \left. \right]. \end{aligned} \quad (\text{B5})$$

Here we have used the relation

$$\begin{aligned} \mathbf{v}_{n0} &= \frac{i}{2\hbar} \langle n | H_D \mathbf{R} - \mathbf{R} H_D | 0 \rangle \\ &= \frac{i}{2\hbar} (W_n - W_0) \mathbf{R}_{n0} \end{aligned}$$

in Eq. (B5).

Substituting Eq. (B4) into Eq. (B5), we obtain

$$\begin{aligned} \kappa &= \frac{|W_0|}{\pi \hbar c} \int_0^\infty dk \left[k^2 \left[\frac{W_k + |W_0|}{|W_0|} \right] \ln \left[\frac{W_k + |W_0|}{|W_0|} \right] \right. \\ &\quad \times \left. \left\{ \int_0^\infty dR [R^2 j_1(kR) \phi_S] \right\}^2 \right]. \end{aligned} \quad (\text{B6})$$

APPENDIX C

In this appendix we derive the expression for L_2 given in Eq. (2.36) starting from Eq. (2.35). To begin with we have

$$L_2 = \frac{1}{\hbar} \sum_n \hat{\mathbf{z}} \cdot \mathbf{M} \mathbf{v}_{0n} \times \mathbf{R}_{n0} \left[\ln \left[\frac{W_n - W_0}{|W_0|} \right] / \left[\frac{W_n - W_0}{|W_0|} \right] \right] = \left[\frac{-iM |W_0|}{2\hbar^2} \right] \sum_n \hat{\mathbf{z}} \cdot \mathbf{R}_{0n} \times \mathbf{R}_{n0} \ln \left[\frac{W_n - W_0}{|W_0|} \right]. \quad (\text{C1})$$

Taking intermediate nuclear states as free states, we can replace the summation by integration as shown in Appendix B. Equation (C1) can be written as

$$L_2 = \left[\frac{-iM|W_0|}{2\hbar^2} \right] \frac{V}{(2\pi)^3} \hat{z} \cdot \int_0^\infty k^2 dk \int d\Omega_{\hat{k}} \sum_{m_s} \langle 0 | \mathbf{R} | n \rangle \times \langle n | \mathbf{R} | 0 \rangle \ln \left[\frac{W_k + |W_0|}{|W_0|} \right]. \quad (C2)$$

In L_2 the S component of the deuteron wave function does not contribute, and therefore we simply write

$$\langle \mathbf{R} | 0 \rangle = \frac{\phi}{R} \sum_{M_s} \langle 21(1-M_s)M_s | 11 \rangle Y_2^{1-M_s}(\hat{\mathbf{R}}) \chi_1^{M_s}, \quad (C3)$$

$$\langle \mathbf{R} | n \rangle = \frac{4\pi}{\sqrt{V}} \sum_{l,m} i^l j_l(kR) Y_{lm}^*(\hat{\mathbf{k}}) Y_{lm}(\hat{\mathbf{R}}) \chi_1^{m_s}, \quad (C4)$$

where $\phi = \sin\omega\phi_D$, and all other symbols stand for their usual meanings. Using Eqs. (C3) and (C4), we write

$$\begin{aligned} \langle n | R_q | 0 \rangle &= \left[\frac{64\pi^3}{3V} \right]^{1/2} \sum_{l,m,M_s} \left[\langle i^l j_l(kR) Y_{lm}^*(\hat{\mathbf{k}}) Y_{lm}(\hat{\mathbf{R}}) \chi_1^{m_s} | R Y_{1q}(\hat{\mathbf{R}}) | \frac{\phi}{R} \langle 21(1-M_s)M_s | 11 \rangle Y_{2(1-M_s)}(\hat{\mathbf{R}}) \chi_1^{M_s} \rangle \right] \\ &= \langle 21(1-m_s)m_s | 11 \rangle \left[\frac{64\pi^3}{3V} \right]^{1/2} \sum_{l,m} \left\{ (-i)^l Y_{lm}(\hat{\mathbf{k}}) \left[\frac{15}{4\pi(2l+1)} \right]^{1/2} \langle 21(1-m_s)q | lm \rangle \right. \\ &\quad \left. \times \langle 2100 | 10 \rangle \int_0^\infty dR [R^2 j_l(kR) \phi] \right\}. \end{aligned} \quad (C5)$$

With the help of Eq. (C5), we obtain

$$\begin{aligned} \langle \mathbf{R} | n \rangle \times \langle n | \mathbf{R} | 0 \rangle &= \left[\frac{64\pi^3}{3V} \right] \langle 21(1-m_s)m_s | 11 \rangle^2 \\ &\quad \times \left\{ \sum_{l,m,q} \sum_{l',m',q'} (-1)^l (i)^{l+l'} \left[\left[\frac{15}{4\pi} \right]^2 \frac{1}{(2l+1)(2l'+1)} \right]^{1/2} \right. \\ &\quad \times \left[\int_0^\infty dR R^2 j_l(kR) \phi \right] \left[\int_0^\infty dR R^2 j_{l'}(kR) \phi \right] \\ &\quad \times [\langle 21(1-m_s)q | lm \rangle \langle 21(1-m_s)q' | l'm' \rangle \langle 2100 | 10 \rangle \langle 2100 | l'0 \rangle] \\ &\quad \left. \times [(-1)^q (\xi_{q'} \times \xi_{-q}) Y_{l'm'}^*(\hat{\mathbf{k}}) Y_{lm}(\hat{\mathbf{k}})] \right\} \\ &= i \left[\frac{240\pi^2}{3V} \right] \xi_0 \sum_{l,m} \left\{ \frac{1}{(2l+1)} \left[\int_0^\infty dR R^2 j_l(kR) \phi \right]^2 \langle 2100 | 10 \rangle^2 | Y_{lm}(\hat{\mathbf{k}}) |^2 \right. \\ &\quad \times \langle 21(1-m_s)m_s | 11 \rangle^2 \\ &\quad \left. \times [\langle 21(1-m_s)-1 | lm \rangle^2 - \langle 21(1-m_s)1 | lm \rangle^2] \right\}. \end{aligned} \quad (C6)$$

Summing over intermediate spin states and performing angular integration over the directions of $\mathbf{k}$, we get

$$\sum_{m_s} \int d\Omega_{\hat{k}} \langle 0 | \mathbf{R} | n \rangle \times \langle n | \mathbf{R} | 0 \rangle = i \xi_0 \left[\frac{24\pi^2}{5V} \right] \left[\left[\int_0^\infty dR [R^2 j_1(kR) \phi] \right]^2 - \left[\int_0^\infty dR [R^2 j_3(kR) \phi] \right]^2 \right]. \quad (C7)$$

Substituting from Eq. (C7) into Eq. (C2), we obtain

$$L_2 = \left[\frac{3 |W_0| M}{10\pi\hbar^2} \right] \left[\int_0^\infty dk k^2 \ln \left[\frac{W_k + |W_0|}{|W_0|} \right] \right] \left\{ \left[\int_0^\infty dR R^2 j_1(kR) \phi_D \right]^2 - \left[\int_0^\infty dR R^2 j_3(kR) \phi_D \right]^2 \right\} \sin^2 \omega, \quad (C8)$$

which is the same as given in Eq. (2.36).

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- ¹E. Fermi, *Z. Phys.* **60**, 320 (1930).
²W. H. Parker, B. N. Taylor, and D. N. Langenberg, *Phys. Rev. Lett.* **18**, 287 (1967).
³P. Kusch, *Phys. Rev.* **100**, 1188 (1955).
⁴A. Bohr, *Phys. Rev.* **73**, 1109 (1948).
⁵F. Low, *Phys. Rev.* **77**, 361 (1950).
⁶F. E. Low and E. E. Salpeter, *Phys. Rev.* **83**, 478 (1951).
⁷D. A. Greenberg and H. M. Foley, *Phys. Rev.* **120**, 1684 (1960).
⁸P. S. Signell and R. E. Marshak, *Phys. Rev.* **106**, 832 (1957).
⁹R. V. Reid, *Ann. Phys. (N.Y.)* **50**, 411 (1968).
¹⁰R. De Tourreil and D. W. L. Sprung, *Nucl. Phys.* **A201**, 193 (1973).
¹¹R. De Tourreil, B. Rouben, and D. W. L. Sprung, *Nucl. Phys.* **A242**, 445 (1975).
¹²M. Lacombe, B. Loiseau, J. M. Richard, R. Vinh Mu, J. Cote, P. Pires, and R. De Tourreil, *Phys. Rev. C* **21**, 861 (1980).
¹³N. K. Glendenning and G. Kramer, *Phys. Rev.* **126**, 2159 (1962).
¹⁴T. Hamada and I. D. Johnston, *Nucl. Phys.* **34**, 382 (1962).
¹⁵We assume an uncertainty of ± 10 ppm in Δ_{Low} due to the inaccuracies involved in our calculations.
¹⁶C. K. Iddings and P. M. Platzman, *Phys. Rev.* **113**, 192 (1959); A. C. Zemach, *ibid.* **104**, 1771 (1956); A. M. Sessler and R. L. Mills, *ibid.* **110**, 1453 (1958).
¹⁷S. J. Brodsky and S. D. Drell, *Annu. Rev. Nucl. Sci.* **20**, 147 (1970).
¹⁸C. Greifinger, Ph.D. thesis, Cornell University, 1954.
¹⁹G. Höhler, E. Pietarinen, I. Sabba-Stefanescu, F. Borkowski, G. G. Simon, V. H. Walther, and R. D. Wendling, *Nucl. Phys.* **B114**, 505 (1976).
²⁰G. G. Simon, Ch. Schmitt, F. Borkowski, and V. H. Walther, *Nucl. Phys.* **A333**, 381 (1980).
²¹The Δ_2 is contained in Δ_{Low} as well as in the recoil correction calculated by Greifinger and by Greenberg and Foley. Therefore, to avoid double counting, the Δ_{recoil} is estimated by subtracting Δ_2 from the recoil correction given by Greenberg and Foley. We take $\Delta_2 = (-21 \pm 7)$ ppm, where an uncertainty of ± 3 ppm is due to the spread in its values for different potentials considered and the remaining uncertainty is due to the assumptions made in our calculation.
²²A. M. Sessler and H. M. Foley, *Phys. Rev.* **110**, 995 (1958).
²³D. W. L. Sprung, *Nucl. Phys.* **A242**, 141 (1975); R. R. Sheerbaum, *Phys. Rev. C* **11**, 255 (1975).
²⁴R. K. Bhaduri and Y. Nogami, *Phys. Lett.* **152**, 35 (1985); Y. E. Kim and X. Orłowski, in *Hadron Substructure in Nuclear Physics (Indiana University, 1983)*, edited by W. Y. P. Hwang and M. H. Macfarlane (AIP Conf. Proc. No. 110) (AIP, New York, 1984), p. 271.
²⁵R. R. Roy and B. P. Nigam, *Nuclear Physics* (Wiley, New York, 1967).