Multiquark compound states and the triton charge form factor

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The ^{3}H charge form factor is investigated based on the Reid soft-core potential model and the relativistic harmonic oscillator quark model. A reasonable agreement with the experimental data is obtained for the momentum transfer up to ~ 1.0 GeV/c with 2.7% six-quark compound state confined within a radius of ~ 0.9 fm.

In the past, there have been many calculations[1,2] of the charge form factors, $F_{\rm ch}^{^3\rm He}(Q^2)$ and $F_{\rm ch}^{^3\rm H}(Q^2)$, of ^{3}He and ^{3}H using "realistic" nucleon-nucleon (NN) potentials. All of the calculated results for $F_{\rm ch}^{^3\rm He}(Q^2)$ and $F_{\rm ch}^{^3\rm H}(Q^2)$ based on "realistic" NN potential models so far reproduce approximately the experimental data for $F_{\rm ch}^{^3\rm He}(Q^2)$ and $F_{\rm ch}^{^3\rm H}(Q^2)$ at lower values of the momentum transfer squared, $Q^2 \lesssim 9$ fm^{-2}, but disagree badly at higher Q^2 beyond $Q^2 \gtrsim 9$ fm^{-2}. The effects of the meson exchange currents[2,3] and three-body forces[2,4] have been shown to improve the calculated results for $Q^2 \gtrsim 9$ fm^{-2} somewhat, but not enough to agree with the experimental data.

In an attempt to make an improvement over the conventional nonrelativistic hadronic description of the ^{3}He charge form factor, Namiki, Okano, and Oshimo (NOO) (Ref. 5) used a hybrid quark-hadron model in which the relativistic harmonic oscillator quark model (RHOM) of Fujimura, Kobayashi, and Namuki[6] is used to supplement the contribution of the Reid soft-core potential[7] as calculated by Brandenburg, Kim, and Tubis.[8] Justifications of the RHOM from more fundamental points of view have been extensively discussed by Kim *et al.*[9] Recently we have calculated the ^{3}He charge form factor, $F_{\rm ch}^{^3\rm He}(Q^2)$, based on the hybrid model of Namiki *et al.* but with an explicit separation of the interior and exterior regions which allows us to determine the probabilities of the multiquark compound (MQC) states as a function of the separation radius, r_0.[10] With a parametric value of $r_0 \approx 0.9$ fm corresponding to 2.7% for the six-quark MQC states, we have obtained a surprisingly reasonable description[10] of $F_{\rm ch}^{^3\rm He}(Q^2)$ for $Q^2 \lesssim 100$ fm^{-2}. Since the recent experimental measurements[11] of $F_{\rm ch}^{^3\rm H}(Q^2)$ have recently been extended to $Q^2 \lesssim 25$ fm^{-2} from $Q^2 \lesssim 8$ fm^{-2},[12,13] it is interesting to see whether the same model with the same set of parameters[10] used successfully for $F_{\rm ch}^{^3\rm He}(Q^2)$ can describe the experimental data for $F_{\rm ch}^{^3\rm H}(Q^2)$ equally as well. In this paper, we present the results of such calculations of $F_{\rm ch}^{^3\rm H}(Q^2)$ for $Q^2 \lesssim 28$ fm^{-2} and compare them with the experimental data for $F_{\rm ch}^{^3\rm H}(Q^2)$.[11-13]

In our model, we decompose the totally antisymmetric ^{3}H wave function as

$$\Psi(\mathbf{r}^{(1)},\boldsymbol{\rho}^{(1)})=\tilde{\phi}(r^{(1)},\rho^{(1)})+\chi(\mathbf{r}^{(1)},\boldsymbol{\rho}^{(1)}) , \tag{1}$$

where $\tilde{\phi}(r^{(1)},\rho^{(1)})$ are the S-state components of the ^{3}H wave function and $\chi(\mathbf{r}^{(1)},\boldsymbol{\rho}^{(1)})$ represents other higher partial wave components. The S-state wave components $\tilde{\phi}(r^{(1)},\rho^{(1)})$ are further decomposed as

$$\tilde{\phi}(r^{(1)},\rho^{(1)})=\tilde{\phi}_{\rm int}^{6q}(r^{(1)},\rho^{(1)})+\tilde{\phi}_{\rm int}^{9q}(r^{(1)},\rho^{(1)})+\tilde{\phi}_{\rm ext}(r^{(1)},\rho^{(1)}) , \tag{2}$$

with

$$\tilde{\phi}_{\rm int}^{6q}(r^{(1)},\rho^{(1)})=\phi_{\rm int}^{6q}(r^{(1)},\rho^{(1)})\Theta(r_0-r^{(1)})\Theta(r^{(2)}-r_0)\times\Theta(r^{(3)}-r_0) , \tag{3}$$

$$\tilde{\phi}_{\rm int}^{9q}(r^{(1)},\rho^{(1)})=\phi_{\rm int}^{9q}\Theta(r_0-r^{(1)})\Theta(r_0-r^{(2)})\Theta(r_0-r^{(3)}) , \tag{4}$$

and

$$\tilde{\phi}_{\rm ext}(r^{(1)},\rho^{(1)})=\sum_{\alpha=1,2}\tilde{\phi}_{\rm ext}^{(\alpha)}(r^{(1)},\rho^{(1)}) =\sum_{\alpha=1,2}\phi_{\rm ext}^{(\alpha)}(r^{(1)},\rho^{(1)})\Theta(r^{(1)}-r_0)\times\Theta(r^{(2)}-r_0)\Theta(r^{(3)}-r_0) , \tag{5}$$

where α labels the S-wave components of the exterior states, r_0 is a cutoff radius in the pair coordinate, and Θ denotes the Heaviside unit function. The coordinate variables are the Lovelace variables defined as $(i,j,k,$ cyclic):

$$\mathbf{r}^{(i)}=\mathbf{r}_j-\mathbf{r}_k , \quad \rho(i)=\frac{1}{\sqrt{3}}(\mathbf{r}_j+\mathbf{r}_k-2\mathbf{r}_i) , \quad R=\sqrt{2/3}(\mathbf{r}_1+\mathbf{r}_2+\mathbf{r}_3) . \tag{6}$$

We note that the Jacobi coordinates are given by

$$\mathbf{x}^{(i)}=\mathbf{r}_j-\mathbf{r}_k=\mathbf{r}^{(i)} ,$$
$$\mathbf{y}^{(i)}=\mathbf{r}_{(i)}-(\mathbf{r}_j+\mathbf{r}_k)/2=\frac{-\sqrt{3}}{2}\boldsymbol{\rho}^{(i)} ,$$
$$\mathbf{R}'=\tfrac{1}{3}(\mathbf{r}_1+\mathbf{r}_2+\mathbf{r}_3)=\frac{1}{\sqrt{6}}\mathbf{R} .$$

The corresponding probabilities for $\phi_{\rm int}^{6q}$, $\phi_{\rm int}^{9q}$ and $\phi_{\rm ext}^{(\alpha)}$ (we drop the superscripts for $r^{(1)}=r$ and $\rho^{(1)}=\rho$ from now on) are given by

$$\langle \phi_{\rm int}^{6q} | \phi_{\rm int}^{6q} \rangle = \sum_{\alpha=1,2} \int_0^{r_0} r^2 dr \int_\rho^\infty \rho^2 d\rho \, | \mathscr{U}^{(\alpha)} |^2 , \tag{7}$$

$$\langle \phi_{\rm int}^{9q} | \phi_{\rm int}^{9q} \rangle = \sum_{\alpha=1,2} \left[\int_0^{r_0} r^2 dr \int_0^{\rho'} \rho^2 d\rho + \int_0^{r_0} \rho^2 d\rho \int_{r_0}^{r'} r^2 dr \right] | \mathscr{U}^{(\alpha)} |^2 , \tag{8}$$

and

$$\langle \phi_{\rm ext}^{(\alpha)} | \phi_{\rm ext}^{(\alpha)} \rangle = \left[\int_{r_0}^\infty r^2 dr^2 \int_{r_0}^\infty \rho^2 d\rho + \int_0^{r_0} \rho^2 d\rho \int_{r'}^\infty r^2 dr \right] | \mathscr{U}^{(\alpha)} |^2 , \tag{9}$$

respectively, where $\mathscr{U}^{(\alpha)}(r,\rho)$ refers to two S-state components of the ^{3}H wave function of Brandenburg *et al.*,[8] the $\alpha=1$ case for the pair spin $S=1$ and pair isospin $T=0$ and the $\alpha=2$ case for $S=0$ and $T=1$. The integration limits, r' and ρ', are given by

$$(r')^2 = 4(r_0^2 - \tfrac{3}{4}\rho^2)$$

and

$$(\rho')^2 = \frac{4}{3} \left[r_0^2 - \frac{r^2}{4} \right] .$$

Using the ^{3}H wave function of Brandenburg *et al.*,[8,14] we can calculate the probabilities of the states $\phi_{\rm int}^{6q}$, $\phi_{\rm int}^{9q}$, and $\phi_{\rm ext}^{(\alpha)}$ [given by Eqs. (7), (8), and (9), respectively], as a function of r_0. Our calculated probabilities with $r_0 = 0.9$ fm are

$$\langle \phi_{\rm int}^{6q} | \phi_{\rm int}^{6q} \rangle \cong 2.7 \times 10^{-2}$$

and

$$\langle \phi_{\rm int}^{qq} | \phi_{\rm int}^{qq} \rangle \cong 9.2 \times 10^{-4} .$$

We note that $r_0 = 0.9$ fm is very close to $(1.04 a_{\rm rms})$ used by Jaffe *et al.*[15] with $a_{\rm rms} \cong 0.88$ fm (the rms charge radius of the proton) for the case of the Reid-soft-core correlation function used in their analysis of the European Muon Collaboration (EMC) effect.[16]

For our interior multiquark compound states, $\phi_{\rm int}^{6q}$ [Eq. (3)] and $\phi_{\rm int}^{9q}$ [Eq. (4)] we use the relativistic harmonic oscillator quark model of Fujimura, Kobayashi, and Namiki,[6] with the probabilities of each state given by Eqs. (7)–(9). For the external three-nucleon wave components, we use the ^{3}H wave function of Brandenburg *et al.* generated from the Reid soft-core potential, with the cutoff for two S-wave components ($\alpha=1$ and 2) and without the cutoff for the other partial wave components ($\alpha \neq 1$ and $\alpha \neq 2$). With the ^{3}H wave function described above, $F_{\rm ch}^{^3{\rm H}}(Q^2)$ can be written as

$$F_{\rm ch}^{^3{\rm He}}(Q^2) = \cos^2\theta_1 [F_{\rm H}(Q^2) + F_\pi(Q^2)] + \sin^2\theta_1 [\cos^2\theta_2 F_{6q-3q}(Q^2) + \sin^2\theta_2 F_{9q}(Q^2)] , \tag{10}$$

where $F_{\rm N}(Q^2)$, $F_\pi(Q^2)$, $F_{6q-3q}(Q^2)$, and $F_{9q}(Q^2)$ are contributions from the exterior impulse approximation, exterior meson-exchange charge density operators, and interior six-quark and nine-quark charge density operators, respectively. The trigonometric functions in Eq. (10) are related to the probabilities

$$\langle \tilde{\phi}_{\rm int}^{6q} | \tilde{\phi}_{\rm int}^{6q} \rangle = \sin^2\theta_1 \cos^2\theta_2 ,$$

$$\langle \tilde{\phi}_{\rm int}^{9q} | \tilde{\phi}_{\rm int}^{9q} \rangle = \sin^2\theta_1 \sin^2\theta_2 ,$$

and

$$\sum_\alpha \langle \phi_{\rm ext}^{(\alpha)} | \phi_{\rm ext}^{(\alpha)} \rangle + \langle \chi(\mathbf{r},\boldsymbol{\rho}) | \chi(\mathbf{r},\boldsymbol{\rho}) \rangle = \cos^2\theta_1 .$$

The contribution from the impulse approximation, $F_{\rm N}(Q^2)$, can be written as[10]

$$F_{\rm N}(Q^2) = \sum_T A_T^N \left[\sum_\alpha \int \int \tilde{\phi}_{\rm ext}^{(\alpha)*}(r,\rho) j_0 \left[\frac{\rho Q}{\sqrt{3}} \right] \tilde{\phi}_{\rm ext}^{(\alpha)}(r,\rho) r^2 dr\, \rho^2 d\rho + \int \int \chi^*(\mathbf{r},\boldsymbol{\rho}) e^{-i\mathbf{Q}\cdot\boldsymbol{\rho}/\sqrt{3}} \chi(\mathbf{r},\boldsymbol{\rho}) d\mathbf{r}\, d\boldsymbol{\rho} \right] , \tag{11}$$

where T is the pair isospin, A_T^N is given by $A_0^N = \frac{3}{2}[G_E^S(Q^2) - G_E^V(Q^2)]$, and $A_1^N = \frac{1}{2}[3G_E^S(Q^2) + G_E^V(Q^2)]$. $\tilde{\phi}_{\rm ext}^{(\alpha)}$ represent two S-state components ($\alpha=1$ and 2) of $\tilde{\phi}_{\rm ext}$ given by Eq. (5). $G_E^S(Q^2)$ and $G_E^V(Q^2)$ are the Sachs form factors normalized as $G_E^S(0) = G_E^V(0) = 1$.

For the meson-exchange current contribution, the zero (charge) component of the four-vector pair current leads to[10]

$$F_\pi(Q^2) = \sum_{\alpha=1,2} A_\alpha^\pi \left[-\frac{3Q}{16\pi} \right] \left[\frac{g^2}{m^3} \right] \int \int \tilde{\phi}_{\rm ext}^{(\alpha)*}(r,\rho)(1+\mu r) \frac{e^{-\mu r}}{r^2} j_0 \left[\frac{\rho Q}{2\sqrt{3}} \right] j_1 \left[\frac{rQ}{2} \right] \tilde{\phi}_{\rm ext}^{(\alpha)}(r,\rho) r^2 dr\, \rho^2 d\rho , \tag{12}$$

where A_α^π is given by $A_1^\pi = G_M^S(Q^2)$ and

$$A_2^\pi = G_M^S(Q^2) - \tfrac{1}{3} G_M^V(Q^2) .$$

The Sach form factors are normalized as $G_M^V(0) = 4.7$ and $G_M^S(0) = 0.88$. The πN coupling constant is taken to be $g^2/4\pi = 14$, and m is the nucleon mass.

For the contribution of the interior states to $F_{\rm ch}^{^3{\rm He}}(Q^2)$, we use the results of the relativistic harmonic oscillator model[6] and identify $F_{6q-3q}(Q^2)$ and $F_{9q}(Q^2)$ as

$$F_{6q-3q}(Q^2)=\langle \tilde{\phi}^{6q}_{\rm int} | \mathcal{O}^{6q}_{\rm em}(2,3)+\mathcal{O}^{N}_{\rm em}(1) | \tilde{\phi}^{6q}_{\rm int} \rangle \qquad (13)$$

and

$$F_{9q}(Q^2)=\langle \tilde{\phi}^{9q}_{\rm int} | \mathcal{O}^{9q}_{\rm em}(1,2,3) | \tilde{\phi}^{9q}_{\rm int} \rangle \ , \qquad (14)$$

where the operators $\mathcal{O}^{6q}_{\rm em}(2,3)$, $\mathcal{O}^{9q}_{\rm em}(1,2,3)$, and $\mathcal{O}^{N}_{\rm em}(1)$ represent the zeroth (charge) components of the electromagnetic currents for six quarks (for a pair of nucleons 2 and 3), nine quarks, and a single nucleon (nucleon 1), respectively. The explicit expressions for $F_{6q-3q}(Q^2)$ and $F_{9q}(Q^2)$ in Eqs. (13) and (14) are given in Ref. 10.

For calculating $F^{^3\rm H}_{\rm ch}(Q^2)$, Eq. (10), we use the five-parameter dipole fits of Iachello *et al.*[17] for the electromagnetic nucleon form factors G_E^S, G_E^V, G_M^S, and G_M^V. Our calculated results are shown schematically and compared with the experimental data[11–13] for $F^{^3\rm H}_{\rm ch}(Q^2)$ in Fig. 1. The meson-exchange contribution, $F_\pi(Q^2)$, is found to be rather small, as expected from the fact that the cutoff of the meson-exchange operators at $r_0=0.9$ fm eliminates a substantial part of these operators which have the pion range of 1.4 fm. The nine-quark contribution $F_{9q}(Q^2)$ to $F^{^3\rm H}_{\rm ch}(Q^2)$ is also found to be small for $Q^2 \lesssim 28$ fm^{-2}. Therefore, the contributions from both $F_\pi(Q^2)$ and $F_{9q}(Q^2)$ are not shown for the total contribution shown in Fig. 1. As can be seen from Fig. 1, the calculated result of the impulse approximation, $F_{\rm N}(Q^2)$, does not agree with the experimental data for the momentum transfer, $Q \gtrsim 3$ fm^{-1}. Our calculated result of $F^{^3\rm H}_{\rm ch}(Q^2)$ with the addition of the six-quark contribution, $F_{6q-3q}(Q^2)$, to the impulse approximation, $F_{\rm N}(Q^2)$, substantially improves the agreement with the experimental data.

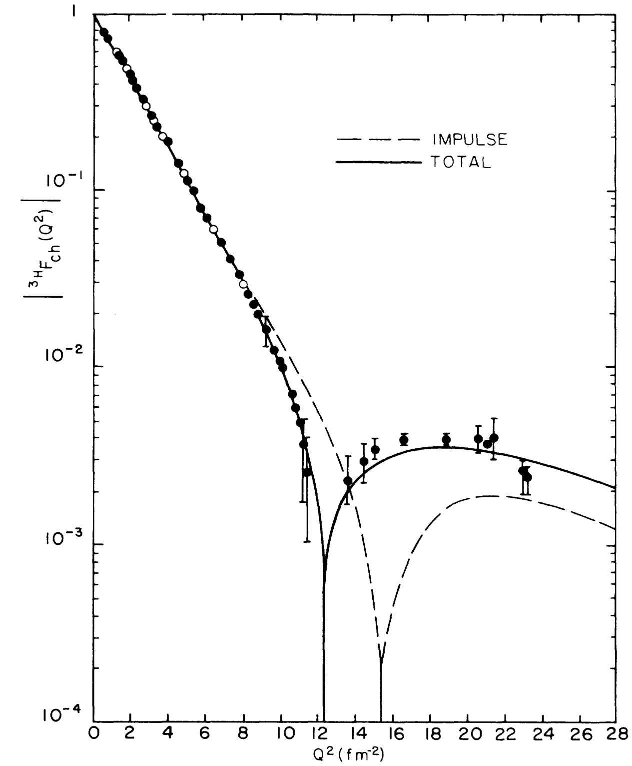

FIG. 1. Comparison of the calculated results of the ^{3}H charge form factor with the experimental data. Solid circles are from Ref. 11, and open circles are from Refs. 12 and 13.

In summary, we have calculated the ^{3}H charge form factor using the RHOM for the interior state and a modified Reid soft-core ^{3}H wave function for the exterior state (with cutoff S states). The probabilities of the interior six-quark and nine-quark states are determined from the missing part of the original Reid soft-core ^{3}H wave function. We find a reasonable fit of our calculated $F^{^3\rm H}_{\rm ch}(Q^2)$ to the experimental data for $Q^2 \lesssim 25$ fm^{-1}, with $r_0=0.9$ fm and with a small probability of the six-quark states (2.7%). The agreement of our calculated results for $F^{^3\rm H}_{\rm ch}(Q^2)$ with the experimental data is as good as and comparable to that of the $F^{^3\rm He}_{\rm ch}(Q^2)$ case[10] in which the same model and parameters have been used.

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