Resolution of the magnetic moment problem in relativistic theories

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A Landau-Migdal approach to relativistic mean field theory of nuclear matter is used to define the single particle isoscalar current. By virtue of the near cancellation of scalar and vector potentials, this current is very close to the standard nonrelativistic isoscalar current; and hence the single particle isoscalar magnetic moment operator gives results in agreement with the nonrelativistic shell model. Calculations of magnetic moments for closed shell plus (or minus) one nucleon using this effective current operator recover the Schmidt values, thus resolving a longstanding problem with relativistic models of nuclear structure.

A long-standing problem with relativistic approaches to nuclear structure[1] has been their failure to predict magnetic moments.[2] For closed shell plus or minus one nucleon where the traditional isoscalar Schmidt values are in agreement with the observed moments, this failure is particularly glaring given the elegant simplicity with which the same relativistic model gives so many other static nuclear properties. (The isovector moments are strongly renormalized by theoretically uncertain meson exchange effects, and are not in agreement with the Schmidt values. The isovector moments represent an interesting and special theoretical challenge in their own right, but will not be considered further here.) Miller[2] has emphasized the difficulties with relativistic models in fitting both the spin-orbit splitting and the magnetic moment. Relativistic models of nuclear structure are characterized by a strong attractive Lorentz scalar potential and a repulsive timelike vector potential. In Walecka's mean field approach, for example, these potentials are generated by the mean scalar σ and vector ω meson densities. One can understand how the binding energy and spin-orbit splitting arise from the scalar and vector potentials by recasting the Dirac equation for the nucleon field in Schrödinger form for the upper component. The resulting Schrödinger-type central potential is given roughly by the sum of the scalar and vector potentials while the spin-orbit potential is proportional to the difference. Fitting the binding energy and saturation density fixes the scalar and vector potentials in a way consistent with the spin-orbit splittings. Typically the scalar potential strength is about -400 MeV while the vector strength is about $+350$ MeV. The magnetic moment difficulty can be traced to the enhanced currents of the bound relativistic Dirac four-spinors. Essentially the effective nucleon mass is reduced by the strong attractive scalar potential, $m \rightarrow m^* \simeq 0.6m$, thereby increasing the velocity and associated current. The tensor anomalous moment is not sensitive to the strong scalar-vector dynamics; so we need consider only the isoscalar Dirac current. In addition to the isoscalar magnetic moment there are other observables which are sensitive to this enhancement, most notably the transverse isoscalar electromagnetic response function,[3] certain inelastic proton induced transitions at intermediate energy,[4–6] and to a lesser extent weak interaction induced processes.[7,8] For relativistic models of nuclear structure to be taken seriously the role of such strong (dynamical) relativistic effects must be addressed in a way consistent with the successful predictions of the nonrelativistic shell model of the measured isoscalar magnetic moments.

In this note the Landau-Migdal quasiparticle approach to relativistic nuclear matter[9] is used to motivate an effective density-dependent isoscalar current operator which incorporates the nontrivial response of the "spectators" to the single quasiparticle motion, i.e., the so-called "backflow."[10] The resulting current operator operator is significantly suppressed from its traditionally enhanced value. This suppression reduces part of the interparticle vector interaction as shown by Matsui.[9] Similar vector induced suppression has recently been reported by Kurasawa and Suzuki[11] in the context of a random-phase approximation (RPA) approach. One can understand the need for such backflow corrections from a consideration of the Ward identity.[12] In studying isoscalar magnetic moments here, we focus on the transverse response function in the zero momentum transfer limit. Isoscalar magnetic moment calculations for single nucleon valence states based on this new current operator agree well with the data and the traditional Schmidt values without upsetting the spin-orbit splitting. The method can be generalized to other operators as well.

The approach taken is to examine the single particle current in nuclear matter and apply the result to finite nuclei by using a local density approximation. We first review the relativistic mean field theory of Walecka[1] and the Landau treatment of Matsui.[9] We use the conventions of Bjorken and Drell.[13] Walecka's simplest relativistic model for nuclear matter consists of three fields, the nucleon field Ψ and two isoscalar meson fields σ, a Lorentz scalar, and ω, a Lorentz vector. The Lagrangian density is given by

$$\begin{aligned}\mathscr{L}=&-\tfrac{1}{4}(\partial_\mu V_\nu-\partial_\nu V_\mu)^2+\tfrac{1}{2}m_\omega^2 V_\mu V^\mu+\tfrac{1}{2}\partial_\mu\Phi\partial^\mu\Phi\\&-\tfrac{1}{2}m_\sigma^2\Phi^2+\overline{\Psi}(i\gamma_\mu\partial^\mu-m)\Psi+g_\sigma\overline{\Psi}\Psi\Phi\\&-g_\omega\Psi\gamma_\mu\Psi V^\mu\ ,\end{aligned}\tag{1}$$

where V is the vector (ω) field with mass m_ω, Φ is the scalar (σ) field with mass m_σ, and Ψ is the nucleon field with mass m. Isospin indices are suppressed. For bulk (long wavelength) properties and low-lying excitations it is sufficient to average the meson source terms yielding the following static mean fields:

$$\overline{\Phi}=\frac{g_\sigma}{m_\sigma^2}\langle\overline{\Psi}\Psi\rangle\ ,\tag{2}$$

$$\overline{V}_\mu=\frac{g_\omega}{m_\omega^2}\langle\overline{\Psi}\gamma_\mu\Psi\rangle\ ,\tag{3}$$

where the brackets indicate the ground state expectation value. The nucleon field equation then becomes the self-consistent Dirac equation

$$[\gamma_\mu(i\partial^\mu-\lambda_\omega\langle\overline{\Psi}\gamma^\mu\Psi\rangle)-(m-\lambda_\sigma\langle\overline{\Psi}\Psi\rangle)]\Psi=0\ ,\tag{4}$$

where $\lambda_\sigma=(g_\sigma/m_\sigma)^2$ and $\lambda_\omega=(g_\omega/m_\omega)^2$. The mean-field Dirac equation has the following quasiparticle solutions in an arbitrary frame:

$$\Psi_i=\left[\frac{E_i+m^*}{2E_i}\right]^{1/2}\times\left[\begin{matrix}1\\ \sigma\cdot(\mathbf{k}_i-\lambda_\omega\mathbf{j}_B)/(E_i+m^*)\end{matrix}\right]\chi_i e^{ik_ix_i}\ ,\tag{5}$$

where χ is the Pauli two-spinor and

$$m^*=m-\lambda_\sigma\langle\overline{\Psi}\Psi\rangle\ ,\tag{6}$$

$$j_B^\mu=(\rho_0,\mathbf{j}_B)=\langle\overline{\Psi}\gamma^\mu\Psi\rangle\ ,\tag{7}$$

$$E_i=[(\mathbf{k}_i-\lambda_\omega\mathbf{j}_B)^2+(m^*)^2]^{1/2}\ .\tag{8}$$

Note that this is in fact a self-consistency condition in that the solution for Ψ involves Ψ itself. The energy density for a zero temperature Fermi liquid filled to Fermi momentum k_F is

$$\mathscr{E}=\frac{\lambda_\omega}{2}(\rho_0^2+\mathbf{j}_B\cdot\mathbf{j}_B)+\frac{1}{2\lambda_\sigma}(m-m^*)^2+\sum_i n_iE_i\ ,\tag{9}$$

where $n_i=\theta(k_F-|\mathbf{k}_i|)$.[14] Note that the energy density is stationary under variations in m^* and $\mathbf{j}_B$, and ρ_0 is by normalization. Fixing the ratios, λ_σ and λ_ω, to the binding energy per particle and saturation density yields the large scalar and vector potentials quoted earlier which are in turn consistent with the spin-orbit splitting in finite nuclei.

We come now to the critical question: What is the single particle current? Or more generally: How do we define any single particle property in this dense, strongly interacting many body context? Landau and Migdal answer these questions by calculating how the system as a whole responds when one quasiparticle is removed. For example, the single particle energy is defined by

$$\epsilon_i\equiv\frac{\delta\mathscr{E}}{\delta n_i}=E_i+\lambda_\omega\rho_0\tag{10}$$

in the nuclear rest frame. Despite the complicated density dependences in $\mathscr{E}(m^*,\mathbf{j}_B)$ the simplicity of this result follows from the stationary nature of $\mathscr{E}$ under variations in m^* and $\mathbf{j}_B$. This definition implicitly assumes that the system has time to rearrange itself to adjust to the quasiparticle's absence. This definition is only appropriate, therefore, for energies near the Fermi surface. In similar fashion we propose defining the single quasiparticle current as the difference in the total baryon current when the particle is removed:

$$\tilde{\mathbf{j}}_i\equiv\frac{\delta\mathbf{j}_B}{\delta n_i}\ .\tag{11}$$

This not only accounts for the missing particle but also for the current due to the rest of the medium adjusting to that particle, i.e., the backflow. Using the self-consistent $\mathbf{j}_B$, Eq. (7), one finds

$$\mathbf{j}_B=\sum_j\frac{n_j}{E_j}(\mathbf{k}_j-\lambda_\omega\mathbf{j}_B)\ ,\tag{12}$$

which, upon variation with respect to the single particle density and evaluated in the nuclear rest frame, yields the effective single particle current

$$\tilde{\mathbf{j}}_i=\left[\frac{\delta\mathbf{j}_B}{\delta n_i}\right]_{\mathbf{j}_B\to 0}\ ,$$

$$\tilde{\mathbf{j}}_i=\frac{\mathbf{k}_i}{[k_i^2+(m^*)^2]^{1/2}}\left[1+\lambda_\omega\sum_j n_j\frac{[\frac{2}{3}k_j^2+(m^*)^2]}{[k_j^2+(m^*)^2]^{3/2}}\right]^{-1}\ .\tag{13}$$

The first term is the usual relativistically enhanced (m/m^*) current, but here we see that the self-consistency condition yields a reduction factor first noted by Matsui for the momentum-dependent part of the interparticle vector interaction.[9]

One can show that the isoscalar current, Eq. (13), is the appropriate one to couple to an external magnetic field. In fact the single quasiparticle electromagnetic spacelike current in this model is

$$(\tilde{\mathbf{j}}_{\rm em})_i=\frac{\delta\mathbf{J}_{\rm em}}{\delta n_i}\ .$$

Evaluating this in the nuclear rest frame yields

$$(\tilde{\mathbf{j}}_{\rm em})_i=\frac{1}{2}\left[\left[1+\lambda_\omega\sum_j n_j\frac{\frac{2}{3}k_j^2+(m^*)^2}{[k_j^2+(m^*)^2]^{3/2}}\right]^{-1}+\tau_3^i\right]\frac{\mathbf{k}_i}{E_i}\ .\tag{14}$$

The backflow renormalizes only the isoscalar part of the electromagnetic current because the model contains only isoscalar mesons. Simply stated, the electromagnetic field can interact with the quasiparticle either by coupling directly to it or to the correlated backflowing nuclear matter which is inextricibly associated with the single particle current in this dense strongly interacting context.

In order to estimate the effect of our definition for the single particle current, Eq. (13), we make a local density approximation to the vector suppression term. The Fermi sum can be done for the zero temperature case, yielding the isoscalar current

$$(\tilde{\mathbf{j}}_i)_{T=0}=\frac{\mathbf{k}_i}{[k_i^2+(m^*)^2]^{1/2}} \times\left[1+\frac{2\lambda_\omega k_F^3}{3\pi^2[k_F^2+(m^*)^2]^{1/2}}\right]^{-1}, \tag{15}$$

where a degeneracy of 4 (2 spin and 2 isospin) is assumed. Bentz *et al.*[12] have argued that this result follows from a consideration of the Ward identity. For a self-energy calculated in Hartree (or mean field) the isoscalar (transverse) vertex correction obtained from the Ward identity is just the (transverse) polarization insertion obtained in random phase approximation. The suppression factor in Eq. (15) is the $\mathbf{q}\to 0$ then $q_0\to 0$ limit of this transverse polarization insertion.[15]

From the rest frame solution for the nucleon field, Eq. (5) with $\mathbf{j}_B=0$, this current can be obtained from the effective density-dependent Dirac operator:

$$\tilde{\gamma}(r)=\gamma\{1+\lambda_\omega\rho_0[(\tfrac{3}{2}\pi^2\rho_0)^{3/2}+(m^*)^2]^{-1/2}\}^{-1}, \tag{16}$$

where the local values for ρ_0 and m^* are now used. In this $\mathbf{q}\to 0$ limit γ is transverse only. Table I displays isoscalar magnetic moment calculations based on this effective single particle operator compared with the standard relativistically enhanced value and the Schmidt value. The relativistic wave functions used in these calculations were obtained from a relativistic Woods-Saxon well with parameters adjusted to give the separation energy and elastic electron scattering form factor (essentially identical wave functions are obtained from relativistic Hartree calculations). We see that the large Dirac current effect is virtually eliminated when the effective current is used, bringing the relativistic magnetic moment calculations into agreement with the successful Schmidt values.

It is no numerical accident that the cancellation of the scalar enhancement due to the sigma and the vector suppression due to the omega is so complete. It is precisely this high degree of cancellation which gives the relatively shallow, 50 MeV, central well depth of the Schrödinger equation form of the Dirac equation. In nonrelativistic Landau liquid theory one expects the velocity corrections to be the order of the binding energy over rest mass. Expanding the effective isoscalar current operator in powers of k^2 demonstrates this dependence explicitly for the present relativistic example. By neglecting terms of order k_i^4 and higher, terms proportional to $(k_F^2-k_i^2)$ and terms of order $\lambda_{\omega,\sigma}\rho_{0,s}/m$, from Eq. (15) we obtain

$$(\tilde{\mathbf{j}}_i)_{T=0}\simeq\mathbf{k}_i\left[m+\lambda_\omega\rho_0-\lambda_\sigma\rho_s+\frac{k_i^2}{2m}\right]^{-1}. \tag{17}$$

The familiar m^* enhancement from the scalar potential is now largely canceled by the vector potential. This cancellation of vector and scalar potentials which here restores the current to roughly its nonrelativistic form is the same cancellation which has kept relativity hidden in low energy nuclear structure. When combined with the kinetic energy term, the potential terms give a small binding energy correction and Eq. (17) can be written

$$(\tilde{\mathbf{j}}_i)_{T=0}\simeq\frac{\mathbf{k}_i}{m}\left[1-\frac{|E_B|}{m}\right]^{-1}, \tag{18}$$

where E_B is the binding energy.

An examination of other observables believed to be sensitive to the relativistic isoscalar current enhancement is underway. For higher energy processes where the characteristic interaction energy is larger than the Fermi energy, this quasiparticle approach cannot be applied. For intermediate energy proton scattering the successful free impulse results apply. The importance of relativistic dynamics in the transition region, for example quasifree knockout where the struck nucleon is bound but the outgoing nucleon is fast, is still an open question. Finally we note that so long as the energies are near the Fermi surface the Landau-Migdal approach to defining single particle properties can be used. The generalization to operators which induce inelastic processes is underway.

TABLE I. Two relativistic isoscalar magnetic moment calculations of closed shell plus or minus one nucleon are shown along with the experimental values and the Schmidt values. All theoretical values include the anomalous moment without modification from its free space value. The γ-labeled column includes the traditional m^* enhancement to the current, the first term of Eq. (13). The $\tilde{\gamma}$-labeled column is based on the effective current operator of this work, Eq. (13). Δ_1 is the difference between the γ result and Schmidt, while Δ_2 is the difference the $\tilde{\gamma}$ result and Schmidt. The difference between the relativistic calculations and the Schmidt values is substantially reduced by the backflow correction.

		Isoscalar magnetic moments				
Mass no.	Expt.	Schmidt	γ	Δ_1	$\tilde{\gamma}$	Δ_2
15	0.218	0.187	0.297	0.11	0.192	0.005
17	1.414	1.44	1.57	0.13	1.41	−0.03
39	0.706	0.636	0.918	0.282	0.645	0.009
41	1.918	1.94	2.24	0.30	1.91	−0.03

In this note we have used the Landau-Migdal approach to define an effective single particle isoscalar current in the relativistic mean field model. In a self-consistent treatment the vector interaction cancels the enhancement due to the scalar, so that to leading order the current operator reduces to the nonrelativistic form. Thus the longstanding problem with the isoscalar magnetic moments in a single particle relativistic shell model is resolved.

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