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Electromagnetic form factors of ^{3}He and ^{3}H and current conservation

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Previous calculations of the electric and magnetic form factors of ^{3}He and ^{3}H have included various theoretical ingredients (nucleon form factors, mesonic exchange currents, pion-nucleon vertices, etc., . . .). The bearing of these ingredients upon the nuclear current conservation equation is investigated.

I. INTRODUCTION

Recent measurements of the electric and magnetic charge form factors of the triton by the Saclay group[1] are interesting in view of the existing data on ^{3}He. These two nuclei form an isodoublet which provides a good framework in which to investigate the isospin properties of their electromagnetic charge and current distribution. The present paper is an extension of calculations carried out using the three-nucleon Faddeev wave function and the nucleon-meson operators.[2–5] Following the Hadjimichael-Bornais-Goulard (HBG) publications, the authors of Refs. 3–5 computed the electromagnetic form factors of ^{3}He and ^{3}H. In these computations, they placed a special emphasis on the requirements of the current conservation equation and its bearing on the structure of the nuclear magnetic current.

The theoretical description of the electric form factor is known to be more uncertain than that of the magnetic form factor. Calculations of the magnetic form factor are reasonably convergent; both the impulse approximation (IA) and the dominant mesonic exchange currents (MEC's) are of the same relativistic order ($1/m_{\rm N}$), and are well defined. They are both shown in Fig. 1. Furthermore, the above mentioned calculations are in reasonable agreement with the old ^{3}He and new ^{3}H measurements.[1] However, considerations of gauge invariance requirements led the authors of Ref. 3 and 5 to substitute the nucleonic Dirac form factor F_1^V (used by HBG) for the nucleonic Sachs form factor G_E^V. Such a substitution upsets the

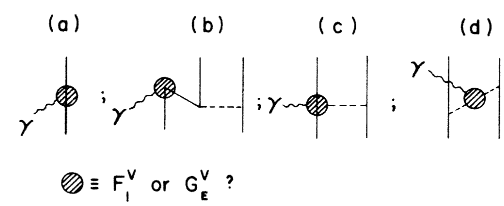

FIG. 1. Diagrams describing (a) the impulse approximation (IA), (b) the pion-pair current, (c) the seagull term, and (d) the pion in flight term. The blob represents the electromagnetic form factors. The blob in (c) is taken to be the axial nucleonic form factor $F_A(q^2)$ on the basis of algebra of currents, while the blob in (b) is the pionic form factor F_π. Assuming these are the only diagrams, current conservation leads to the overall equality of these form factors, i.e., $F_A = F_\pi = F_1^V$ or G_E^V.

otherwise good overall agreement with the experimental data and suggests that the use of F_1^V causes a basic violation of the current conservation equation.

The electric form factor, as mentioned above, provides a less satisfactory situation; more specifically, the overall consistency of the charge operator and the wave function it acts upon is not clear because of the nonrelativistic character of the Faddeev wave function coupled with the charge operator of relativistic order ($1/m_{\rm N}^2$). In the latter case, use of current conservation as a constraint would require a consistent description of the magnetic longitudinal current to the order ($1/m_{\rm N}^3$). However, estimates of the dominant mesonic exchange density, i.e., the pionic pair contribution, moves the theoretical form factor, which is based on IA, toward an overall agreement with the experimental data on ^{3}He, as illustrated in Fig. 2. The same term does not, however, predict a significant shift of the dip in the triton case which would be necessary to fit the recent Saclay result on the triton, as shown in Fig. 3.

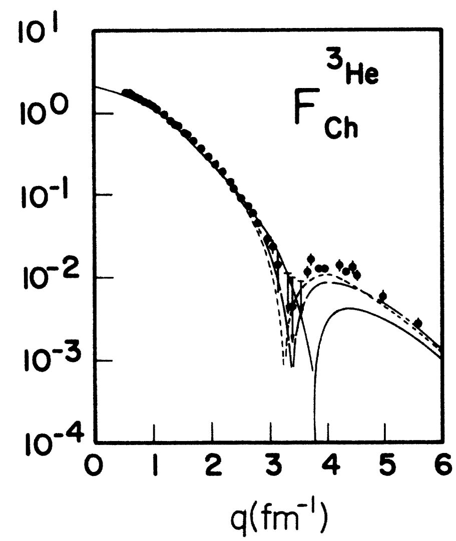

FIG. 2. ^{3}He electric charge form factor. Faddeev wave function based on a Reid soft core potential, Blatnik nucleonic form factors, and physical hadronic vertices. (——) IA contribution; (-----) total charge form factor with pseudovector coupling in the pion pair contribution; (— — —) total charge form factor with pseudoscalar coupling in the pion pair contribution; (●) experimental points.

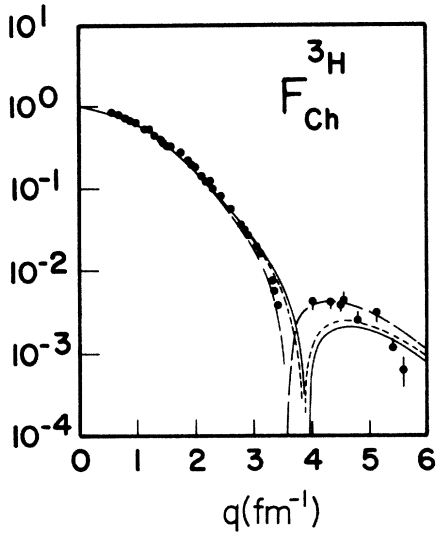

FIG. 3. ^{3}H electric charge form factor. Conventions identical to those of Fig. 2.

The aims of the present work are (i) to show that the substitution of F_1^V by G_E^V is not made necessary by gauge invariance; the expressions used by HBG will be shown to be consistent with the current conservation equation within specific limitations; and (ii) to draw attention to the consequences of the choice of the pion-nucleon coupling on the calculated value of the electric charge form factor.

II. CURRENT CONSERVATION AND MAGNETIC FORM FACTORS

The most general conservation equation satisfied by the nuclear current $J_\mu^N = (\mathbf{J}^N, J_0^N)$, i.e.,

$$\partial_\mu J_\mu^N = 0 \tag{1}$$

is usually written as,

$$\nabla \cdot \mathbf{J}^N(x) + i[\mathcal{H}, J_0^N(x)] = 0 , \tag{2}$$

$J_0^N(x)$, $\mathbf{J}^N(x)$, and $\mathcal{H}$ being, respectively, the nonrelativistic part of the nuclear charge density, the nuclear current density (to the order $1/m_N$), and the nuclear Hamiltonian. This can be rewritten as

$$\nabla \cdot \mathbf{J}_{IA}^N(x) = -i[T, J_0^N] , \tag{3}$$

$$\nabla \cdot \mathbf{J}_{MEC}^N(x) = -i[V, J_0^N] , \tag{4}$$

where $\mathbf{J}_{IA}$ and $\mathbf{J}_{MEC}$ are the nuclear current in the IA framework and the MEC contribution, respectively, and $V\ (=V_0 + V_\tau \tau_1 \cdot \tau_2)$ is the nucleon-nucleon interaction; Eq. (4) originates from the isospin component (V_τ) which requires the introduction of two mesonic exchange currents in order to preserve the nuclear current conservation equation i.e., the true pion-exchange current J_π and the pion pair current $J_{\pi NN}$ (or the seagull term J_c), all of which are represented in Fig. 1. The current conservation equation derived in this way for point nucleons and mesons is no longer correct once nucleonic form factors are incorporated into those currents. In order to preserve the current conservation equation, it has been suggested[7,8] that the same nucleonic form factor should appear in each of the IA and MEC contributions listed in Fig. 1. It is at this point that the authors of Refs. 3–5 differ with HBG. The former take G_E^V, claiming it to be necessary to preserve gauge invariance, while the latter take F_1^V to factorize the graphs of Fig. 1. In the following we will show the overall consistency of the HBG approach, together with its limitations.

A. Impulse approximation

The single nucleon current J_μ^N can be written in terms of the Dirac form factors $F_{1,2} = F_{1,2}^S + \tau_3 F_{1,2}^V$; the superscripts refer to the isoscalar and the isovector:

$$J_\mu^N = F_1(q^2)\gamma_\mu + F_2(q^2)\sigma_{\mu\nu} q_\nu . \tag{5}$$

The conservation equation ($q_\mu J_\mu = 0$) leads to

$$q_\mu q_\nu \sigma_{\mu\nu}(F_2^S + \tau_3 F_2^V) \equiv 0 , \tag{6}$$

$$q_\mu \gamma_\mu (F_1^S + \tau_3 F_1^V) = 0 , \tag{7}$$

so that the F_2 component of the current is identically divergenceless.[9] In a consistent nonrelativistic expansion, this identity should be reflected to any order in $1/m_N$. To illustrate how the argument works at the level $1/m_N$ of the conservation equation, let us write J^N to the order $1/m_N$ and J_0 to the order $1/m_N^2$ as they appear in HBG, remembering that the kinetic energy $T = q_0 \simeq O(1/m_N)$,

$$\begin{aligned} \mathbf{J}_{IA}^N &\approx \mathbf{J}_{IA}^N\left[\frac{1}{m_N}\right] + \mathbf{J}_{F_1}^N\left[\frac{1}{m_N^3}\right] + \mathbf{J}_{F_2}^N\left[\frac{1}{m_N^3}\right] \\ &\approx \frac{1}{i}\frac{2\mathbf{p}-\mathbf{q}}{2m_N}F_1(q^2) + \frac{F_1(q^2)+F_2(q^2)}{2m_N}\sigma\times\mathbf{q} \\ &\quad + \mathbf{J}_{F_1}^N\left[\frac{1}{m_N^3}\right] + \mathbf{J}_{F_2}^N\left[\frac{1}{m_N^3}\right] , \end{aligned} \tag{8}$$

$$\begin{aligned} J_0^N &\approx J_0^N\left[\frac{0}{m_N}\right] + J_{0F_1}^N\left[\frac{1}{m_N^2}\right] + J_{0F_2}^N\left[\frac{1}{m_N^2}\right] \\ &\approx \frac{1}{i}\left[F_1(q^2) - \frac{\mathbf{q}^2}{8m_N^2}(F_1+2F_2)\right] \\ &\quad + \frac{\mathbf{q}\cdot\sigma\times\mathbf{p}}{4m_N^2}(F_1+2F_2) , \end{aligned} \tag{9}$$

where $\mathbf{J}_{F_1,F_2}^N(1/m_N^3)$, $J_{0F_1,F_2}^N(1/m_N^2)$ refer to contributions of order $(1/m_N^2)$ and higher order, for instance:

$$\mathbf{J}_{F_2}\left[\frac{1}{m_N^3}\right] = -\frac{\mathbf{q}^2}{4m_N^2}\frac{(2\mathbf{p}-\mathbf{q})}{m_N}F_2 + \cdots . \tag{10}$$

The currents $\mathbf{J}_{IA}^N(1/m_N)$, $J_{0F_1}^N(0/m_N)$ follow the conservation equation to the order $(1/m_N)$:

$$q_\mu J_\mu \cong \mathbf{q}\cdot\mathbf{J}^{\mathrm{N}} - q_0 J_0^{\mathrm{N}}$$

$$\cong \frac{1}{i}\left[\frac{\mathbf{p}'^2-\mathbf{p}^2}{2m_{\mathrm{N}}} - \frac{\mathbf{p}'^2-\mathbf{p}^2}{2m_{\mathrm{N}}}\right]F_1 + O\left[\frac{1}{m_{\mathrm{N}}^3}\right] \cong 0 . \quad (11)$$

The sum of terms containing F_2 to a specific order of $1/m_{\mathrm{N}}$ in $O(1/m_{\mathrm{N}}^3)$ is ignored not only because of its higher order, but because it should disappear identically to all orders of $1/m_{\mathrm{N}}$ as a consequence of Eq. (6). Thus, it is claimed that, at the level $1/m_{\mathrm{N}}$, the current $(\mathbf{J}^{\mathrm{N}}, J_0^{\mathrm{N}})$ taken by HBG satisfies gauge invariance; furthermore the terms containing F_2 are conserved independently. From the point of view of our approach, the choice of the Sachs form factor $G_E=[F_1-(\mathbf{q}^2/4m_{\mathrm{N}}^2)F_2]$ instead of F_1 amounts to picking up the second term in J_0^{N}, i.e., $[-(\mathbf{q}^2/4m_{\mathrm{N}}^2)F_2]$ and the term shown in Eq. (10) for $\mathbf{J}^{\mathrm{N}}$, i.e., $(-\mathbf{q}^2/4m_{\mathrm{N}}^2)[(2\mathbf{p}-\mathbf{q})/m_{\mathrm{N}}]F_2$ (despite the fact that they are the next higher order and contain F_2); these two terms will then add up to the expression of a current which already satisfies the conservation equation (to the order $1/m_{\mathrm{N}}$). In order to prove that with the substitution $F_1 \to G_E$ the current and charge densities satisfy gauge invariance to a higher relativistic order, it would be necessary to calculate *all* the terms in $\mathbf{J}^{\mathrm{N}}_{F_1,F_2}(1/m_{\mathrm{N}}^3)$, and show that they yield a divergenceless current to the order $1/m_{\mathrm{N}}^3$.

Rather than investigating these terms we have chosen to rewrite the nucleon current in relativistically covariant terms using Sachs form factors. The following expression is equivalent to the expression based on Dirac form factors provided nucleons are on the mass shell:[10]

$$\langle p' | J_\mu^{\mathrm{N}} | p\rangle = \left[1-\frac{q^2}{4m_{\mathrm{N}}^2}\right]^{-1} \left\langle p' \left| G_E \frac{p_\mu + p'_\mu}{2m_{\mathrm{N}}} - G_M \gamma_5 \epsilon_{\mu\nu\rho\sigma} \frac{p_\nu + p'_\nu}{2m_{\mathrm{N}}} q_\rho \gamma_\sigma \right| p \right\rangle . \quad (12)$$

This expression separates in an invariant way the contribution of the charge density from the magnetic current density. The part of the magnetic current density containing G_M is identically divergenceless. Indeed, the substitution of the Dirac form factors for the Sachs form factors

$$\left[F_2 \to F_1 + F_2,\ F_1 \to F_1 - \frac{\mathbf{q}^2}{4m_{\mathrm{N}}^2}F_2\right]$$

amounts to reshuffling the divergenceless form factor F_2.

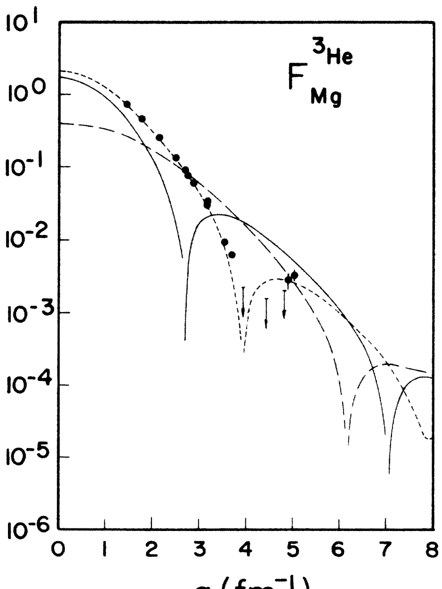

FIG. 4. ^{3}He magnetic form factor with $F_\pi = F_1^V$. As in Figs. 2 and 3, the Faddeev wave function is based on a Reid soft core potential, Blatnik nucleonic form factors, and physical hadronic vertices. (——) IA contribution; (– – –) exchange current contribution; (-----) total magnetic form factor; (●) experimental points.

Thus, as far as gauge invariance is concerned, the use of Dirac or Sachs form factors does not make any difference. However, the expansion to order $1/m_{\mathrm{N}}$ used by the authors of Refs. 3 and 5 yields

$$\mathbf{J} = \frac{1}{i}G_E\frac{\mathbf{p}+\mathbf{p}'}{2m_{\mathrm{N}}} + G_M\frac{\sigma\times\mathbf{q}}{2m_{\mathrm{N}}} , \quad (13)$$

$$\rho = \frac{1}{i}G_E . \quad (14)$$

Such an expansion of the right-hand side of Eq. (12) both omits the factor $[1-(q^2/4m_{\mathrm{N}}^2)]^{-1}$ and incorporates a correction to the order $(1/m_{\mathrm{N}}^2)$ under the cover of the new form factor G_E. It is then not surprising that the current $\mathbf{J}$ of Eq. (13) leads to a magnetic form factor different from the HBG result.

To be specific, the dip in the ^{3}He magnetic form factor (see Fig. 4) shows up at $q \cong 4$ fm^{-1}. The use of G_E^V instead of F_1^V would shift this dip to lower momentum transfer (around 3.5 fm^{-1}) and farther from experimental data. This reshuffling of terms to order $(1/m_{\mathrm{N}}^2)$ reminds us that the internal structure of the nucleons, as expressed by form factors in terms of $[1+(\mathbf{q}^2/m_B^2)]^{-1}$ with $m_B \cong 1$ GeV, yields corrections of a relativistic nature over which we have no basic theoretical control. This point will be briefly commented upon in the conclusion.

B. Pionic exchange currents

When systems with more than one nucleon are considered, the identity (6) involving F_2 is not affected by the inclusion of an isospin dependent nucleon-nucleon interaction; thus, the MEC's needed to preserve the conservation equation (2) will be added to the F_1 component of the free nucleon current J_μ^{N}. then, provided that the expansion in $1/m_{\mathrm{N}}$ terms is correct we have Eqs. (3) and (4) redistributed into the following equations:[11]

$$\nabla\cdot\mathbf{J}^{\mathrm{IA}}_{(F_1)} = -i[T, \rho^{\mathrm{IA}}_{(F_1)}] , \quad (15)$$

$$\nabla\cdot\mathbf{J}^{\rm IA}_{(F_2)}=-i\,[T+V,\rho^{\rm IA}_{(F_2)}]\ , \tag{16}$$

$$\nabla\cdot\mathbf{J}^{\rm MEC}=-i[V,\rho^{\rm IA}_{(F_1)}]\ . \tag{17}$$

Indeed, the true pion exchange current $\mathbf{J}_\pi$ and the nucleon-pair pion exchange current $\mathbf{J}_{\pi\rm NN}$ were first introduced into Eq. (4) together with the pion exchange potential $V_{\rm OPE}$ in order to ensure current conservation within the framework of point particles. Our concern in the present section is to explore the extent to which the successive shifts from the limit of point particles to structured particles affect current conservation as expressed by Eq. (2).

The first step is expressed by Eqs. (3) and (4) in the limit of structureless particles and the one-pion exchange potential (OPE),

$$\nabla\cdot(\mathbf{J}^{\rm IA}+\mathbf{J}^{\rm MEC}_{\pi,\pi\rm NN})_{\rm point}=-i\,[T+V_{\rm OPE},\rho^{\rm IA}_{\rm point}] \tag{18}$$

with the general expression for the MEC,

$$\mathbf{J}_{\pi\rm NN}=\frac{g^2_{\pi\rm NN}}{4m^2_{\rm N}}\frac{F^2_{\pi\rm NN}}{\mathbf{k}_2+m^2_\pi}(-i)(\tau_1\times\tau_2)F^V_1(q^2)$$
$$\times\boldsymbol{\sigma}_1(\boldsymbol{\sigma}_2\cdot\mathbf{k}_2)+1\leftrightarrow2\ , \tag{19}$$

$$\mathbf{J}_\pi=\frac{g^2_{\pi\rm NN}}{4m^2_{\rm N}}\frac{F_{\pi\rm NN}}{\mathbf{k}^2_1+m^2_\pi}\frac{F_{\pi\rm NN}}{\mathbf{k}^2_2+m^2_\pi}F_\pi(q^2)$$
$$\times(\tau_1\times\tau_2)\boldsymbol{\sigma}_1\cdot\mathbf{k}_1\boldsymbol{\sigma}_2\cdot\mathbf{k}_2(\mathbf{k}_2-\mathbf{k}_1)\ , \tag{20}$$

and the expressions $\mathbf{J}^{\rm MEC}_{\pi(\rm point)}$, $\mathbf{J}^{\rm MEC}_{\pi\rm NN(point)}$, and $\rho^{\rm IA}_{\rm point}$ correspond to $\mathbf{J}^{\rm MEC}_\pi$, $\mathbf{J}^{\rm MEC}_{\pi\rm NN}$, and $\rho^{\rm IA}$ with $F_{\pi\rm NN}(q^2)=F_{\pi\rm NN}(0)=1$, $F_\pi(q^2)=F_\pi(0)=1$, and $F^V_1(q^2)=F^V_1(0)=1$. In this limit, current conservation is explicitly satisfied with the following one-pion exchange potential:

$$V_{\rm OPE}=\frac{g^2_{\pi\rm NN}}{4m^2_{\rm N}}\tau_1\cdot\tau_2\frac{\boldsymbol{\sigma}_1\cdot\mathbf{k}_1\boldsymbol{\sigma}_2\cdot\mathbf{k}_2}{q^2+m^2_\pi}\ . \tag{21}$$

The second step is precisely the factorization of the left-hand side of Eq. (18) by the electromagnetic form factor F^V_1 which yields current conservation expressed as

$$\nabla\cdot(\mathbf{J}^{\rm IA}_{(F_1)}+\mathbf{J}^{\rm MEC}_{(F_1)})=-i\,[T+V_{\rm OPE},\rho^{\rm IA}_{F_1}] \tag{22}$$

with $\mathbf{J}^{\rm MEC}_{(F_1)}=\mathbf{J}^{\rm MEC}_{\rm point}\cdot F^V_1(q^2)$.

Furthermore, the use of the seagull graph [Fig. 1(d)] which is based on considerations of partially conserved axial current (PCAC) current algebra, led to the postulation that this graph should be proportional to the nucleon axial form factor $F_A(q^2)$.[14] In other words, when both current conservation and structured nucleons are required, a specific constraint is imposed:

$$F_\pi(q^2)=F^V_1(q^2)=F_A(q^2)\ . \tag{23}$$

When such a constraint was suggested a few years ago,[8] the existing data were consistent with the vector dominance model which predicts equality of the three form factors of Eq. (23). Nowadays, new measurements of the pion form factor,[15] the proton charge radius,[16] and the axial form factor[17] imply that the statement of Eq. (23) is imperfect. As a consequence of this, if one retains the assumption that the two processes corresponding to Figs. 1(b) and (c) [or (d)] still satisfy current conservation, then one has to assume that the free electromagnetic and weak form factors of Eq. (23) are slightly distorted by the nuclear dynamics.

The consideration of finite size nucleons has also led to the incorporation of the pion-hadron vertex $F_{\pi\rm NN}(q^2)$ in the expression of the currents $\mathbf{J}_{\pi\rm NN}$ and $\mathbf{J}_\pi$. It is introduced through a phenomenological form factor $F_{\pi\rm NN}=\Lambda^2_\pi-m^2_\pi/\Lambda^2_\pi+q^2$. Its incorporation in Figs. 1(b) and (d) yields a shift in the q dependence of the electric charge and the magnetic trinucleon nuclear form factors. Illustrations of these shifts are shown in Figs. 17 and 18 in HBG. The effect on the electric charge form factor is modest, around the commonly picked value for Λ_π (1.2 GeV), while the effect on the magnetic form factor is more significant.

When this vertex $F_{\pi\rm NN}(q^2)$ is included on the left-hand side of the current conservation equation, it can be shown that in the absence of the electromagnetic nucleon size effect ($F^V_1=F_\pi=F_A=1$), a modified version of $V_{\rm OPE}$, i.e.,

$$V_{\rm OPE}=\frac{g^2_{\pi\rm NN}}{4m^2_{\rm N}}\tau_1\cdot\tau_2\frac{\boldsymbol{\sigma}_1\cdot\mathbf{k}_1\boldsymbol{\sigma}_2\cdot\mathbf{k}_2}{q^2+m^2_\pi}\left[\frac{\Lambda^2_\pi-m^2_\pi}{\Lambda^2_\pi+q^2}\right]^2\ , \tag{24}$$

can restore a strict current conservation.[18]

However, when both pion-hadronic form factors and nucleon electromagnetic form factors are present in the current conservation equation, it is necessary[19] to introduce off mass-shell effects and new pionic exchange currents over and above the two currents $\mathbf{J}_\pi$ and $\mathbf{J}_{\pi\rm NN}$ to preserve current conservation. To our knowledge, no complete calculations have been carried out along these lines.

In a perfectly consistent calculation, it is anticipated that deviation from point-particle behavior on the left-hand side of Eq. (18) will be properly compensated by a corresponding modification of the right-hand side. The problem is that available nucleon-nucleon interaction are of a phenomenological character and do not necessarily guarantee such a compensation.

C. Numerical studies

Since the HBG calculations (as well as Refs. 3—5) included precisely limited relativistic expansions, pion vertices in the MEC expressions, and a nucleon-nucleon interaction which is not necessarily compatible with the MEC structure, it is not possible to state the extent of current conservation violation in an analytical form. However, we can estimate how Eq. (17) (involving operators) is violated once the various contributions mentioned above have been included, by computing their expectation value over one of the Faddeev three-nucleon wave functions, using a specific set of the theoretical ingredients adopted by HBG for this numerical study.

The prescription for determining how well current conservation is satisfied is thus to evaluate (i) the variation of the total charge with time, and (ii) the quantities $\delta^{\rm IA}(r)$

and $\delta^{MEC}(r)$, which are related to Eqs. (15) and (17), respectively, and are defined as follows:

$$\dot{Q} = \int d\mathbf{r} \langle \psi | \nabla \cdot \mathbf{J}^{IA}(\mathbf{r}) + i[T, \rho^{IA}(r)] | \psi \rangle + \int d\mathbf{r} \langle \psi | \nabla \cdot \mathbf{J}^{MEC}(\mathbf{r},\mathbf{r}') + i[V(\mathbf{r},\mathbf{r}'), \rho^{IA}(r)] | \psi \rangle ,$$

$$= \int d\mathbf{r}\, \delta^{IA}(\mathbf{r}) + \int d\mathbf{r}\, \delta^{MEC}(\mathbf{r}) \tag{25}$$

$$= 4\pi \int dr\, r^2 \delta^{IA}(\mathbf{r}) + 4\pi \int dr\, r^2 \delta^{MEC}(\mathbf{r}) ; \tag{26}$$

r being the radial distance from the nuclear center.

A nonzero $\dot{Q}$ measures the degree to which the average charge over the nuclear volume is not constant (global conservation) while nonzero $\delta^{IA}(r)$ and $\delta^{MEC}(r)$ measure the degree to which the single-nucleon and the two-nucleon currents are not locally conserved.

Figures 5–8 correspond to ^{3}He with a Reid-soft core (RSC) nucleon-nucleon interaction, together with other ingredients included in HBG, see their Figs. 15 and 16 (Blatnik nucleon form factor, pionic-nucleon vertex with $\Lambda_\pi = 6.0$ fm^{-1}). F_π has been taken equal to F_1^V (Ref. 20). In the present section, each step will be a numerical evaluation of one of the more general steps considered in Eqs. (15)–(24). As a first step, the numerical accuracy of the proposed testing procedure was checked by evaluating the first term on the right-hand side of Eq. (25), which as indicated in connection with Eq. (15) should be explicitly equal to zero. In Fig. 5, the dotted curve represents $4\pi r^2 \delta^{IA}(r)$ and its two parts, $4\pi r^2 \nabla \cdot \mathbf{J}^{IA}$ and $4\pi r^2 [T, \rho^{IA}]$. We checked that $4\pi r^2 \delta^{IA}(r)$ is zero for all values of r, demonstrating both global charge conservation and local current conservation. As a second step, we evaluate $4\pi r^2 \delta^{MEC}(r)$ for one-pion exchange (OPE) nucleon-nucleon potential and MEC's with no nucleonic form factor, i.e., the expectation value of the second term on the right-hand side of Eq. (25), in the limit of point nucleons. This equation is explicitely satisfied, as shown in Fig. 6. The numerical value of $4\pi r^2 \delta^{MEC}(r)$ is zero for all values of r. Hence, we conclude that the numerical accuracy of our procedure is satisfactory. Furthermore, comparison of Eq. (18) with Eq. (19) very clearly shows that the multiplication of both sides of Eq. (18) by the same form factor F_1^V (or G_E^V) will not alter the explicit result obtained in the limit of point nucleons.

Then, the final step is to proceed to the "realistic" case

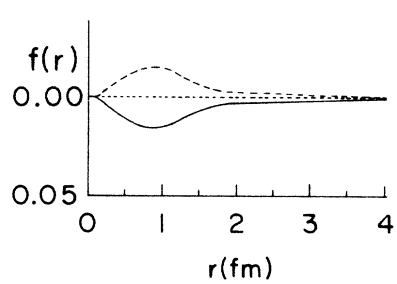

FIG. 5. Current conservation equation for ^{3}He (impulse approximation only). (-----) $4\pi r^2 \delta_{IA}(r)$; (— — —) $4\pi r^2 \nabla \cdot \mathbf{J}_{IA}$; (——) $4\pi r^2 i[T, \rho_{IA}]$.

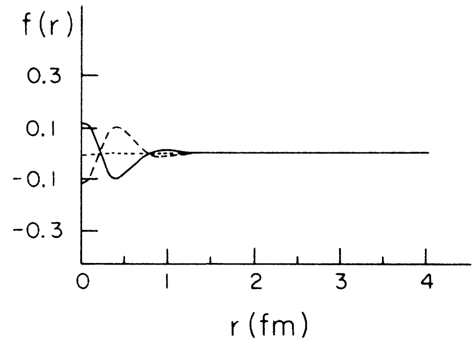

FIG. 6. Current conservation equation for ^{3}H (pionic exchange current and point nucleus only). (-----) $\delta_{MEC}(r)$; (——) $\nabla \cdot (\mathbf{J}^{MEC}_{\pi,\pi NN})_{point}$; (— — —) $-i[V_{OPE}, \rho^{IA}_{point}]$.

where the current $\mathbf{J} = \mathbf{J}^{IA} + \mathbf{J}^{MEC}$ includes both F_1^V and $F_{\pi NN}$ together with the RSC interaction in the right-hand side of Eq. (17). It is recalled that Eqs. (17) and (26) only involve the F_1 component of the charge density, while the F_2 component follows the independent Eq. (16). While the δ^{IA} part is the same as in Fig. 5, the expression for $4\pi r^2 \delta^{MEC}(r)$ is shown in Fig. 7(a). Here again, we note two separate contributions, i.e., the divergence term (solid line) and the commutator term (dashed line). A totally consistent set of ingredients should lead to a zero sum. In fact, such a sum (dotted line) shows local current nonconservation, but, globally, the average charge is conserved, i.e.,

$$\dot{Q} = 4\pi \int dr\, r^2 [\delta^{IA}(r) + \delta^{MEC}(r)] = 4\pi \int dr\, r^2 \delta^{MEC}(r) = 0 , \tag{27}$$

as can be seen by inspection of Fig. 7(b). This result illustrates the overall self-consistency of Eq. (17) when incorporating realistic contributions. Global charge conservation which precludes a gross violation of some basic principles in the HBG work then coexists with a slight deviation from local current conservation. This is not surpris-

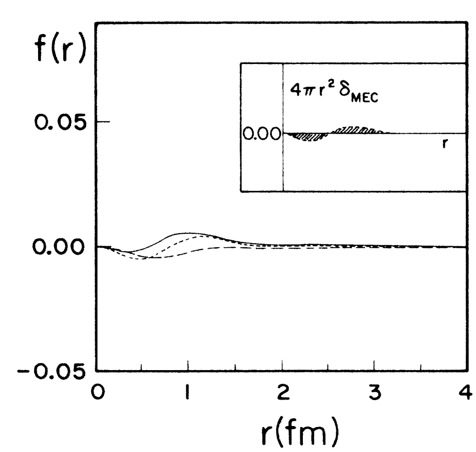

FIG. 7. (a) Current conservation equation for pionic exchange current in a realistic case, i.e., Faddeev wave function with Reid soft core potential, Blatnik nucleonic form factor, and physical hadronic form factor. (b) The hatched area represents the time derivative of the total charge $\dot{Q}$.

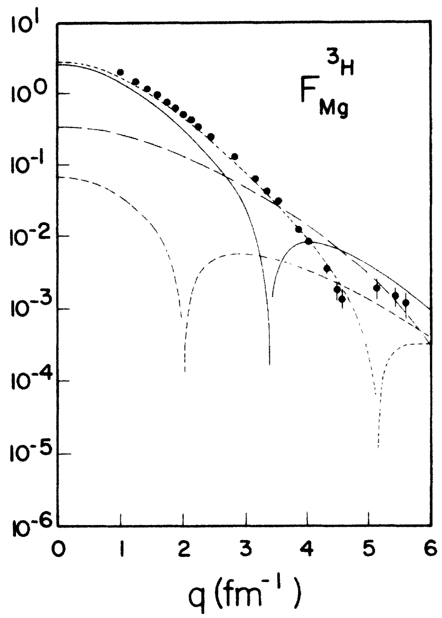

FIG. 8. ^{3}H magnetic form factor in the soft pion approximation. Faddeev wave function based on a Reid soft core potential, Blatnik nucleonic form factors, and point hadronic vertices. (——) IA contribution; (——) soft pion-pair contribution; (– – –) pionic contribution; (-----) total magnetic form factor; (●) experimental points.

ing in view of the shortcomings of the traditional framework.

This numerical version of the current conservation equation includes explicit pionic currents only, while the two-nucleon densities involved in the HBG work contained more mesonic exchange densities. This limitation to pionic currents is quite justified on the basis of the pion-pair dominance in the momentum transfers of interest to us (see Fig. 9 of HBG). Indeed, it is of interest to look at a description of MEC's in terms of soft pions [this description amounts to considering only the pion currents shown in Figs. 1(b)—(d) with $F_{\pi NN}=1$] which yields a theoretical magnetic form factor which closely reproduces the experimental data up to the dip (Fig. 8). However, the breakdown of this soft pion description with increasing momentum transfers occurs sooner than for the deuteron case,[21] probably reflecting the fact that nucleons in ^{3}He and ^{3}H are more closely packed than they are in the deuteron.[22]

III. ELECTRIC CHARGE FORM FACTOR

Since the triton, for which new data are now available,[1] and ^{3}He form an isodoublet and since the pionic pair process yields the dominant exchange contribution, it is worth reanalyzing the pion-pair density $\rho_{\pi NN}$ from the point of view of its isospin structure. The isoscalar and isovector parts have the same sign in ^{3}He, and the opposite sign in ^{3}H. This feature shows up in the expression for the pion pair density, Ref. 12,

$$\rho_{\pi NN}=\frac{g^2_{\pi NN}}{8m^3_N}\frac{K_{\pi NN}}{q^2+m^2_\pi}[(F^S_1+F^S_2)\tau_1\cdot\tau_2 +(F^V_1+F^V_2)\tau_{2z}]\sigma_1\cdot\mathbf{q}\sigma_2\cdot\mathbf{k}_2 \ . \quad (28)$$

The pseudoscalar (PS) case, which was considered by HBG (and by the authors of Ref. 4), is characterized by the presence of $F^{S,V}_2$ in Eq. (3), which gives rise to approximately equal isoscalar and isovector contributions. The switch to pseudovector (PV) coupling amounts to the deletion of the anomalous part of the nucleon magnetic moment.[23,24] This deletion, i.e., the substitutions $F^V_1+F^V_2\to F^V_1$ and $F^S_1+F^S_2\to F^S_1$ in Eq. (24), drastically decrease the isovector term and does not significantly alter the isoscalar term. The charge form factors of ^{3}He and ^{3}H are shown in Figs. 2 and 3; note the location of the dip and the height of the second maximum which are crucial features of these curves. For ^{3}He, the pion exchange current based on PS coupling induces a strong upward shift toward the second maximum and pushes the dip a little too far to the left, while the exchange current based on PV coupling produces a lower second maximum and a better overlap with the experimental dip. For ^{3}H, the pion exchange charge density based on PS coupling has practically no effect (as expected from the near equality of the isoscalar and isovector parts which tend to cancel) while PV coupling yields a very good overall theoretical agreement with experiment. It should be remembered that in the present work, the effect of the three-nucleon forces investigated by HBG (Ref. 2) have been ignored in order to concentrate on the effect of the pionic nucleon pair current upon the charge form factor. Indeed, the effects of the three-nucleon forces upon the charge form factors is to slightly increase the height of the second maximum and shift the dip to the right.[2] Finally, Figs. 9 and 10 show the isoscalar and isovector form factor of the three-nucleon systems:

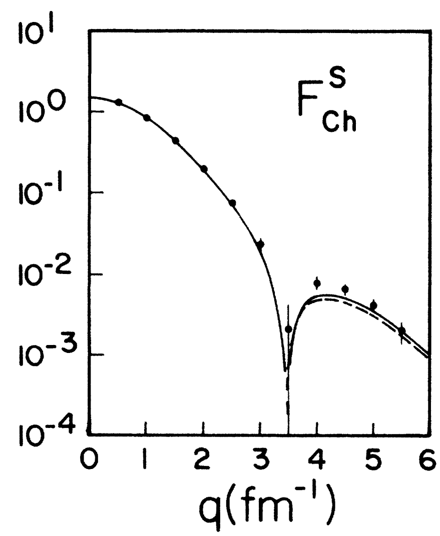

FIG. 9. Isoscalar charge form factor. (——) PV coupling; (– – –) PS coupling; (●) experimental points.

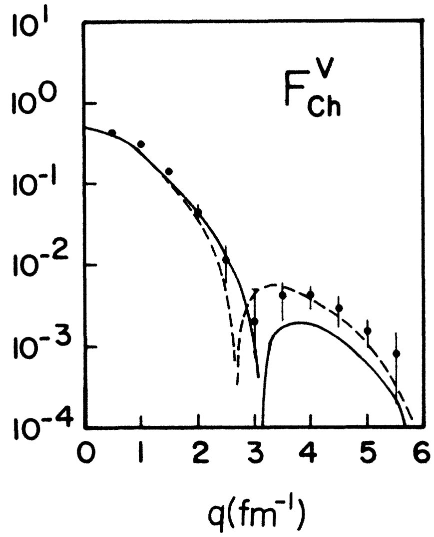

FIG. 10. Isovector charge form factor. Conventions identical to those of Fig. 9.

$$F^S_{\rm ch}=\frac{(F^{^3{\rm He}}_{\rm ch}+F^{^3{\rm H}}_{\rm ch})}{2}\,,\quad F^V_{\rm ch}=\frac{(F^{^3{\rm He}}_{\rm ch}-F^{^3{\rm H}}_{\rm ch})}{2}\,. \tag{29}$$

The isoscalar form factor does not depend on the choice of PS or PV coupling in a significant way. Such a result is not surprising on the basis of the comment following Eq. (28). This isoscalar component satisfactorily reproduces the experimental data for low momenta up to the dip at ≈ 3.5 fm^{-1}. The isovector part presents more serious difficulties because PS coupling yields a dip which is pushed much too far toward low momenta. PV coupling, which yields a satisfactory shape from low momenta, up to the dip at (≈ 2.5 fm^{-1}), is however significantly below the experimental result beyond momenta ≈ 2.5 fm^{-1}.

Arguments which favor the use of PV coupling for the charge exchange density follow two trends; the first emphasizes that the equivalence principle between PV and PS coupling undergoes a breaking which involves the nucleon anomalous magnetic moment,[23] PV coupling being preferable because of its easy incorporation of PCAC effects;[24] the second argument insists that in a chiral invariant formulation, PS coupling can be used, provided an extra term is added which precisely cancels the equivalence breaking term,[25] and brings back the equivalence principle. Also, a comparison of theory with experiment for the forward photodisintegretion of the deuteron at low energy indicated the advantage of PV coupling.[26,27] Along these lines (keeping in mind the uncertainties plaguing the charge density corrections), the new data from Saclay on the triton represent a nice example of the need for a more careful assessment of chiral symmetry in the domain of photopion physics at higher momenta.[1]

Finally, it is of interest to compare this switch from PS to PV coupling within the above mentioned traditional approach with the approach based on a constituent quark model (CQM) which has been put forward by Beyer *et al.*[28] These authors consider that the photon impinges on an individual quark rather than on a nucleon in the traditional approach. The resulting exchange charge density is still given by Eq. (28) where the substitutions $(F^S_1+F^S_2)\to F^S_1$ and $(F^V_1+F^V_2)\to\frac{9}{25}(F^V_1+F^V_2)$ reproduce the qualitative trend of the switch from PS to PV coupling. Thus, the observed disagreement between the CQM model and the traditional method when the latter was used with PS coupling does not persist when PV coupling is adopted.[29]

IV. CONCLUSION

The problem of obtaining the ^{3}He (^{3}H) electromagnetic form factors by calculating various corrections (hadronic form factors, . . .) which are added to the conventional model of the nucleus (considered as a system of nucleons interacting via a static potential) has been previously addressed by HBG. In the present work, our problem has been to evaluate the extent to which current conservation retains its validity while those theoretical corrections are added to the conventional expression of the ^{3}He (^{3}H) electromagnetic form factors. Starting in Sec. II A with a free nucleon current, we have checked that the nonrelativistic expansion of (i) the magnetic current to the order $1/m_{\rm N}$, (ii) the nucleon energy to the order $1/m_{\rm N}$, and (iii) the electric charge to the order $0/m_{\rm N}$, satisfy the current conservation equation to the order $1/m_{\rm N}$. It should also be noticed that the terms involving F_2 are separately divergenceless. This is the reason why the replacement of the Dirac form factor F_1 with the Sachs form factor $G_E=F_1+(\mathbf{q}^2/4m^2_{\rm N})F_2$ does not change the gauge invariance properties of the electromagnetic current. However, as a comparison of Eqs. (12) and (13) illustrates, the expansion to order $1/m_{\rm N}$ of the magnetic current, described by $[G_E,G_M]$, introduces terms of order $1/m^2_{\rm N}$ through the interchange of form factors $F_1\to G_E$. In order to substantiate our claim that F^V_1 is more appropriate than G^V_E, a truly relativistic theory should be employed. At present, the magnetic current (assuming point nucleons) is described to order $1/m_{\rm N}$, the internal structure of nucleons is phenomenologically described to the order $\mathbf{q}^2/m^2_B\simeq\mathbf{q}^2/m^2_{\rm N}$ and the Faddeev wave function is nonrelativistic. In Sec. II A which deals with pionic exchange currents, the consequences of the departure from the limit of pointlike particles upon the validity from current conservation have been reminded. This outline shows that in the traditional framework (i) a strict gauge invariance is not possible, and (ii) an analytical calculation of the deviation from validity is also not possible. With such a limitation in mind, the extent to which the current conservation equation is satisfied by the pionic exchange densities over a specific Faddeev wave function has been explored in Sec. II B. Calculations were carried out using a specific set of parameters which had been previously employed by HBG and the results presented graphically. Figure 7, which shows that the electric charge of the three-nucleon system is globally conserved, also exhibits some variance from local conservation. This comes as no surprise in view of the overall nonrelativistic expansion, the introduction of πNN vertices and of realistic nucleon-nucleon forces characterizing the HBG approach (as well as those of Refs. 3 and 5). It is realized that the present numerical analysis does not provide a stringent test; however, it is felt that this approach is the best possible one within the present nonrelativistic description of the nucleus. Further investigations on form factors of ^{3}H and ^{3}He should be based upon a nucleon-nucleon interaction more directly connected with mesonic exchange currents.[30,31] A better assessment of the role of genuine three-nucleon forces[32] would also lead to significant progress. However, going further will necessitate a consistent theory of composite systems which includes relativistic corrections, mesonic exchanges, and hadronic sizes.

Finally, electric charge form factors were the subject of Sec. III. The shift from PS to PV coupling improves the agreement between theory and experiment (i) for ^{3}H, and (ii) for ^{3}He in the sense that the shift from the IA result leads to a location of the dip in good agreement with experimental data, even if the theoretical height of the second maximum is significantly lowered. The description in isospin terms shows that (i) the isoscalar form factor $F_{\rm ch}^S$ is not very sensitive to the nature of the pion-nucleon coupling (Fig. 9 shows that disagreement with experimental data resides in the area of the second maximum), and (ii) the situation is less satisfactory for the isovector component $F_{\rm ch}^V$. Substitution of PS coupling for PV coupling leads to a location of the dip which is in better agreement with experimental data as illustrated by Fig. 10. However, the agreement is significantly poorer for the part of the curve on the right-hand side of the dip. These difficulties are probably one more indication that a fully relativistic treatment of the problem is necessary.

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