Radiative pion capture on ^{13}C

M. K. Singham

Theoretical and Medium Energy Physics Divisions, Los Alamos National Laboratory, Los Alamos, New Mexico 87545

F. Tabakin

Department of Physics and Astronomy, University of Pittsburgh, Pittsburgh, Pennsylvania 15260

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It is found that the differential cross sections for ${}^{13}C(\pi^+,\gamma){}^{13}N_{g.s.}$ for 115.5 MeV kinetic energy pions can be understood within the framework of a distorted wave impulse approximation, suggesting that core polarization and precursor effects give small contributions. It is further found that, within the limits of current nuclear photopion theory, the data for the radiative capture process are incompatible with earlier measurements for the inverse reaction ${}^{13}C(\gamma,\pi^-){}^{13}N_{g.s.}$. It is shown that the reconciliation of the two sets of data places stringent demands on possible modifications to the theory.

I. INTRODUCTION

Recent measurements by Martoff et al.¹ on the reaction ${}^{13}C(\pi^+,\gamma){}^{13}N_{g.s.}$ for in-flight capture of pions of 115.5 MeV kinetic energy have shed interesting light on what is one of the most perplexing problems in photopion nuclear physics. This is the inability of existing theoretical calculations to reproduce satisfactorily the unusually small measured angular cross sections for the reaction $^{13}C(\gamma,\pi^{-})^{13}N_{g.s.}$ for $T_{\pi}=49$ MeV.^{2,3} The original motivation for the (π^{+},γ) experiment had been based on predictions that pion condensation or precritical phenomena would greatly enhance, and drastically change, the angular distribution for this reaction.^{4,5} The measured (π^+, γ) cross sections were, however, a factor of 50 smaller than those predictions, suggesting that the precursor effects were either absent or subject to delicate cancellations. The calculations by Delorme⁴ and Giraud and Delorme⁵ involved standard distorted wave Born approximation (DWBA) techniques as well as more complicated ones involving medium polarizations, zero-range repulsions, and Δ -isobar effects. All these calculations are compared with the data in Fig. 7 of Ref. 1. Martoff et al. found that no calculations to that date, with or without the precursor effects, were able to predict cross sections small enough to fit the data. They raised the possibility that the fault lay in the inadequacy of the transition operator or in the nuclear physics input.¹

In this study we find that the approximate size and shape of the measured (π^+,γ) cross sections can be well reproduced (to within two standard deviations) by a standard, local distorted wave impulse approximation (DWIA) calculation with no adjusted parameters. We further argue that the results of the (π^+,γ) experiment are incompatible with the results of previous experiments for ${}^{13}C(\gamma,\pi^-){}^{13}N_{g.s.}$ for $T_{\pi}=49$ MeV, within the framework of existing theory. We show that, on the theoretical side, the resolution of this inconsistency places stringent constraints on possible modifications to the photopion transition operator.

II. FORMULAE FOR CROSS SECTIONS

The formula for the differential cross section for the (π, γ) reaction in the center-of-mass frame is given by

$$\sigma(\theta) = \frac{\pi}{2} \frac{k}{q} \frac{E_i E_f}{W^2} F_{\text{c.m.}} \frac{1}{2J_i + 1} \sum_{\lambda M_i M_f} |T_{\pi\gamma}|^2 , \qquad (1)$$

where

$$T_{\pi\gamma} \equiv \langle J_f M_f : T_f \tau_f \mid H_{\pi\gamma} \mid J_i M_i : T_i \tau_i \rangle \tag{2}$$

and

$$H_{\pi^{\pm}\gamma} = \sum_{j=1}^{A} \mathbf{J}_{\pi\gamma}^{5} \cdot \hat{\boldsymbol{\epsilon}}(\mathbf{k}\lambda)^{*} \phi_{\mathbf{q}}^{(+)}(\mathbf{r}_{j}) e^{-i\mathbf{k}\cdot\mathbf{r}_{j}} \delta(\mathbf{r}-\mathbf{r}_{j}) \tau_{\pm}^{j} .$$
(3)

 $\phi_{\mathbf{q}}^{(+)}(\mathbf{r})$ is the wave function of a pion with asymptotic three momentum \mathbf{q} and appropriate incoming boundary conditions denoted by (+) (for plane wave pions, $\phi_{\mathbf{q}}^{(+)}(\mathbf{r}) \rightarrow [1/(2\pi)^{3/2}]e^{i\mathbf{q}\cdot\mathbf{r}})$, $\hat{\boldsymbol{\epsilon}}(\mathbf{k}\lambda)$ is the polarization vector for a plane wave of momentum \mathbf{k} and helicity λ , $\mathbf{J}_{\pi\gamma}^5$ is the pseudovector (π^{\pm},γ) transition operator, $F_{c.m.}$ $(=e^{Q^2b^2/2A})$ is the usual correction factor which adjusts for the lack of translational invariance of the shell model wave functions that we will be using to describe the nuclear states, \mathbf{Q} is the momentum transfer to the nucleus, and E_i , E_f , and W are the center-of-mass energies of the initial nucleus, final nucleus, and total system, respectively.

In the calculations we assume that the initial and final nuclear states are eigenstates of angular momentum (J) and isospin (T) with quantum numbers $(J_iM_i:T_i\tau_i)$ and $(J_fM_f;T_f\tau_f)$, respectively. The experiments deal with unpolarized target and photons and so we sum over final, and average over initial, spin states. Since we will be using the impulse approximation with a transition operator $J_{\pi\gamma}^5$ obtained from the radiative capture of charged pions on free nucleons, $H_{\pi\gamma}$ is a one-body operator and τ_{\pm} is the appropriate isospin raising or lowering operator.

Since we will be comparing the (π^{\pm}, γ) results to those obtained for the inverse (γ, π^{\pm}) process, it is helpful to write the corresponding expressions for that process. These are given by

$$\sigma(\theta) = \frac{\pi}{2} \frac{q}{k} \frac{E_i E_f}{W^2} F_{\text{c.m.}} \frac{1}{2(2J_i + 1)} \sum_{\lambda M_i M_f} |T_{\gamma \pi}|^2 , \qquad (4)$$

where

$$T_{\gamma\pi} \equiv \langle J_f M_f : T_f \tau_f \mid H_{\gamma\pi} \mid J_i M_i : T_i \tau_i \rangle$$
(5)

and

$$H_{\gamma\pi^{\pm}} = \sum_{j=1}^{A} \phi_{\mathbf{q}}^{*(-)}(\mathbf{r}_{j}) \mathbf{J}_{\gamma\pi}^{5}(\mathbf{r}_{j}) \cdot \widehat{\boldsymbol{\epsilon}}(\mathbf{k}\lambda) e^{i\mathbf{k}\cdot\mathbf{r}_{j}} \delta(\mathbf{r}-\mathbf{r}_{j}) \tau_{\mp}^{j} .$$
(6)

The differences in these expressions over the (π, γ) ones are (i) an additional factor $\frac{1}{2}$ coming from the photon spin being now averaged over in the initial state; (ii) a new operator $\mathbf{J}_{\gamma\pi}^5$; (iii) different pion waves $\phi_{\mathbf{q}}^{*(-)}(\mathbf{r})$; and (iv) the inverse of the kinematic factor q/k.

III. TRANSITION OPERATOR J⁵

For the transition operator $J_{\gamma\pi}^5$, we will use the form derived by Blomqvist and Laget⁶ (BL), which is obtained by a semirelativistic reduction of the lowest order Feynman diagrams contributing to the elementary process. The parameters in their amplitude are fitted using pion-nucleon scattering and the elementary (γ,π) data. These parameters are then used *unchanged* in our calculations, in the spirit of the impulse approximation. For applications in nuclei, we use a local coordinate space version of this operator.⁷ This operator is not without certain deficiencies,^{8,9} but it is useful and has, for the most part, enjoyed considerable success in applications to low energy nuclear charged pion photoproduction reactions.⁷

For the inverse (π, γ) operator, Reynaud and Tabakin¹⁰ have used time-reversal and detailed balance arguments to obtain an operator $J_{\pi\gamma}^5$ starting from the Blomqvist-Laget form $J_{\gamma\pi}^5$. It is important to bear in mind that, in principle, these arguments apply to any operator. That is, once one has fixed the form of the operator in one direction, the operator for the reverse process on the nucleon is determined by time reversal and detailed balance. The importance of this point is discussed in Sec. VII when the ${}^{13}C(\gamma,\pi^{-}){}^{13}N_{g.s.}$ data for experimental and ${}^{13}C(\pi^+,\gamma){}^{13}N_{g.s.}$ are compared. In this paper we will be describing the two nuclear reactions as one being the inverse of the other. However, we are not implying that the two nuclear reactions are connected by time reversal, since this is clearly not the case.

IV. PION WAVE FUNCTION $\phi_q(\mathbf{r})$

The distorted pion wave functions are obtained by solving a Klein-Gordon equation with an optical potential using a modified form of the computer code PIRK.¹¹ For 50 MeV pions, we use the MSU (1982) optical potential¹² which gives very good agreement with a range of pion nucleus elastic scattering data, particularly with the 50 MeV π^{+} -¹³C elastic scattering data of Dytman *et al.*¹³ This agreement, and its equally satisfactory applications in ${}^{13}C(\gamma,\pi^{+}){}^{13}B_{g.s.},{}^{14,15}$ suggest that the pion wave functions used at 50 MeV are rather tightly constrained.

For 115.5 MeV pions we use the older MSU (1979) optical potential,¹⁶ which gives good agreement with pionnucleus elastic scattering data for a range of heavier nuclei at this energy. However, no data on ¹³C exist, and hence there is some room for uncertainty in our knowledge of the distorted pion wave functions at this energy. It seems unlikely that the uncertainties in the pion wave functions will produce errors of more than 50% in the calculated (π, γ) cross sections at $T_{\pi} = 115.5$ MeV. This point will be discussed further in Sec. VII.

V. NUCLEAR WAVE FUNCTIONS

In any nuclear transition involving one-body operators, the nuclear structure input in a shell model is given by the quantity $\phi_{JT}^{fi}(ab)$, where

$$\phi_{JT}^{fi}(ab) \equiv \left[\frac{2(2j_a+1)}{(2J+1)(2T+1)}\right]^{1/2} \times \langle X_f J_f T_f || |(a_a^{\dagger} \times \tilde{a}_b)_{JT} || |X_i J_i T_i \rangle .$$
(7)

and by the shape of the single particle radial wave functions. Here, a_a^{\dagger} and a_b are the creation and annihilation operators for single nucleons with the quantum numbers $a = \{n_a, l_a, j_a\}$ and $b = \{n_b, l_b, j_b\}$, respectively, and

$$\tilde{a}_{bm_b\tau_b} \equiv (-1)^{j_b - m_b + 1/2 - \tau_b} a_{b - m_b - \tau_b} .$$
(8)

Here, m_b is the third component of j_b , and τ_b is the third component of isospin; the matrix elements are reduced in spin and isospin¹⁷ and X_i and X_f represent internal quantum numbers of the nuclear states. The numbers $\phi_{JT}^{fi}(ab)$ are usually obtained from shell-model calculations within a given model space using an effective interaction. For 1p-shell nuclei like ¹³C the standard effective interaction is that of Cohen and Kurath (CK).¹⁸ We use their (8-16) POT results (as tabulated by Lee and Kurath¹⁹) as one set of ¹³C nuclear wave functions in our calculations. [Recall that we need only the ground state wave functions of ¹³C, since ${}^{13}N_{g.s.}$ is an isospin analog state of ${}^{13}C_{g.s.}$. We neglect the small differences in $\phi_{IT}(ab)$ due to Coulomb effects.] The CK matrix elements agree with the measured static properties of the mass-13 system to within \sim 15%, but, when supplemented with harmonic oscillator radial wave functions, give predictions that are $\sim 50\%$ too large for the elastic²⁰⁻²² and inelastic^{23,24} electron scattering form factors in the momentum transfer (Q)range 1-2 fm⁻¹. They also predict cross sections which are too large for the reactions ${}^{13}C(\gamma, \pi^-){}^{13}N_{g.s.}$ and ${}^{13}C(\gamma, \pi^+){}^{13}B_{g.s.}$ in the same momentum transfer range.¹⁴ The ${}^{13}C(\pi^+, \gamma){}^{13}N_{g.s.}$ data at $T_{\pi} = 115.5$ MeV also span this same momentum transfer range, indicating that the CK matrix elements and oscillator wave functions may not be the most appropriate ones for the analysis of this data.

An alternative phenomenological set of nuclear shell

model matrix elements for the mass-13 system has been obtained by fitting to accurately measured weak and electromagnetic observables. In that study²⁵ the states ${}^{13}C_{g.s.}$ and ${}^{13}N_{g.s.}$ were treated as an isodoublet while ${}^{13}B_{g.s.}$, ${}^{13}C(15.11 \text{ MeV})$, ${}^{13}N(15.06 \text{ MeV})$, and ${}^{13}O_{g.s.}$ were treated as an isoquartet. The wave functions of the isodoublet and isoquartet states were obtained by requiring them to reproduce exactly the accurately measured observables involving these two states. The observables selected were the magnetic dipole moments, β -decay rates, and the minimum of the elastic electron scattering M1 form factor. The resulting matrix elements combined with oscillator radial wave functions predict good agreement with the measured elastic and inelastic electron scattering form factors and with ${}^{13}C(\gamma, \pi^+){}^{13}B_{g.s.}$ for $Q \leq 2.2 \text{ fm}^{-1}$ (Refs. 14 and 25). The fact that these wave functions (which we call set 1) give good agreement with a wide spectrum of observables suggests that they should be suitable for use in the ${}^{13}C(\pi^+,\gamma){}^{13}N_{g.s.}$ reaction as well. In Table I we give values of $\phi_{IT}^{fi}(ab)$ for CK and set 1 wave functions for the transition ${}^{13}C \leftrightarrow {}^{13}N_{g.s.}$. For the single particle radial wave functions we use a harmonic oscillator basis with length parameter b = 1.59 fm for CK and b = 1.73 fm for set 1 wave functions. These values were fixed to reproduce the minimum of the elastic electron scattering M1 form factor.²³ All calculations were done within the 1p-shell model space.

VI. RESULTS AND DISCUSSION

The results of the present calculation for ${}^{13}C(\pi^+,\gamma){}^{13}N_{g.s.}$ for $T_{\pi} = 115.5$ MeV are shown in Fig. 1. The calculations used the MSU (1979) pion distortions (Sec. IV) and the time reversed form of the BL pion-photoproduction operator (Sec. III). We tried the calculation with both sets of nuclear wave functions, CK (dashed line) and set 1 (solid line).²⁵ We see that both calculations at 37° exceed the data point by a factor of about 5. At the larger angles both calculations agree with the data at the two standard deviation level (2σ) , with the CK wave functions giving slightly better results.

TABLE I. Reduced density matrices $\phi_{JT}^{fi}(ab)$ for the transition ${}^{13}C_{g.s.} \rightarrow {}^{13}N_{g.s.}$ [Eq. (7)].

$\xrightarrow{\text{tion}} C_{g.s.} \xrightarrow{\longrightarrow} P_{g.s.} [Eq. (7)].$			
	CK	Set 1	
$\phi_{01}^{fi}(rac{1}{2}rac{1}{2})$	0.652	0.292	
$\phi_{01}^{fi}(rac{1}{2}rac{3}{2})$	0	0	
$\phi_{01}^{fi}(rac{3}{2}rac{1}{2})$	0	0	
$\phi_{01}^{fi}(rac{3}{2} rac{3}{2})$	0.348	0.708	
$\phi_{11}^{fi}(rac{1}{2}rac{1}{2})$	0.796	0.625	
$\phi_{11}^{fi}(rac{1}{2}rac{3}{2})$	0.023	0.027	
$\phi_{11}^{fi}(rac{3}{2}rac{1}{2})$	-0.033	-0.037	
$\phi_{11}^{fi}(rac{3}{2} rac{3}{2})$	-0.142	-0.158	
<i>b</i> (fm)	1.59	1.73	

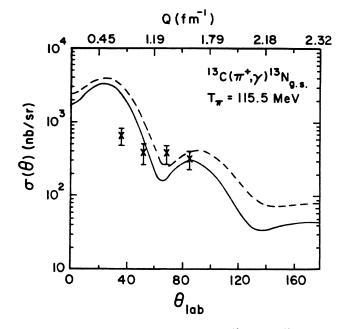


FIG. 1. Differential cross sections for ${}^{13}C(\pi^+,\gamma){}^{13}N_{g.s.}$ for 115.5 MeV pions. The solid line is calculated with set 1 nuclear wave functions (Ref. 25), while the dashed line is calculated with CK (8-16) POT wave functions (Ref. 19). Both calculations used the time reversed form of the BL transition operator (Ref. 10) and MSU (1979) pion distortions (Ref. 16). The data are from Martoff *et al.* (Ref. 1).

To test the sensitivity of the reaction to changes in the photoproduction operator, the same calculations were carried out, but with s-channel Δ -isobar term in the BL operator omitted. We call this modified form of the operator BLN. (It should be noted that the omission of the isobar term means that the only parameter in the BLN operator is the pion-nucleon coupling constant g, which is fixed by elastic scattering to have the value $g^2/4\pi = 14.5.$) The presence of this isobar term in the BL operator has been criticized previously on general theoretical grounds as leading to possible double counting,^{7,26} and its omission has led to better agreement with the data for the reactions ${}^{13}C(\gamma,\pi^-){}^{13}N_{g.s.}$ (Refs. 26 and 27) and ${}^{13}C(\gamma,\pi^+){}^{13}B_{g.s.}$ (Ref. 14). The results are shown in Fig. 2 for set 1 a wave functions (solid curve) and CK wave functions (dashed curve). We see that the agreement is now better with all the data being reproduced, at least up to the 2σ level. The omission of the Δ brings the forward peak down.

The effect of pion distortions is shown by the dotted curve in Fig. 2, which represents the result using set 1 nuclear wave functions, the BLN operator, and plane wave pions. We see that the effect of the pion distortions is to reduce the cross section around the second maximum by a factor of about 3.

In summary, Figs. 1 and 2 show that the data on this reaction can be roughly reproduced by standard DWIA calculations with no adjustable parameters. This suggests that pion condensation and core polarization effects are

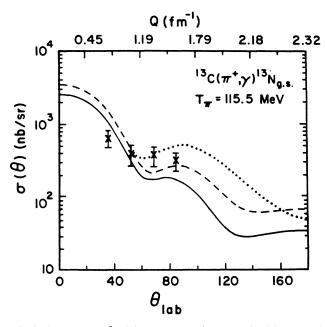


FIG. 2. Same as for Fig. 1, except that now the BLN transition operator is used throughout. The dotted line represents calculations using the set 1 nuclear wave functions (Ref. 25) and plane wave pions.

rather small. The level of precision of the data is not sufficient to really discriminate between the two sets of nuclear wave functions that were used.

It is now interesting to compare the results of this reaction with that of ${}^{13}C(\gamma,\pi^-){}^{13}N_{g.s.}$ at $T_{\pi}=49$ MeV. The data and calculations for this reaction are shown in Fig. 3. The data are from Stoler *et al.*² Our best calculation (solid line) is obtained using the BLN operator, set 1 nuclear wave functions, and the MSU (1982) optical potential. For comparison purposes, we also show other calculations where we use (i) the full BL operator, set 1, and MSU (1982) (dotted line), (ii) CK nuclear matrix elements, BLN, and MSU (1982) (dashed-dotted line), and (iii) plane wave pions, set 1, and the BLN operator (dashed line).

We see that using the BLN operator and set 1 wave functions with MSU (1982) pion distortions gives the best results. (Actually, the plane wave calculation gives the best results, but this is not realistic.) But even this result is still a factor of 6 larger than the data. [In an earlier work we obtained results that were a factor of "only" 3 larger than the data. This was because we had used the older MSU (1979) pion optical potential in the earlier calculations. However, as we have discussed in Sec. IV, the MSU (1982) potential is to be preferred.]

If we look at the data for the (π^+, γ) and the (γ, π^-) reactions as a function of momentum transfer Q to the nucleus, we see that both sets of data span roughly the same range, Q=0.8-1.6 fm⁻¹. Since the initial and final nuclear states are identical for each reaction, we see that the same region of the nuclear form factor is being probed for each reaction, even though the other kinematic quantities are different. Hence the nuclear structure input is essentially the same for both reactions.

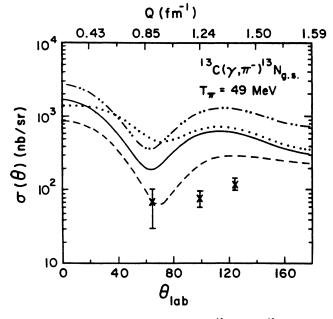


FIG. 3. Differential cross sections for ${}^{13}C(\gamma, \pi^{-}){}^{13}N_{g.s.}$ for 49 MeV pions with (i) the BLN operator, set 1 nuclear wave functions (Ref. 25), and MSU (1982) pion distortions (Ref. 12) (solid line); (ii) same as for (i), but with the full BL operator (dotted line); (iii) same as for (i), but with CK (8-16) POT nuclear wave functions (Ref. 19) (dashed-dotted line); (iv) same as for (i), but with plane wave pions (dashed line). The data are from Stoler *et al.* (Ref. 2).

The transition operators J^5 for each reaction are also essentially the same, with one the time-reversed form of the other. Of course, the operators have an energy evolution which makes the $T_{\pi} = 115.5$ MeV operator different from that at $T_{\pi} = 50$ MeV. However, this energy evolution is built into the theory and is not adjustable within the framework of the calculation. In addition, most of the changes in the operator due to energy variations comes from the presence of the isobar term.⁷ The BLN operator is relatively insensitive to these energy changes. Hence, fixing the transition operator for one process essentially fixes it for the inverse, even allowing for the 65 MeV difference in the pion energy. (We will discuss this point further in the next section.)

Given the similarities in the input to the two reactions, it is indeed surprising that the DWIA calculations agree with the data for the (π^+, γ) reaction but are a factor of 6 larger than the (γ, π^-) data. Clearly, there is a problem here and we will examine possible reasons for this discrepancy in the next section.

VII. POSSIBLE REASONS FOR DISCREPANCY

The sources of the discrepancy could be in one or more of the following:

(a) *Nuclear structure*. While there does exist a certain element of uncertainty in our knowledge of the nuclear structure of the states involved (in particular, the size of

core polarization contributions), this is unlikely to be the source. This is because both the (π^+, γ) and (γ, π^-) data involve the same initial and final nuclear states in the same range of momentum transfer. Hence, any changes that could be made in the nuclear structure to scale down the (γ, π^-) calculations by a factor of 6 would very probably bring down the (π^+, γ) cross sections too, since a very similar transition operator is used there. This would destroy the agreement that currently exists for the latter reaction.

(b) Transition operator. In principle, this is the most likely candidate. The results of Toker and Tabakin²⁶ and Tiator and Wright²⁷ predict that the nonlocal effects in the 50 MeV (γ, π^{-}) reaction could reduce those cross sections by a factor of 2, which would still leave a factor of 3 discrepancy. In order to resolve this, one would need additional modifications to the transition operator that produce a further factor of 3 reduction in the (γ, π^{-}) cross section at 50 MeV but leave the (π^+, γ) cross sections at 115.5 MeV relatively unchanged for the same nuclear structure. These modifications also should not destroy the relative success that similar calculations have had on other (γ, π) reactions.⁷ To date, there has been no evidence for such a strong energy dependence of the medium modifications. Another possible consideration is the effect of the nuclear medium on the propagation of virtual pions which have the ability to excite the nucleon- and isobarhole states. This medium effect has been studied by Dytman and Tabakin.²⁸

(c) Pion distortions. For both reactions, the pion waves that have been used were generated by optical potentials that gave good results for pion-nucleus elastic scattering. It should be borne in mind that this in itself is not sufficient to judge the quality of the pion wave functions used, since the elastic scattering cross sections depend only on the asymptotic form of the wave functions, while the (π,γ) and (γ,π) reactions depend on the nature of the pion wave function within the nucleus. Hence it is possible that the optical potentials could correctly predict elastic scattering while giving wrong results for (γ,π) and (π,γ) . However, the studies by Keister on phase-shiftequivalent potentials^{29,30} have shown that, even in very favorable cases, the allowed variation in the photopion cross section is only about 50%, and hence this is not likely to be sufficient to explain the present anomaly.

(d) Normalization of (γ, π) data. The overall normalization of the experimental data for these reactions is a very difficult problem. This is indicated by the fact that the data of Stoler *et al.*² for ${}^{13}C(\gamma, \pi^{-}){}^{13}N_{g.s.}$ is in itself a factor of 2 larger than the earlier data of LeRose *et al.*³ A factor of about 3 error in the (γ, π^{-}) data would bring about consistency with both sets of data and the calculations. This is an extremely unlikely event, but such a discovery would, in fact, remove one of the most intransigent problems in the field of photopion physics.

It would take an extremely delicate combination of theoretical modifications in (b) and (c) in order to produce changes in the calculations of the type needed to reconcile the two sets of data. The two sets of data thus provide very stringent constraints on the type of modifications that can be made in the transition operator.

VIII. SUMMARY

We have analyzed the experimental data for the reactions ${}^{13}C(\pi^+,\gamma){}^{13}N_{g.s.}$ at $T_{\pi}=115.5$ MeV and ${}^{13}C(\gamma,\pi^-){}^{13}N_{g.s.}$ at $T_{\pi}=49$ MeV. We find that the (π^+,γ) data can be satisfactorily reproduced to within two standard deviations by a local DWIA calculation with no core polarization effects, while a similar calculation for the (γ,π^-) reaction is a factor of 6 too large. We argue that the present level of theory of nuclear photopion reactions cannot simultaneously account for the two sets of data. The resolution of this discrepancy places very stringent constraints on the type of modifications that can be made to the transition operator. Hence the satisfactory, simultaneous reproduction of these two sets of data could prove to be the yardstick by which the quality of nuclear photopion reaction theories are measured.

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