Energy variation of the matrix element in the statistical breakup of light nuclei below the pion production threshold

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(Received 7 February 1986)

The energy dependence of the part of inclusive spectra corresponding to statistical breakup has been explained modifying the Fermi model for pion production. The modification consists of introducing an energy variable volume of interaction that is dependent on the de Broglie wavelength of the incident channel. The comparison with experiments is satisfactory.

The breakup of the light nuclei systems may be divided into three separate categories:

(1) Breakup resulting from quasifree scattering of incident particles off clusters in light nuclei. The final state shall in the general case consist of two colliding particles and the spectator.

(2) Sequential breakup where one of the particles leaves the residual nucleus in a resonant state that shall subsequently decay in usually two, or mostly three fragments.

(3) The statistical (simultaneous) breakup that can be pictured from phenomenological observations as occurring in the following manner: (a) the energy of the incident particle is absorbed in the composite system (incident particle plus target); (b) the composite system decays by simultaneous emission of particles whose energies are determined by the available phase space.

The data that have been analyzed so far, using the phase space (PS) distributions to explain the inclusive particle spectra obtained in the breakup of light nuclei, point towards the conclusion[1,2] that the observed mechanism is complementary to the peripheral processes (like direct reactions, quasifree scatterings, and reactions). A simple model[3] and exact calculations[4,5] have shown that low momenta of the incident particles do not contribute appreciably to quasifree processes. However, for the inclusive spectra, integrated as statistical breakup contributions, there is no quantitative proof about the contribution of low incident momenta. Analyzing the possibilities to check on the contribution of low momenta we concluded that the dependence of the PS process on the incident energy may give information on the momenta involved.

First Fermi has suggested that the pion production might be explained using a simple model[6] where the energy of the collision of two nuclei may be assumed to be concentrated in a small volume Ω, called the interaction volume. Considering a strong enough interaction, it is to be expected that the concentrated energy shall be rapidly distributed among the available degrees of freedom, according to statistical laws. The statistical assumption is based on the postulate that the square of the transition matrix element M_{if} is solely dependent on the probability that for a given state all particles should be at the same time within the volume Ω. For n independent particles with momenta $p_1,\ldots,p_n$ this probability is $(\Omega/V)^n$ where V is the large normalizing volume. The first-order perturbation theory gives for the total cross section

$$\sigma=\frac{2\pi}{\hbar}\frac{1}{|\text{flux}|}\int\cdots\int|M_{if}|^2dR_n\ , \tag{1}$$

where $|\text{flux}|=v_a/V$ (v_a is the velocity of incident particles), and dR_n is the phase space element for n particles in the final state given by

$$dR_n(m_1,\ldots,m_n,E,\mathbf{P})=\left[\frac{V}{(2\pi\hbar)^3}\right]^{n-1}\prod_{i=1}^{n}d^3p_i\delta^3\left[\sum_{i=1}^{n}\mathbf{p}_i-\mathbf{P}\right]\delta\left[\sum_{i=1}^{n}E_i-E\right]\ , \tag{2}$$

where $\mathbf{P}$ and E are, respectively, the total momentum and total kinetic energy to be shared between n particles, E_i, $\mathbf{p}_i$, and m_i are, respectively, the kinetic energy, momentum, and mass of particle i. If one out of the n particles is detected in a reaction $A(a,m_n)m_1,m_2,\ldots,m_{n-1}$, and assuming, following the model, that the average value of the matrix squared ($|\overline{M_{if}}|^2$) may be brought outside the integral, the double differential cross section takes the form

$$\frac{d^2\sigma}{d\Omega_n dE_n}=\frac{2\pi}{\hbar v_a}\frac{p_n m_n}{(2\pi\hbar)^{3(n-1)}}V^n|\overline{M_{if}}|^2R^*_{n-1}\times(m_1,\ldots,m_{n-1},E-E_n,\mathbf{P}-\mathbf{p}_n)\ ,$$

$$R^*_{n-1}=\frac{(2\pi)^{(3/2)(n-2)}}{\Gamma\left[\dfrac{3(n-2)}{2}\right]}\frac{\left[\prod_{i=1}^{n-1} m_1\right]^{3/2}}{\left[\sum_{i=1}^{n-1} m_i\right]^{3/2}}$$

$$\times\left[E-E_n-\frac{(\mathbf{P}-\mathbf{p}_n)^2}{2\sum_{i=1}^{n-1} m_i}\right]^{3n-8/2}, \qquad (3)$$

where

$$|\overline{M_{if}}|^2=C\left[\frac{\Omega}{V}\right]^n \qquad (4)$$

and C is constant.

In the original work Fermi indicates an exponent $(n-1)$ in relation (4). However, to get dimensional agreement in relation (3) it is obvious that the exponent should be n. Since Fermi, and later authors[7–9] did not calculate the cross section but rather the relative probability of individual final states, the difference in the exponent did not influence their results.

To introduce a dynamical behavior of $|\overline{M_{if}}|^2$, keeping the Fermi approach, it is necessary to assume an energy variable volume of interaction, a feature that was neither introduced by Fermi nor by later authors. To be able to predict the energy behavior of $|\overline{M_{if}}|^2$ we have followed the following phenomenological explanation of relation (4). The amplitude of the interaction of a particle with a given system is proportional to the product of the probabilities: P_1 represents the incident particle within the interaction volume Ω, P_2 represents the center of mass of the target system within Ω, and P_3 represents the particles of the target system within Ω.

The matrix element squared (4) takes thus the form

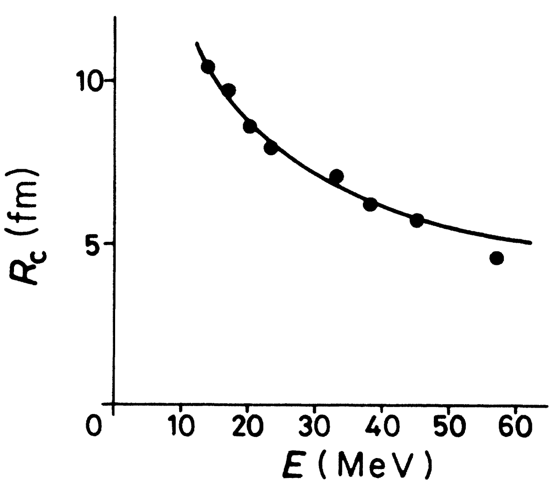

FIG. 1. Dependence of the cutoff radius R_c on the incident energy for the D(p,pp)n reaction under the QFS condition. The points are experimental data and the solid line is the behavior of the de Broglie wavelength of the incident channel with energy.

$$|\overline{M_{if}}|^2\sim\prod_{j=1}^{3}P_j=\left[\frac{\Omega}{V}\right]^2\frac{1}{V^{(l-1)}}\prod_{i=2}^{l}\int_\Omega|\psi(\mathbf{r}_{i(i-1)})|^2 \times d\mathbf{r}_{i(i-1)}, \qquad (5)$$

where $P_1=P_2=\Omega/V$ and the remainder represents P_3, l is the number of fragments interacting with the incident particle, $\psi(\mathbf{r}_{i(i-1)})$ is the wave function of the relative motion of the ith particle and the center of mass of the remaining $(i-1)$ particles.

In order to predict the energy behavior of the matrix element (5), we have learned from the results of the analysis of quasifree scattering (QFS) in the frame of the modified simple impulse approximation (MSIA). The only consistent energy dependence for quasifree scattering was done[10] for the case of p-p QFS on deuterons. The analysis in the frame of the MSIA has resulted in the dependence of the cutoff radius R_c on the incident proton energy, shown in Fig. 1. Analyzing this dependence we have observed that the trend of decrease in the value R_c is proportional to the de Broglie wavelength $\lambda\!\!\!^-$ of the incident channel, with a coefficient of proportionality $k=5.73$. This result is not totally unexpected since it means that QFS shall occur only in those cases when $\lambda\!\!\!^-$ is much smaller than the distance between the participants, as predicted by the simple impulse approximation (SIA). The trend of $\lambda\!\!\!^-$ with energy is shown superimposed on experimental data in Fig. 1. Starting with the above result for k we precisely assume that the volume of the system forbidden to SIA processes (for $r<R_c$) is the one that contributes to the statistical breakup. Consequently, we define that the radius of Ω is proportional to $\lambda\!\!\!^-$ ($R=k\lambda\!\!\!^-$), where k should be the same as extracted from QFS (i.e., $R=R_c$). We can then rewrite Eq. (5),

$$|\overline{M_{if}}|^2\sim\lambda\!\!\!^{-6}\prod_{i=2}^{l}\int_0^{k\lambda\!\!\!^-}|\psi(\mathbf{r}_{i(i-1)})|^2 r^2_{i(i-1)}dr_{i(i-1)}\,. \qquad (6)$$

To compare prediction (6) we have reanalyzed the data of ^{9}Be(p,α) and $\alpha(\alpha$,p) reactions[1,2] in order to extract the amplitudes $|\overline{M_{if}}|^2_{\rm exp}$ of individual phase space contributions in the final channel. In these reactions many final channels are open. We have treated in detail only those channels for which intercluster wave functions were readily available, namely the α^5Hep and ptα channels in the ^{9}Be + p and $\alpha+\alpha$ reactions, respectively. For the intercluster wave function of the relative motion of p and t in α, and α and ^{5}He in ^{9}Be we used[11,12]

$$\psi_{\rm p,t}(r)=\left[\frac{\alpha}{2\pi 8!\beta^8}\prod_{n=1}^{8}(2\alpha+n\beta)\right]^{1/2}\frac{e^{-\alpha r}}{r}(1-e^{-\beta r})^4\,,$$

$$\psi_{\alpha,{}^5{\rm He}}(r)=\left[\frac{\gamma\delta(\gamma-\delta)}{2\pi(\gamma-\delta)^2}\right]^{1/2}\frac{(e^{-\gamma r}-e^{-\delta r})}{r}\,,$$

with $\alpha=0.84262$ fm^{-1}; $\beta=1.2$ fm^{-1}, $\gamma=0.367$ fm^{-1}, and $\delta=0.7$ fm^{-1}.

The graphical representation of $r^2|\psi(r)|^2$ and P_3 is shown for both wave functions in Figs. 2(a) and (b), respectively.

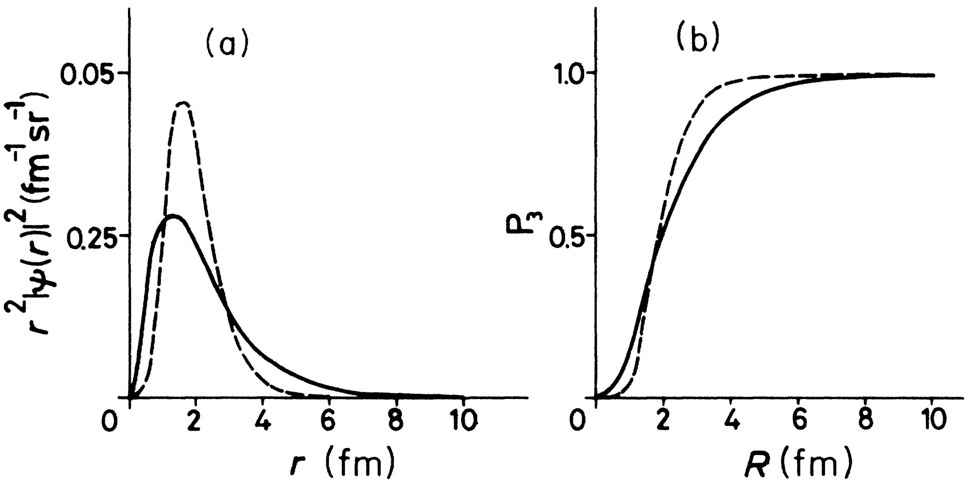

FIG. 2. Graphical representation of $r^2 |\psi(r)|^2$ (a) and of the possibility of finding two particles within Ω (b) for (pt) in α (dotted line) and ($^5He\alpha$) in ^9Be (solid line).

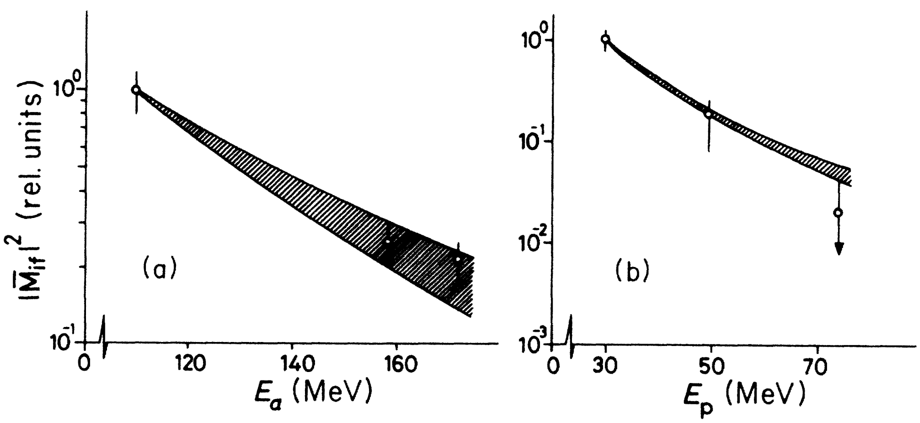

FIG. 3. Relative values of $|M_{if}|^2_{exp}$ for the channels ptα (a) and α^5Hep (b), respectively. The shaded area corresponds to the predictions using $k=5.73\pm2$.

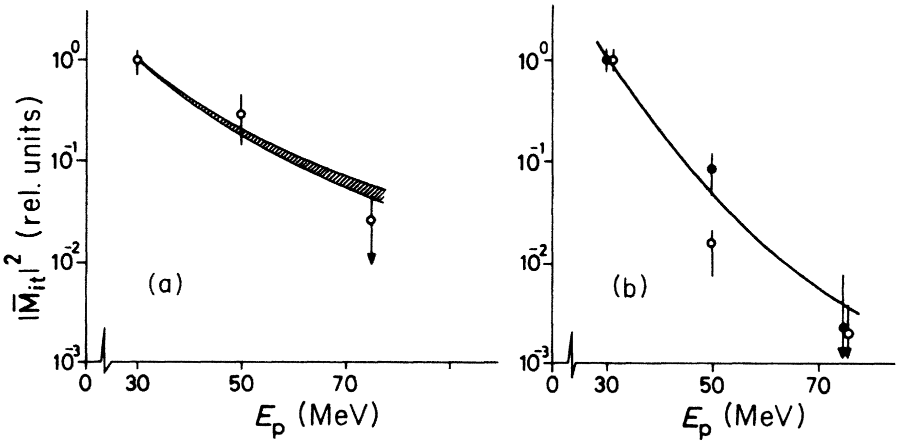

FIG. 4. Relative values of $|M_{if}|^2_{exp}$ for the channel $\alpha\alpha$d (a) and for the channels $\alpha\alpha$np (dots) and αtdp (circles) (b). For an explanation of the theoretical curves see the text.

Figures 3(a) and (b) show the relative values of the $|\overline{M_{if}}|^2_{\rm exp}$ in the function of incident energy, for the ptα and α^5Hep channels. In the same figures we show the shaded area that corresponds to the predictions of our model, [Eq. (6)], assuming $k=5.73\pm2$. This range of k satisfies most experiments on light nuclei fitted with MSIA in this energy range. We observe a very good agreement with the experimental data.

We did not try to apply the model in full for $\alpha\alpha$d, $\alpha\alpha$np, and αtdp channels in the ^{9}Be + p reaction, because we run into difficulties defining the wave functions that enter into play, and additionally, some of the processes are complex since they involve a rearrangement of particles (e.g., the $\alpha\alpha$d channel). For the $\alpha\alpha$d channel data presented in Fig. 4(a) we show that the same theoretical prediction as for the α^5Hep channel fits data reasonably well. This result is not surprising if one considers the rearrangement having occurred between the incident proton and the ^{5}He cluster. For four-body exit channels the situation is more complex as visible from Eq. (5). In our calculation we have assumed, that in the energy range (or cutoff radius range) investigated, the wave functions in play do not change appreciably, so that the probability P_3 of finding three particles within Ω is proportional to Ω^2, hence

$$|M_{if}|^2 \sim \Omega^4 \sim (k\lambda\!\!\!^-)^{12} . \quad (7)$$

In Fig. 4(b) we see that the curve representing Eq. (7) satisfactorily fits the data for $\alpha\alpha$np and αtdp channels.

In summary, the modification of the Fermi model that we have introduced linking the interaction volume to the de Broglie wavelength of the incident channel has exhibited a very satisfactory agreement with experimental results, corroborating the earlier claims that the statistical breakup is confined at low angular momenta in the incident channel and that it is a process complementary to peripheral ones. It is also interesting to note that according to the present modification the matrix element for the four-body breakup channels would, not taking into account the wave function related effects, decrease with energy more rapidly than three-body breakup channels. Of course, in the angle differential cross sections the picture is obscured by the fact that the four-body channels have a much larger phase space available. These conclusions may find application in the studies of competing processes in reactions among light nuclei, and possibly also in heavy ion collisions, when the projectile and target nuclei are treated as colliding systems composed of various subsystems.

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