Nucleus-nucleus potential from fusion and elastic scattering

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An analytical expression for the calculation of the nucleus-nucleus potential has been derived from an empirical formula which describes the fusion excitation function. The nucleus-nucleus potentials calculated for the projectiles ^{6}Li, ^{9}Be, and ^{12}C in the interaction region ~3–6.5 fm by the above procedure agree remarkably well with the real part of the respective optical potentials determined from the analysis of elastic scattering data which exhibit the "rainbow" phenomenon.

In studying nuclear reactions between heavy ions it is of great importance to know the interaction potential acting between the colliding ions as a function of the interaction distance. Information about the nucleus-nucleus potentials has been normally deduced from the analysis of experimental heavy-ion fusion excitation functions[1–3] and elastic scattering data.[4] Several theoretical models[1–10] have also been proposed to calculate the heavy-ion potential. In Ref. 11 we have shown that starting from an empirical expression which describes the fusion excitation function it is possible to derive an analytical expression for the calculation of the total heavy-ion interaction potential at any interaction distance R.

In the last few years, "rainbow" phenomenon has been observed in the elastic scattering of several heavy ions like ^{6}Li (Ref. 12), ^{9}Be (Ref. 13), and ^{12}C (Ref. 14). This being the case it is now possible to determine the real part of the optical potential up to small interaction distances close to the sum of the half-density radii. It seems interesting to compare the nuclear potentials obtained from the analysis of two different reactions, fusion and elastic scattering. Some attempts[15–18] have been made in the past to make this sort of comparison, mainly in the surface region of the interaction. In the present work, we have compared the two potentials for the ions ^{6}Li, ^{9}Be, and ^{12}C over large interaction regions from 3 to 6.5 fm. We define the fusion cross section for the interacting nuclei (Z_P,A_P) and (Z_T,A_T) at the collision energy E (MeV; c.m. system) as

$$\sigma_{\rm fus}({\rm mb})=10\pi\rho(\rho-D) \qquad (1)$$

where

$$\rho({\rm fm})=mE+b$$

and

$$D({\rm fm})=1.44Z_TZ_P/E \ .$$

In the above expression m and b are parameters for a given system. Different prescriptions[19–21] have been given for the calculation of these parameters.

On the basis of classical concepts Bass[1] has shown that given the fusion cross section as a function of E the fusion radius R and the total potential $V_T(R)$ can be determined as

$$\frac{d(E\sigma_{\rm fus})}{dE}=10\pi R^2+\left[\frac{d}{dr}\{10\pi r^2[E-V_T(r)]\}\right]_{r=R}\frac{dR}{dE}=10\pi R^2\ , \qquad (2)$$

$$V_T(R)=E-\frac{E\sigma_{\rm fus}}{10\pi R^2}\ . \qquad (3)$$

Equation (3) can be recast into the classical expression for fusion cross section. The second term of the first line of Eq. (2) is zero even when R is energy dependent as R is defined by the minimization of the curly bracket with respect to r. It may be noted that $R\geq R_{\rm cr}$, where $R_{\rm cr}$ is the critical distance which marks the onset of strong frictional forces. $R_{\rm cr}$ is roughly given by the sum of the half-density radii of the colliding nuclei and in all the calculations reported in the present work we have limited ourselves to R values $\geq 0.75(A_T^{1/3}+A_P^{1/3})$. In fact, Eqs. (2) and (3) are a generalization of the usual intercept and slope method for obtaining the fusion potential.

Bass[1] has used Eqs. (2) and (3) in a graphical manner for deducing the potentials. However, substituting the analytical (empirical) expression (1) for $\sigma_{\rm fus}$ in Eqs. (2) and (3) we have obtained

$$R^2=3\rho^2-2\rho b-mz \qquad (2')$$

and

$$V_T(R)=E-\frac{\rho^2E-\rho z}{R^2}\quad (z=1.44Z_TZ_P)\ . \qquad (3')$$

In our earlier work,[11] using Eqs. (2′) and (3′), we obtained fusion barrier parameters which agreed very well with other determinations. We can get the nuclear potential $V_N(R)$ by subtracting from $V_T(R)$ the Coulomb potential $V_C(R)$. There are various prescriptions[2] given for the calculation of the Coulomb potential. In the present work we have assumed sphere-sphere distribution of charges and used the Coulomb potential expression given in Ref. 22.

$$V_C(R)=\frac{z}{2R_t^3}\left[3\left(R_t^2-\frac{R_p^2}{5}\right)-R^2\right] \quad R\le R_t-R_p$$

$$=\frac{z}{R_t^3R_p^3}\left[\tfrac{1}{32}(R_t-R_p)^4(R_t^2+4R_tR_p+R_p^2)\frac{1}{R}-\tfrac{3}{20}(R_t+R_p)^3(R_t^2-3R_tR_p+R_p^2)\right.$$

$$\left.+\tfrac{9}{32}(R_t^2-R_p^2)^2R-\tfrac{1}{4}(R_t^3+R_p^3)R^2+\tfrac{3}{32}(R_t^2+R_p^2)R^3-\tfrac{1}{160}R^5\right] \quad R_t-R_p\le R\le R_t+R_p$$

$$=z/R \quad R\ge R_t+R_p \tag{4}$$

$R_t=1.3A_T^{1/3}$ (target); $R_p=1.3A_P^{1/3}$ (projectile) .

The Coulomb potential calculated by the above-mentioned procedure is in good agreement with that calculated starting from a more realistic Fermi-Fermi distribution.

We have listed in Table I the best fit "m" and "b" values for the systems ^{6}Li + ^{28}Si (Ref. 23), ^{9}Be + ^{12}C (Ref. 24), and ^{12}C + ^{12}C (Ref. 25) and these parameters are obtained by fitting the fusion excitation functions. The empirical fits to the fusion data are shown in Fig. 1. We have also plotted in Fig. 1 the variation of the interaction distance R with E (dashed line). The R value at each E has been obtained from Eq. (2a). The uncertainty in the R value due to the errors on m and b is given by the shaded region. It is seen that if σ_{fus} data have less errors and are fitted well by expression (1), then R also gets determined in a better way. The spread in R value also increases at higher E (smaller R). The nucleus-nucleus potentials extracted for these systems by the procedure discussed above [using expressions (3a) and (4)] are plotted in Fig. 2, in the interaction region ~3 to 6.5 fm. The uncertainty in the V_N value due to the errors on m and b is shown by the shaded region. It is also found that V_N becomes less certain at higher E (smaller R). As the fusion data for the ^{12}C + ^{12}C system span a large energy region (including high energies) it is possible to determine the V_N values for R up to ~3 fm. This is not the case for the other two systems. As per the present availability of fusion data, it is possible to determine V_N up to ~4.5 fm for the ^{9}Be + ^{12}C system and up to ~6 fm for the ^{6}Li + ^{28}Si system. Both these cases require the extrapolation of the data to higher energies through the use of Eq. (1), to determine the V_N values at smaller distances. There is certainly a need for more data at higher energies for these two systems. In order to reduce the uncertainty on the $V_N(R)$ value, it is necessary to measure the data with better accuracy than available now for the lower energies.

As mentioned earlier, by our technique, it is possible to calculate the potential at any interaction distance R and it

TABLE I. Parameters "m" and "b" determined from the fit to the fusion excitation function data. Note: The errors on R and V_N, viz., ΔR and ΔV_N, are calculated as follows:

$$\Delta R=\left[\left(\frac{\partial R}{\partial m}\right)^2\sigma_m^2+\left(\frac{\partial R}{\partial b}\right)^2\sigma_b^2+2\left(\frac{\partial R}{\partial m}\right)\left(\frac{\partial R}{\partial b}\right)\sigma_m\sigma_b C\right]^{1/2},$$

$$\Delta V_N=\left[\left(\frac{\partial V_N}{\partial m}\right)^2\sigma_m^2+\left(\frac{\partial V_N}{\partial b}\right)^2\sigma_b^2+2\left(\frac{\partial V_N}{\partial m}\right)\left(\frac{\partial V_N}{\partial b}\right)\sigma_m\sigma_b C\right]^{1/2}.$$

System	m (fm/MeV)	σ_m (fm/MeV)	b (fm)	σ_b (fm)	Correlation coefficient (C)
^{6}Li + ^{28}Si	−0.064	0.017	8.93	0.30	−0.98
^{9}Be + ^{12}C	−0.123	0.025	8.57	0.17	−0.96
^{12}C + ^{12}C	−0.084	0.005	8.42	0.09	−0.94

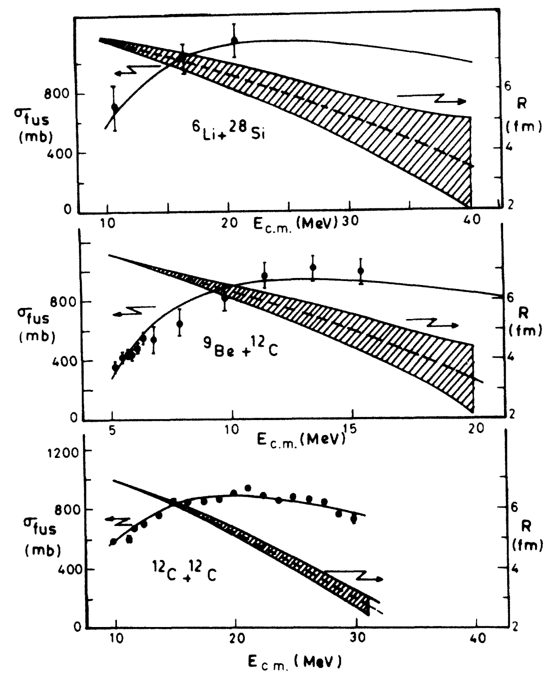

FIG. 1. Plot of σ_{fus} data as a function of E. The continuous lines are the fits to the data using expression (1). In the same figure, the variation of R with E is also shown. The dashed line corresponds to the R values. The shaded region represents the uncertainty on the R values.

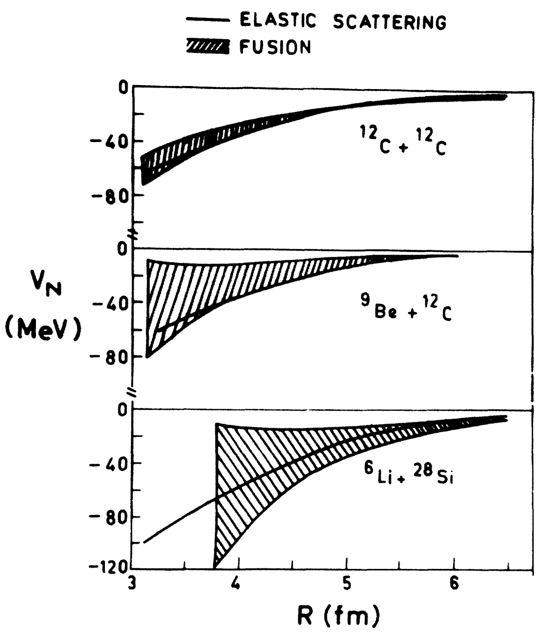

FIG. 2. Plot of nucleus-nucleus potential V_N as a function of the interaction distance R. The continuous lines stand for the V_N values determined from the analysis of elastic scattering. The shaded region corresponds to V_N values obtained from the analysis of fusion data.

can be considered reliable provided at the corresponding E, the σ_{fus} calculated by the empirical procedure is in agreement with the experimental value. From Fig. 2 it is evident that the potential values are fairly large at small interaction distances. Recently, Gomez del Campo and Satchler,[26] from the analysis of fusion cross section at high energies, have deduced V_N and R values for several heavy ion systems from the slope and the intercept of the plot of σ_{fus} vs $1/E$. They also find that at short distances the V_N values are large.

By their method it is possible to get only one set of V_N and R values for a given system. However, the method proposed by us yields V_N as a function of R, for a range of R values.

As stated earlier, the nucleus-nucleus potentials have also been determined from the analysis of the elastic scattering data. With the observation of rainbow phenomenon in the elastic scattering of projectiles ^{6}Li, ^{9}Be, and ^{12}C, it has been possible to obtain the real part of the optical potential for these ions up to very small distances close to the sum of the half-density radii. It is interesting to compare the potential V_N obtained from the analysis of fusion data with the real part of the optical potential deduced from the analysis of the rainbow elastic scattering. In Fig. 2 we have also plotted the real part of the optical potential for the projectiles ^{6}Li, ^{9}Be, and ^{12}C. It is interesting to find that the potentials obtained from the analysis of the two different reactions agree remarkably well with each other in the interaction region ~ 3 to 6.5 fm. We have implicitly made the assumption that the real part of the heavy ion potential deduced does not vary significantly with the variation of the bombarding energy and this may be reasonable.

Though we found the two methods yielding almost identical values for the nucleus-nucleus potential, it is not clear whether the same potential should be operating in both the situations, viz., fusion and elastic scattering. One might expect in the fusion process, due to excitations and rearrangements occurring along the path to fusion, modification of the real part of the potential. The fusion in general can be considered "adiabatic" in character and the elastic scattering is more like a "sudden" process. The other feature which distinguishes between these is the effect of energy dissipation due to friction which is important for understanding the fusion reaction but not the elastic scattering. The fact that we have got for the ions ^{6}Li, ^{9}Be, and ^{12}C potentials from the analysis of fusion and elastic scattering agreeing with each other over a long interaction region may imply that perhaps for these ions the adiabatic and energy dissipation effects are not very large. It will be interesting to extend this type of comparison to heavier ions where we expect the interaction process to be more adiabatic than sudden.

As pointed out earlier, even for the systems ^{6}Li + ^{28}Si and ^{9}Be + ^{12}C, it is necessary to measure the fusion excitation function data up to $E \sim 35$ MeV for the former and up to $E \sim 25$ MeV for the latter with better precision, to verify some of the predictions of the present work regarding the V_N values for $R < 6$ fm for the former and for $R < 4.5$ fm for the latter.

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