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F-spin multiplets and alpha transfer systematics in the interacting boson model

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It is shown that the recently proposed existence of F-spin multiplets in nuclear regions where the nuclei contained in the multiplets differ by four nucleons gives rise to a simple prediction for the relative spectroscopic factors for ground state to ground state alpha transfer reactions among them.

It has been pointed out recently by Harter *et al.*[1] that a connection seems to exist among nuclei with constant $N=N_\nu+N_\pi$ values where N_ν (N_π) is the number of neutron (proton) bosons in the interacting boson model (IBM),[2] to be identified with the number of neutron (proton) pairs in the valence shell of these nuclei. The nuclei associated with such multiplets are either isobaric or differ by four nucleons (two neutrons and two protons), depending on the particle-hole character of the neutron and proton bosons. This connection manifests itself by the presence of an approximate overall constancy in the excitation spectra, particularly in the Xe-Ba region, which Harter *et al.*[1] attribute to the existence of a predominantly F-spin[3] invariant Hamiltonian in the IBA-2. In analogy with the case of isospin, they refer to these nuclei as members of an F-spin multiplet,[1,4] characterized by the same value of F and with different projections $F_0=(N_\pi-N_\nu)/2$.

Although the similarities in the spectra of these nuclei are quite compelling, in order to clearly establish the existence of such multiplets it would be important to have corroborating evidence from other physical observables. The purpose of this short communication is to point out a simple consequence of the existence of F-spin multiplets in the Xe-Ba region, which gives rise to a clearcut prediction for the relative spectroscopic factors for ground state to ground state alpha transfer reactions among nuclei in the multiplets.

Consider an F invariant Hamiltonian H, i.e.,

$$[H,F_\mu]=0\,, \tag{1}$$

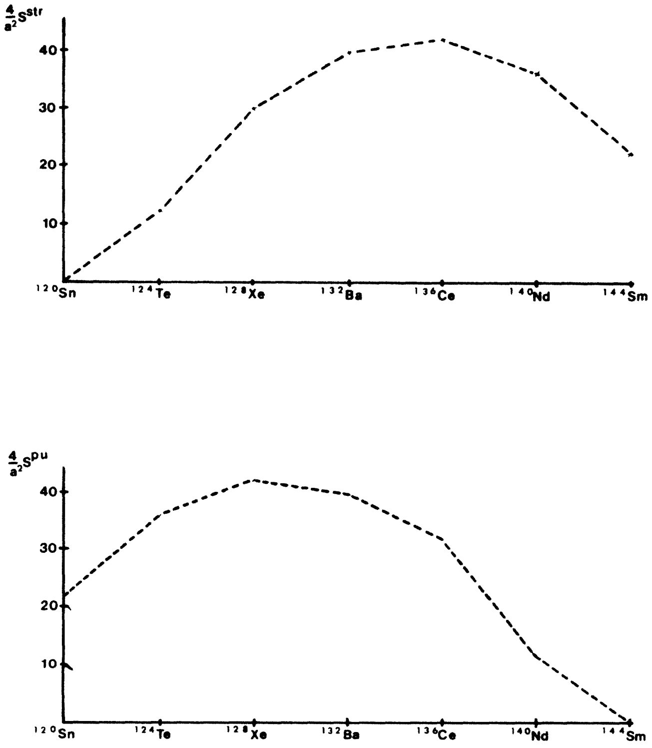

FIG. 1. Predicted behavior of $4S^{\rm str}/a^2$ and $4S^{\rm PU}/a^2$ for the $N=6$ multiplet (arbitrary units). Points are labeled according to residual nucleus.

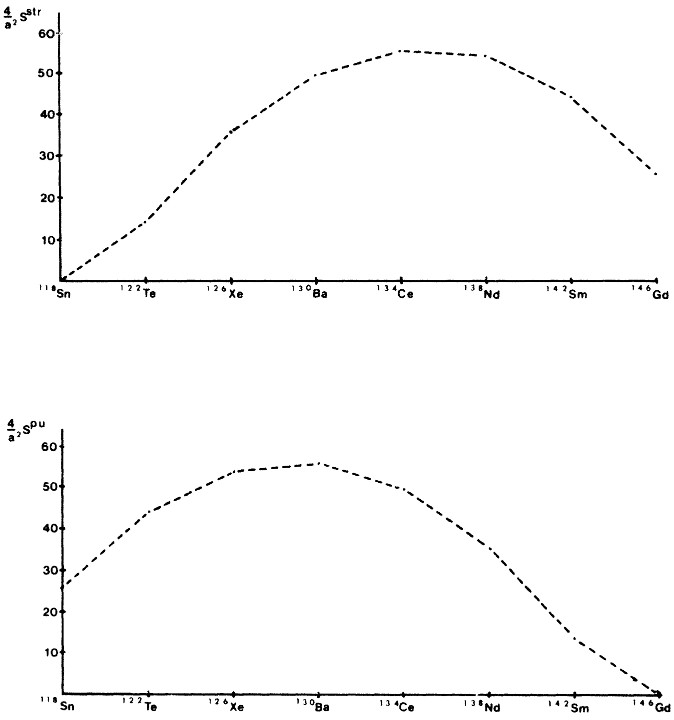

FIG. 2. Predicted behavior of $4S^{\rm str}/a^2$ and $4S^{\rm PU}/a^2$ for the $N=7$ multiplet (arbitrary units). Points are labeled according to residual nucleus.

where $F_\mu, \mu = +,-,0$ are the F-spin generators.[1,4] Denoting the F spin multiplet states by $|\Psi_\sigma^{FF_0}\rangle$, where σ denotes all other quantum numbers corresponding to U(6) and its subgroups, we have

$$H\,|\Psi_\sigma^{FF_0}\rangle = E_\sigma^{FF_0}\,|\Psi_\sigma^{FF_0}\rangle \,, \tag{2}$$

and from (1), it is clear that

$$E_\sigma^{FF_0} = E_\sigma^{FF_0+1} \,, \tag{3}$$

which implies identical excitation spectra for all members of the multiplet.

Consider now the application of F_+ to the states Ψ,

$$F_+\,|\Psi_\sigma^{FF_0}\rangle = \sqrt{(F-F_0)(F+F_0+1)}\,|\Psi_\sigma^{FF_0+1}\rangle \,, \tag{4}$$

a result arising solely from the SU(2) character of the F-spin generators. We also note that, due to the particle and hole character of the proton and neutron bosons, respectively, in the Xe-Ba region, the members of the multiplet differ by two protons and two neutrons, i.e., an alpha particle. This means that a natural interpretation for the rising operator (4) in this case is (up to a constant) an alpha particle creation operator. The alpha particle spectroscopic factor is thus defined by

$$S^{\mathrm{str}}(\mathrm{g.s.}\to\mathrm{g.s.}) = (\langle\Psi_\sigma^{FF_0+1}||aF_+||\Psi_\sigma^{FF_0}\rangle)^2 \,, \tag{5a}$$

$$S^{\mathrm{PU}}(\mathrm{g.s.}\to\mathrm{g.s.}) = (\langle\Psi_\sigma^{F_0+1}||aF_-||\Psi_\sigma^{FF_0+2}\rangle)^2 \,, \tag{5b}$$

for stripping and pickup reactions, respectively. In Figs. 1 and 2 we show the predicted behavior of $4S^{\mathrm{str}}/a^2$ and $4S^{\mathrm{PU}}/a^2$ for the $N=6$ and $N=7$ F multiplets, with $F=N/2$, where points are labeled according to residual nucleus. In Fig. 3 we show the function $S^{\mathrm{str}}/S^{\mathrm{PU}}$ for the same multiplets, which are the ones studied by Harter *et al.*[1] in this region. Unfortunately, there are no systematic experimental data for these nuclei.[5] Previous studies of alpha particle transfer in the interacting boson model[6] make use of rather complicated expressions stemming from an extension of the IBM treatment of two nucleon transfer,[7] and involve several adjustable parameters.

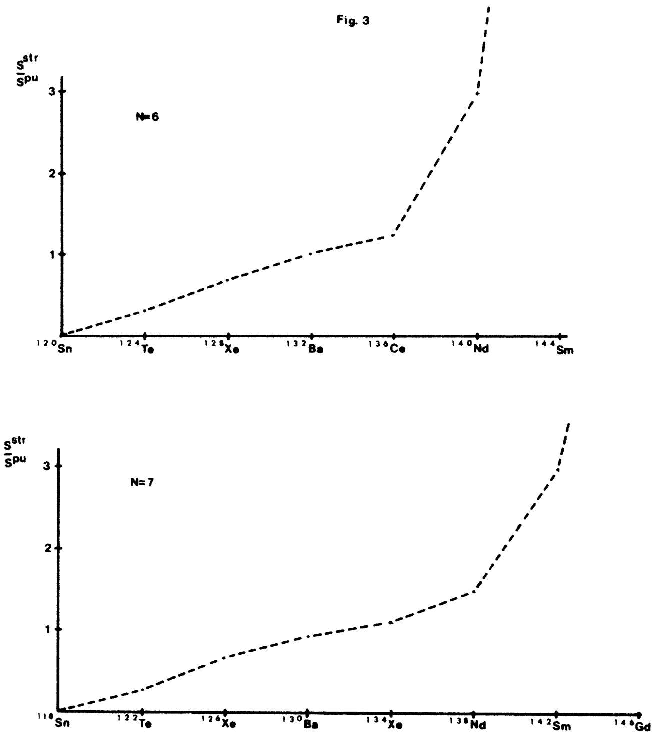

FIG. 3. Predicted behavior for $S^{\mathrm{str}}/S^{\mathrm{PU}}$ for the $N=6$ and $N=7$ multiplets. Points are labeled according to residual nucleus.

It remains to be seen if the additional symmetry associated with the existence of F-spin multiplets and which gives rise to the simple pattern implied by Eqs. (5) is indeed experimentally observed.

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[1] H. Harter, P. Von Brentano, A. Gelberg, and R. F. Casten, Phys. Rev. C **32**, 631 (1985).

[2] See, e.g., *Interacting Bose-Fermi Systems in Nuclei*, edited by F. Iachello (Plenum, New York, 1981).

[3] A. Arima, T. Otsuka, F. Iachello, and I. Talmi, Phys. Lett. **66B**, 206 (1976).

[4] A. Frank and P. Van Isacker, Phys. Rev. C **32**, 1770 (1985).

[5] R. Casten (private communication).

[6] C. L. Bennet and H. W. Fulbright, Phys. Rev. C **17**, 2225 (1978).

[7] A. Arima and F. Iachello, Phys. Rev. C **16**, 2085 (1977).