Test of forbidden state NN potential in the 3N system

S. Nakaichi-Maeda
Institut für Theoretische Physik, Universität Tübingen, 7400 Tübingen, Federal Republic of Germany
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Significant underbinding for the triton of a quantum-chromodynamically motivated realistic NN potential containing a superdeep short-ranged attraction and a forbidden state in the S waves has been reported from a previous Faddeev calculation. In the present paper, this feature is reproduced with a simple S-wave-interaction model of this NN potential, generalized into a wide range in the form of a short distance node, and explained in terms of short-ranged NN D-wave components which the nodal S-wave components introduce inevitably through the three-body kinematics. The strengths of the NN D-wave interactions are estimated to be not sufficient to compensate for the kinetic energies arising from these D-wave components, which means a significant loss for 3N binding. This is in sharp contrast to the strong 3α binding in the forbidden state $\alpha\alpha$ potential model which is supported by strong D- and G-wave interactions. The present study indicates that a noticeable short distance node in the 2N wave function is unlikely.

I. INTRODUCTION

It is well known that the 2N data provide us with limited information on the NN interaction, i.e., the on-shell properties. The electron scattering data also cannot determine the deuteron wave function at relatively small 2N distances. There are, in principle, various ways to account for the sign changes of the S-wave phase shifts. Conventional realistic NN potential models exclusively employ a short ranged repulsive core.[1-3] It has been shown that all of them underbind the triton by 1–2 MeV.[4] On one hand, a possible explanation is the neglect of the three-nucleon interaction.[5] On the other hand, it is an interesting question how such a few-nucleon quantity is sensitive to the off-shell properties of the NN interaction. In particular, one asks to what extent the off-shell variation of the NN potential can remedy the underbinding for the triton already. Many investigations have been made in this direction.[6-8] Up to now no noticeable gain in the triton binding energy has emerged. On the contrary, a short distance node generated by the unitary off-shell transformation of a conventional repulsive core potential tends to decrease the triton binding energy.[8]

Recently, attempts to understand the NN interaction at short range from the use of 6-quark degrees of freedom have been made quite extensively.[9,10] In this line of study, a new realistic NN potential model was proposed which presupposes a short ranged deep attraction with a forbidden state, as well as the one pion exchange tail.[11] The parameters of the short ranged attraction (and the forbidden state) are determined from the 2N data. In its most recent version,[12] which we call the Kukulin-Pomerantsev-Krasnopol'sky-Sazonov (KPKS) potential in the present paper, a set of $^1S_0$ and $^3S_1$-$^3D_1$ potentials has been proposed. They reproduce the experimental $^1S_0$, $^3S_1$, and $^3D_1$ phase shifts up to $E_{\rm lab}=400$ MeV together with the deuteron property with an accuracy similar to those of the conventional realistic NN potentials, but lead to a significant short distance node behavior of the 2N wave function in these partial waves. From the interest in the off-shell property of this newly proposed NN potential, a Faddeev calculation for the triton has been performed,[13] resulting in a significant underbinding, even when compared with the results obtained by the conventional realistic NN potentials. The separable expansions employed for the potential in this Faddeev calculation are rather complicated, and the reason for the "less" triton binding has not been studied.

In the present paper we investigate whether we can confirm the "less" triton binding of the KPKS potential in a second triton calculation and can clarify this feature. For this aim we adopt some simple interaction models. In Sec. II we derive an effective $^3S_1$ potential from the KPKS $^3S_1$-$^3D_1$ potential and another from the Reid soft core[2] (RSC) $^3S_1$-$^3D_1$ potential. These $^3S_1$ potentials reproduce the deuteron binding energies and the $^3S_1$ components of the deuteron wave function determined by the respective exact $^3S_1$-$^3D_1$ potentials. We call the $^3S_1$ component just the deuteron wave function in the present paper. We compare the two $^3S_1$ potentials in a simple S-wave interaction model, in how strongly they bind three nucleons, in order to see whether we can reproduce the "less" triton binding. In Sec. III we present a variation of the KPKS S-wave interaction model through a systematic change in the short distance part of the deuteron wave function. Thereby, we study the influence of short distance node, in general, on the triton binding energy. In Sec. IV we make an additional analysis of previously performed 3α calculations. We will see that strong D- and G-wave interactions were a necessary condition for the strong 3α binding achieved in the forbidden state $\alpha\alpha$ potential model. We then consider the NN D-wave interactions in an explicit way, and estimate whether their strengths are enough to support a "strong" 3N binding. In Sec. V we give a summary and a conclusion.

II. THE S-WAVE INTERACTION MODEL

The KPKS $^3S_1$-$^3D_1$ potential generates two bound states in the free 2N system: the physical deuteron state

(u_d, w_d) and an unphysical deep lying state (u_0, w_0), where the first components in parentheses are the $^3S_1$ components. The deep lying state is taken as the forbidden state to which all physical 2N states in the many-nucleon systems should be orthogonal. We define an effective $^3S_1$ potential $V_S^{(K)}$ by the local potential which gives a 2N bound state with the same wave functional form as u_d, at the energy E_d, where $-E_d$ is the deuteron binding energy the KPKS $^3S_1$-$^3D_1$ potential gives. The construction of this effective $^3S_1$ potential can be done trivially. Its shape is smooth, owing to the local potential model adopted for the $^3S_1$-$^3D_1$ central and tensor potentials. Since u_d has a node, $V_S^{(K)}$ gives another 2N bound state u_f at a lower energy. We found that its energy and its wave functional form are essentially identical to the energy of the coupled deep lying state (u_0, w_0) and the functional form of the $^3S_1$ component u_0, respectively. We define our S-wave interaction model by approximating the nucleons as bosons, by assuming that two nucleons interact by the potential $V_S^{(K)}$ in the S wave but do not interact in the other partial waves, and by assuming that two nucleons in the S wave should not be in the state u_f. We also derive a similar effective $^3S_1$ potential $V_S^{(R)}$ from the RSC $^3S_1$-$^3D_1$ potential, and define a similar S-wave interaction model by this potential, but without any orthogonality condition. We found that the S-wave phase shifts calculated from $V_S^{(K)}$ and $V_S^{(R)}$ simulate the $^3S_1$ phase shifts calculated from the exact KPKS and RSC $^3S_1$-$^3D_1$ potentials rather accurately up to $E_{lab}=200$ MeV, respectively. The effective $^3S_1$ potentials and the deuteron wave functions $u_d^{(K)}, u_d^{(R)}$ are depicted in Fig. 1.

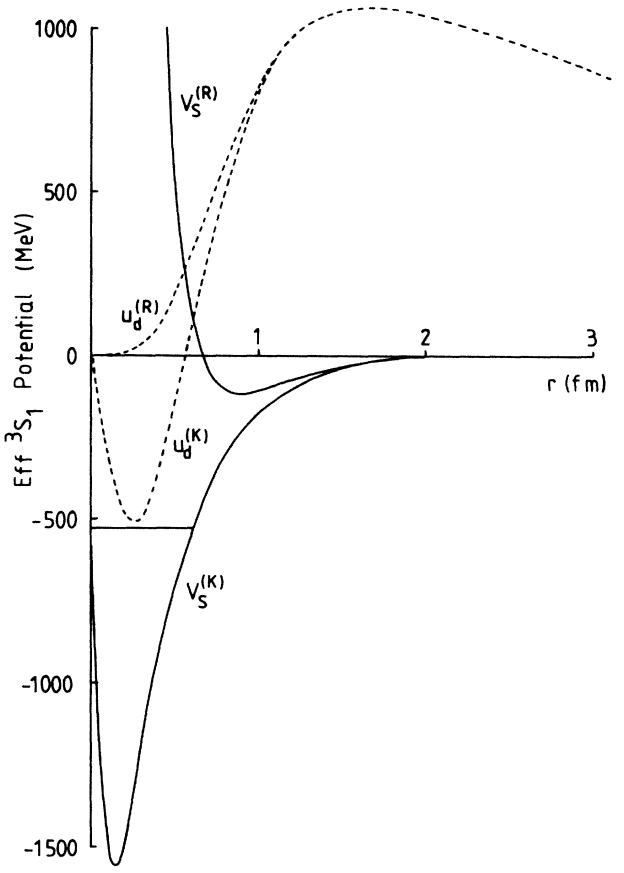

FIG. 1. The effective $^3S_1$ potentials $V_S^{(K)}$ and $V_S^{(R)}$ derived from the KPKS and the RSC $^3S_1$-$^3D_1$ potentials, respectively, and the deuteron wave functions which these $^3S_1$ potentials give. The energy of the deep lying state which $V_S^{(K)}$ gives is denoted by a horizontal line.

We use the two simplified S-wave interaction models, of the KPKS potential and of the RSC potential, to calculate the triton binding energy. Of course, the absolute values of our triton binding energies will be grossly an overestimation of a realistic value, because the most attractive $^3S_1$-$^3D_1$ interaction is taken to construct our (spin-isospin-independent) S-wave potential. However, our interest lies in whether a comparison between the triton binding energies resulting from the two S-wave interaction models can confirm the "less" triton binding of the KPKS potential reported in the previous Faddeev calculation.

To make the three-body calculation simple we adopt the unitary pole approximation[14] (UPA) of the local potentials $V_S^{(K)}$ and $V_S^{(R)}$. To get a rough idea of this rank-1 separable approximation, we compared the triton binding energy $-E_{t,UPA}$ obtained by the UPA of $V_S^{(R)}$ with the triton binding energy $-E_{t,local}$ evaluated by the local form through the optimum choice of the Jastrow test function. This optimum Jastrow wave function can be obtained by an integro-differential Euler-Lagrange equation with iteration. In the case of a simple local potential active only on the S wave, the Jastrow wave function gives an accurate estimate for the three-body ground state energy.[15,16] We found that the difference between $E_{t,UPA}$ and $E_{t,local}$ is within 1%.

The orthogonality condition with respect to u_f of all the 2N states in the triton wave function can be fulfilled by the use of the orthogonalizing pseudopotential[17] (OPP), $\epsilon |u_f\rangle\langle u_f|$, which is also of separable form and can also be handled easily in the Faddeev calculation. The parameter ϵ is taken to be positive and sufficiently large in the final estimate of the triton energy. The behavior of the triton binding energy with respect to this parameter is shown in Table I.

The results for the triton binding energy in the S-wave interaction model presented in this section are listed in Table II, in comparison with the previous results including the $^1S_0$ and $^3S_1$-$^3D_1$ potentials exactly. We see that,

TABLE I. The behavior of the triton binding energy $-E_t$ with respect to the coefficient ϵ of the orthogonalizing pseudopotential.

ϵ (MeV)	$-E_t$ (MeV)
10^3	10.736
10^4	10.536
10^5	10.508
10^6	10.505
10^7	10.504
10^8	10.504

TABLE II. The triton binding energy $-E_t$ (in MeV) calculated from the KPKS and RSC potentials. The previous results, including the $^1S_0$ and $^3S_1$-$^3D_1$ interactions, are from Ref. 13. The value in parentheses for the RSC potential is from Ref. 4(b). The present calculation employs a simple S-wave interaction model. (See the text.)

	Previous	Present
KPKS	4.50	10.50
RSC	7.37 (7.02)	17.87
Ratio	0.61	0.59

again in the present S-wave interaction model, the KPKS potential gives smaller binding for the triton than the RSC potential gives. The ratio of the triton binding energy between the KPKS and RSC potentials essentially does not change from that obtained in the previous calculations. Thus, the present calculation confirms the "less" triton binding feature of the KPKS potential. Conversely, this means that the reason for the "less" triton binding is preserved in our simplified interaction model. This gives us a motive for a further detailed study which will be described in the following sections.

III. VARIATION OF THE SHORT DISTANCE NODE OF THE KPKS DEUTERON WAVE FUNCTION

The unitary pole approximation is useful in studying the correlation between the deuteron wave function and the triton quantities. In this section we study, in the framework of the S-wave UPA plus OPP, a relatively general variation of the short distance behavior of the KPKS deuteron wave function. Thereby we ask whether it is a general feature of short distance node to decrease the triton binding energy. In other words, we are interested in whether there is a form of short distance node which gives a noticeably larger triton binding energy than the RSC value. We make the variation of the deuteron wave function in two directions. In the first variation we amplify/reduce the short distance amplitude with the node radius $r_{\rm node}$ kept fixed. In the second variation we vary $r_{\rm node}$ too. We do not check explicitly the on-shell equivalence between the KPKS S-wave interaction model and its variations. Since we will fix the deuteron binding energy, we expect an approximate on-shell equivalence for low energies. On the other hand, it was reported in a previous triton calculation using rank-2 separable interactions that the dependence of the triton binding energy on the high energy phase shifts is much weaker than its dependence on the deuteron wave function.[7]

We generate the first variation by

$$u'_{\rm d}(r)=h(r)u_{\rm d}^{(K)}(r)\ ,$$
$$h(r)=1+(C-1)\exp[-(r/r_h)^n]\ . \tag{1a}$$

We take $n=8$ and $r_h=r_{\rm node}$. The change of the deuteron wave function is then essentially restricted to the range $r\lesssim r_{\rm node}$. When C is larger than 1 the inner amplitude is amplified. When C is smaller than 1 it is reduced. In accordance with (1a), the orthogonality condition with respect to u_f is replaced by the orthogonality to

$$u'_f(r)=u_f(r)/h(r)\ . \tag{1b}$$

Some examples of the varied deuteron wave functions are shown in Fig. 2(a).

In the second variation, we generate the change in $r_{\rm node}$ by varying the range of the forbidden state. To parametrize this range, we recall a general tendency of deep lying ground states to look like a zero point oscillation. Indeed, we found that the orthogonality condition with respect to u_f can be replaced in a very good approximation by the orthogonality to a Gaussian function

$$u''_f(r)=2(8/\pi)^{1/4}b^{-3/2}\exp[-(r/b)^2] \tag{2}$$

if we take as $b=b_0=0.617$ fm; the values for the deuteron and the triton binding energies essentially do not change from those determined in Sec. II, and the change in the deuteron wave function cannot be seen in a plot like Fig. 1. Then we can change $r_{\rm node}$ by varying b. When b deviates from b_0, it is necessary to weaken the local potential $V_S^{(K)}$ in order to keep the deuteron binding energy unchanged. We do this by a multiplication constant. It should be noted, however, that interaction becomes essentially nonlocal in this case, and $V_S^{(K)}$ (multiplied by a constant) does not represent an equivalent local potential. The resulting variation in the deuteron wave function is such that the inner amplitude increases as $r_{\rm node}$ increases and vice versa. Some examples of the varied deuteron wave functions are shown in Fig. 2(b).

Figures 3(a) and 3(b), respectively, show how the difference of the triton ground state energies,

$$\Delta E_t=E_t(\text{KPKS})-E_t(\text{RSC})\ , \tag{3}$$

changes by the two independent variations of the KPKS deuteron wave function presented in this section. The starting point (3) is denoted by crosses. These figures clearly show that, in order to reduce the loss for the triton binding energy, we must go in the direction of smaller node behavior. On the other hand, in the range of variation adopted in the present study, we could not get a triton binding energy which is larger than the RSC value.

The present variation for the NN interaction starts from the KPKS potential (in its simplified S-wave interaction model) for which the deuteron wave function has already a significant node behavior. To connect these results to the results of the previous studies which start from the RSC potential and employ unitary off-shell transformation, particularly to the results of Ref. 8, we have performed the same unitary off-shell transformation as was done in Ref. 8, on our S-wave interaction model of the RSC potential. To specify the parameters of this off-shell transformation, we note that the deuteron wave function $u_{\rm d}(r)$ changes as

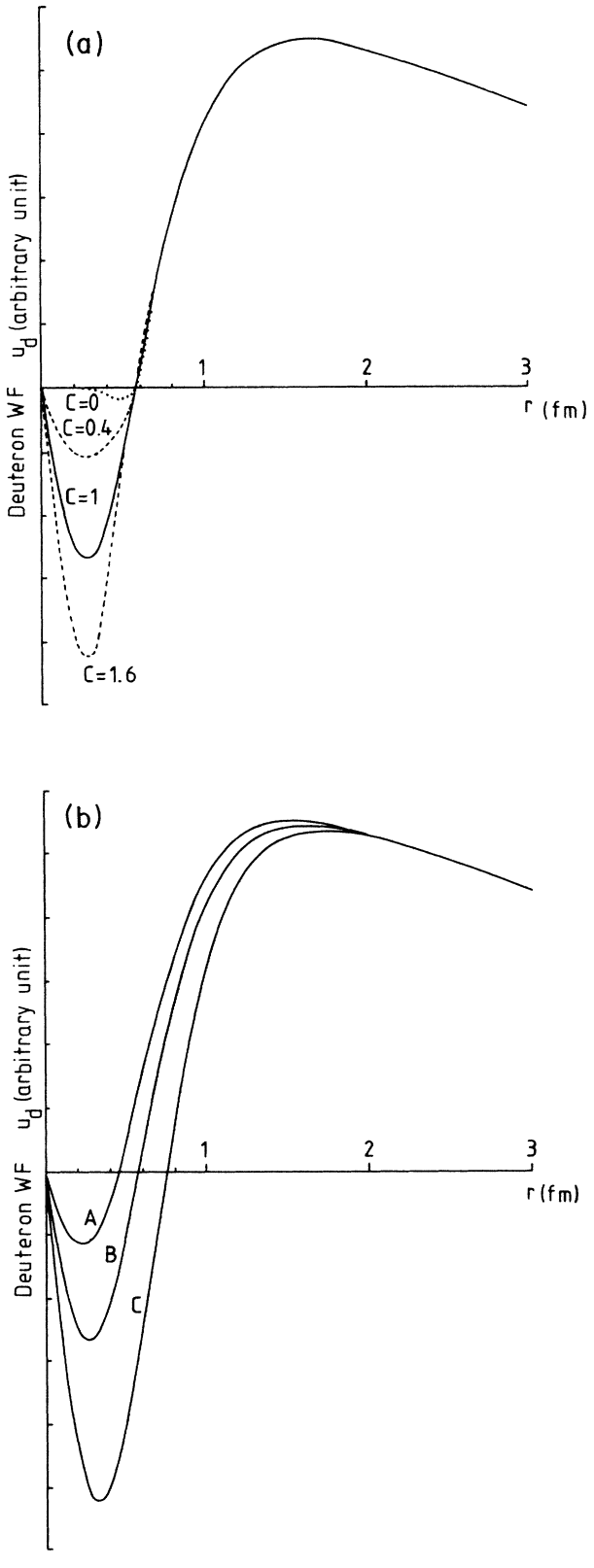

FIG. 2. (a) Examples of varied deuteron wave functions generated from the KPKS deuteron wave function by the first variation scheme. The parameter C is defined by Eq. (1). (b) Examples of varied deuteron wave functions generated from the KPKS deuteron wave function by the second variation scheme, with the node at 0.447 fm (A), 0.517 fm (B), and 0.750 fm (C).

$$\bar{u}_d(r)=u_d^{(R)}(r)-2g(r)\int_0^\infty dr' g(r')u_d^{(R)}(r') ,$$
$$g(r)=C_g r(1-\beta r)\exp(-\alpha r) , \tag{4}$$

through this off-shell transformation, where C_g is a normalization factor. We fix $\alpha=3$ fm^{-1}. At a certain value β_0 of β, which is 0.93 fm^{-1} for this choice of α, the overlap integral of Eq. (4) vanishes and the deuteron wave function does not change. We have varied β around this value. For smaller values of β than β_0, the unitary off-shell transformation (4) induces a node in the deuteron wave function in such a way that as β becomes smaller the node radius r_{node} becomes larger and the inner amplitude also becomes larger. On the other hand, for larger values of β than β_0, $\bar{u}_d(r)$ has no node but has a charac-

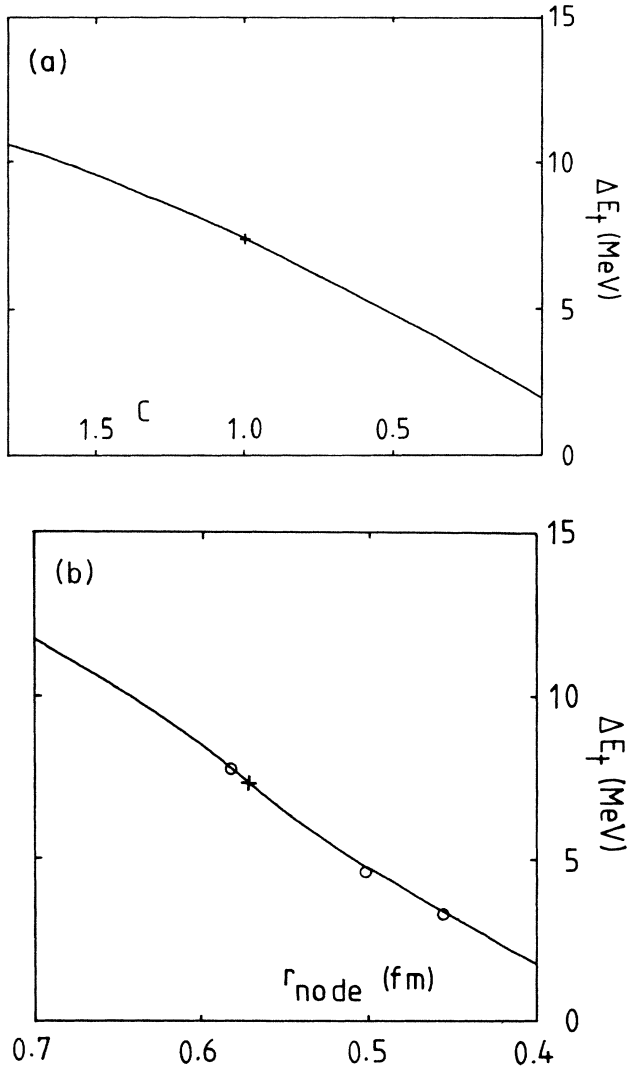

FIG. 3. (a) The behavior of the triton ground state energy ΔE_t as measured from the RSC value with respect to the variation of C [Eq. (1)]. (b) The behavior of ΔE_t with respect to the variation of the node radius r_{node}. The circles are those results obtained by the unitary off-shell transformation (4).

teristic inner bump, as can be seen from the figures of Ref. 8.

For the potential we again employ the UPA form of $V_S^{(R)}$, and make a unitary off-shell transformation of this rank-1 separable potential. The unitary off-shell transformation specified by Eq. (4) means addition of two separable terms; thus we have a rank-3 separable potential. The resulting changes ΔE_t of the triton ground state energy are 2 MeV (0.3 MeV), 0.6 MeV (0.2 MeV), -0.5 MeV (-0.2 MeV), and 3 MeV (0.8 MeV) at $\beta=0.9$, 0.95, 1, and 1.2 fm^{-1}, respectively, where we have shown the results of Ref. 8 in parentheses. The off-shell effect is enhanced in the present calculation when compared with the previous results. This is because in Ref. 8 the triton binding energy was calculated by employing the RSC $^1S_0$ and $^3S_1$-$^3D_1$ potentials and by performing unitary off-shell transformation only on the $^1S_0$ potential. On the other hand, we see that the behavior of ΔE_t against the variation of β is very similar between the present and the previous calculations. In particular, the additional node generally brings about a loss of triton binding energy, and this loss becomes larger for larger node behavior. (It is also clear that the additional node is not the only origin of loss of the triton binding energy.) To achieve a connection with Fig. 3, we have calculated the ΔE_t also at somewhat even smaller values of β, i.e., 0.85, 0.8, and 0.7 fm^{-1}, and plotted them in Fig. 3(b) against r_{node}, by circles. We see that the correlation between ΔE_t and the short distance node shows a similar trend, even in a quantitative sense, whether we start the off-shell variation from the KPKS deuteron wave function or from the RSC deuteron wave function. To summarize, the results of this section indicate that it is a general feature of the short distance node to decrease the triton binding energy.

IV. COMPARISON BETWEEN THE 3N AND 3α SYSTEMS

The off-shell properties of the forbidden state potential and of the repulsive core potential have already been studied extensively in the ground state of the 3α system. It has become clear that the forbidden state potential representation for the $\alpha\alpha$ interaction can reproduce a 3α binding energy comparable to the empirical value,[18–21] or even tends to overbind the system.[22] On the other hand, the repulsive core potential representation leads to a significant underbinding.[23] Indeed, this observation and the underbinding for the triton seen among all conventional realistic NN potentials was a motive for the proposal of the forbidden state NN potential.[11] The characteristic difference between the two kinds of $\alpha\alpha$ potentials was explained in terms of the Hall-Post lower bound.[18] When we go from the two-body system to the three-body system, the effect of the two-body interaction becomes more emphasized. This is because, in the three-body Hamiltonian, the interaction appears three times, in contrast to the kinetic energy, which appears twice. In the repulsive core potential model both the repulsion and the attraction are strengthened, while in the forbidden state potential model only the attraction is strengthened with the repulsive orthogonality condition unmodified. The question is why this energy gain mechanism of the forbidden state potential is realized in the 3α system and not in the 3N system. In this section we investigate this question.

The Ali-Bodmer potential[24] is the most extensively used repulsive core $\alpha\alpha$ potential, while the FBOM (fishbone optical model) potential[25] and the OCM (orthogonality condition model) of Ref. 18 serve as typical examples of the forbidden state $\alpha\alpha$ potentials. Figure 4 shows the behavior of the 3α ground state energy with respect to the sequential inclusion of higher partial wave interactions, obtained in previous calculations. This figure brings out the characteristic difference between the two kinds of potential models most clearly. (i) When only the S-wave interaction is included, or when only the S- and D-wave interactions are included, the forbidden state potentials give smaller 3α binding than the on-shell equivalent repulsive core potential. (ii) When all of the S-, D-, and G-wave interactions are included the situation becomes reversed. The size of the energy gain brought about by the inclusion of the G-wave interaction ($S+D \rightarrow S+D+G$) in the forbidden state potential model is impressively large. On the other hand, the effect of including the next higher partial

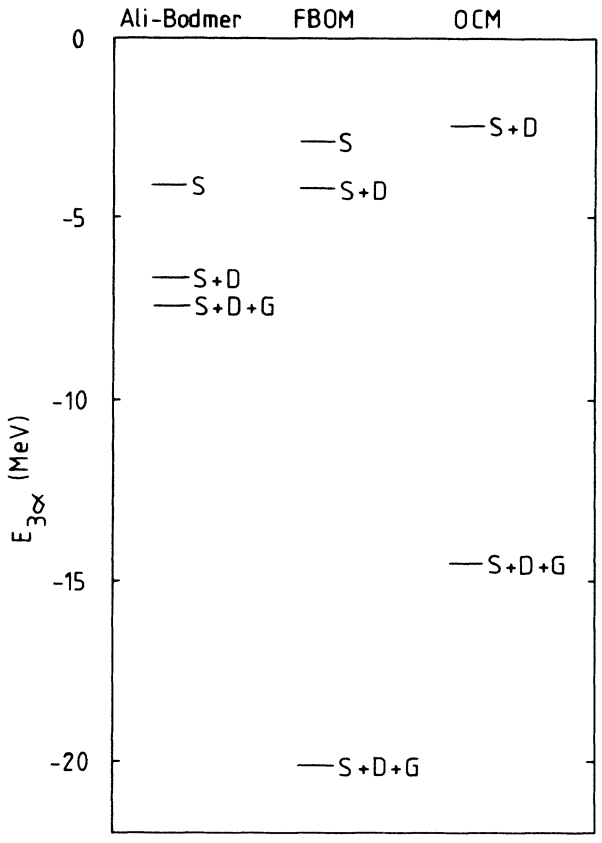

FIG. 4. The behavior of the 3α ground state energy $E_{3\alpha}$ with respect to sequential inclusion of the S-, D-, and G-wave $\alpha\alpha$ interactions. The Coulomb force is not considered. The results are from Refs. 23, 21, and 18 for the Ali-Bodmer potential, the FBOM potential, and the orthogonality condition model, respectively.

wave interaction is negligible.[19]

These features suggest that, in the forbidden state potential model, the D- and G-wave interactions play an essential role even for taking the S-wave interaction properly into account in the three-body system. Such a strong interplay between the S-wave and the neighboring higher partial wave interactions can be understood qualitatively in terms of the harmonic oscillator states. The forbidden states for the $\alpha\alpha$ interaction are represented by the lowest harmonic oscillator states, the $(0S)$ and $(1S)$ states in the S wave and the $(0D)$ state in the D wave. Embedded in the 3α system, a 2α pair tends to be in an S state in order to gain the important S-wave interaction. The lowest possible state consistent with the orthogonality condition is the $(2S)$ state with two nodes. If the two α's occupy this state, the three α's as a whole cannot occupy a lower state than the $(2S\text{-}2S)$ state, where the second "$2S$" refers to the motion of the third α particle with respect to the c.m. of the first two. This is because this three-body state is the lowest possible state consistent with the orthogonality conditions with respect to all three pairs. Now we change the Jacobi coordinates to the ones referring to the pair which consists of the third α particle and one of the first two. Then this three-body state contains a large amplitude in the $(1D\text{-}1D)$ *and* in the $(0G\text{-}0G)$ states as well as in the $(2S\text{-}2S)$ state, according to the Talmi-Moshinsky transformation coefficients. If the D- *or* G-wave interaction is missing, this means a loss of three-body binding because the kinetic energy of the three-body state (corresponding to eight quanta) is partly not compensated for by the interaction in this pair. This energy loss is a rather significant one since four quanta (to be attributed to the pair motion) represents the amount of kinetic energy which almost cancels out the deep S-wave attraction. This means that, in order to take proper account of the S-wave interaction in the three-body system, the D- and G-wave interactions should be present, and that they should be rather strong.

The above arguments also mean that strong D- and G-wave interactions are a necessary condition for realization of the strong 3α coupling by the forbidden state potential. In terms of the interplay between the S-, D-, and G-wave interactions, the most ideal situation is, of course, a common Hooke's law potential where the $(2S)$, $(1D)$, and $(0G)$ states become degenerate. A close physical situation is, e.g., a common Gaussian potential, and many forbidden state $\alpha\alpha$ potential models indeed employ this choice. (See also Ref. 26.)

In the case of repulsive core potential, the repulsive core strongly reduces the amplitude of three-body motion at short two-body distances. This circumstance can be described by generalizing the exponential part of the harmonic oscillator functions into another form of Jastrow function, $u(r_{12})u(r_{23})u(r_{31})$, where the function $u(r)$ is small for small values of r. We found that, by this extension for the $(0S\text{-}0S)$, $(0D\text{-}0D)$, and $(0G\text{-}0G)$ states, the 3α ground state under the Ali-Bodmer potentials can be solved rather accurately.[16] In particular, the (simple) Jastrow function space gives an accurate wave function when only the S-wave interaction is considered, and not when the D- or G-wave interaction is also included. This is because a characteristic feature of the $(0S\text{-}0S)$ state is preserved in this function space, namely, to contain a predominant component only in the S wave in all three pairs, in favor of taking proper account of the S-wave interaction. Our calculation also reproduced a feature of the previous results that the effect of including the interaction on the 3α ground state energy gets smaller as we go from the S- to the D- and to the G-wave interactions.

The important roles of both the D- and G-wave interactions in the forbidden state $\alpha\alpha$ potential model come from the fact that the 2α wave function has two orthogonality nodes. They are taken over by the D-wave interaction in the forbidden state NN potential model. So the question of whether the forbidden state NN potential model can give a strong 3N binding concerns the strengths of the NN D-wave interactions.

To estimate the effect of the D-wave interaction contained in the KPKS $^3S_1$-$^3D_1$ potential, it is not important to treat this potential exactly. The $^3S_1$-$^3D_1$ coupling potential works mainly in producing a predominant $^3S_1$ component in each nucleon pair. If we assume that the triton is mainly in the zero total orbital angular momentum state, such a nodal $^3S_1$ component (and the $^1S_0$ component) of a pair inevitably gives rise to a significant amount of short ranged $^3D_1$ and other D-wave components in the other nucleon pairs. The question is whether the strength of the NN attractions is sufficient to compensate for the kinetic energies coming from these D-wave components. The contribution to the potential energy from the $^3S_1$-$^3D_1$ coupling potential is incorporated into our effective $^3S_1$ potential. What remains to be considered are the interaction in the $^3D_1$ state and the interactions in the other D-wave states. The attraction in the $^3D_1$ state is very small. In the case of the KPKS $^3S_1$-$^3D_1$ potential, the node required in this state represents this situation. This seems to be why Hahn *et al.* obtained a very similar degree of "less" triton binding to ours (of Sec. II).

A typical realistic triton calculation includes the $^1S_0$ and the $^3S_1$-$^3D_1$ potentials only. This is connected to the fact that, in the case of the RSC potential, for instance, inclusion of all the remaining partial wave interactions with $J \leq 2$ ($J \leq 4$) changes the triton binding energy from 7.02 to 7.23 MeV (7.35 MeV) only.[4(b)] But from the argument above, it is possible that such a feature is characteristic of the repulsive core NN potential model. We can expect a larger contribution from these "remaining" partial wave interactions in a NN potential model which assumes a forbidden state in the S-wave states. To estimate an upper bound for the effect of the D-wave interactions on the triton binding energy, we consider the most attractive D-wave interaction. This is the interaction in the $^3D_2$ state. Since KPKS did not determine a potential for this interaction, we adopt the RSC $^3D_2$ potential. We add it in its UPA to the S-wave interaction models of Sec. II. Thereby we assume that it acts to all D-wave states state-independently. Table III shows the results. We see that the inclusion of this D-wave potential brings about a significantly larger gain to the triton binding energy when it is combined with the KPKS $^3S_1$ potential than when it is combined with the RSC $^3S_1$ potential. This shows that

TABLE III. The triton binding energy $-E_t$ (in MeV) calculated in a simple $S+D$ interaction model. (See the text.)

Potential		
S	D	$-E_t$ (MeV)
KPKS($^3S_1$)	RSC($^3D_2$)	14.47
RSC($^3S_1$)	RSC($^3D_2$)	18.80

the interplay between the S- and D-wave interactions is important in the forbidden state NN potential model also. However, we still do not get a stronger 3N binding with the KPKS $^3S_1$ potential than with the RSC $^3S_1$ potential, in contrast to the situation for the 3α system. The two values for the triton binding energy are still far away from each other. Since the RSC $^3D_2$ potential assumes a repulsive core at short range, we also examined a purely attractive $^3D_2$ potential with the two-range Gaussian form. With such a potential shape, however, we could not get a satisfyingly "realistic" fit to the experimental $^3D_2$ phase shift. So we made it a rule that the potential gives a larger phase shift than the experimental one at all energies below $E_{lab}=400$ MeV, and searched some best parameter sets to simulate the experiment. Thus the resulting $^3D_2$ potentials overestimate the effect of the $^3D_2$ interaction rather than underestimate it. However, we could not get a single result which differs from Table III appreciably. These results strongly indicate that the strengths of the NN D-wave interactions are not sufficient to support a "strong" 3N binding in the forbidden state NN potential model. On the contrary, they indicate that a realistic NN potential which induces a considerable short distance node in the 2N wave function will give a significantly smaller triton binding than a phase-equivalent repulsive core NN potential.

V. SUMMARY AND CONCLUSION

The first motivation for the present study was the wish to confirm a previous Faddeev result that a realistic NN potential model presupposing a deep short ranged attraction and a forbidden state leads to a significant underbinding for the triton, even compared with the results obtained by the conventional repulsive core NN potentials. The previous Faddeev calculation is rather complicated due to realistic treatment of the potential and, therefore, highly ranked separable expansions. For this aim we derived an effective $^3S_1$ potential which keeps the deuteron binding energy and (the $^3S_1$ component of) the deuteron wave function which the forbidden state NN potential (the KPKS potential) determines. We also derived a similar $^3S_1$ potential from the RSC potential, and compared these two $^3S_1$ potentials in how strongly they bind three nucleons, in a simple S-wave interaction model. This comparison reproduced the "less" triton binding feature of the KPKS potential. In particular, the ratio of the triton binding energies between the KPKS and the RSC potential (59%) was approximately the same as that obtained in the previous calculation (61%). Thus the previous calculation is confirmed.

Then, to clarify the reason, we asked whether it is a general feature of a short distance node required by the orthogonality condition to decrease the triton binding energy. We varied the KPKS $^3S_1$ potential and the forbidden state such that they generate a systematic change in the short distance behavior of the deuteron wave function with respect to the node position and the inner amplitude. We found that the triton binding energy always gets smaller when we increase the node behavior and vice versa. On the other hand, we could not obtain a triton binding energy larger than the RSC value in a rather wide range of variation. Combined with a similar previous result obtained through an off-shell variation of the RSC potential, these results indicated that the "less" triton binding is essentially connected to the node behavior.

An analysis of previous 3α results indicated that there exists a strong interplay between the S-wave and the neighboring higher partial wave interactions in the forbidden state potential model. Generally, when a two-body system is embedded in a three-body system, the two bodies try to gain the two-body interaction and, consequently, they tend to be in a relative motion state which is not much different from their free motion state. This is particularly so at short distances where the two-body correlation is strongest. When the short ranged repulsion in the S state is represented by orthogonality to a compact state with small radius, the resulting short distance node in the two-body wave function is such that it raises a non-negligible amount of neighboring higher partial wave components of a similar range. This is due to the three-body angular momentum coupling kinematics. These components contribute to the kinetic energy significantly and, in the absence of the interactions in these partial waves, decrease the three-body binding energy. To compensate for these kinetic energies, a similar strength of attraction as the S-wave one is necessary. We can phrase such a circumstance as "strong higher partial wave interactions are a promise for forbidden state S-wave potential."

The question of whether a forbidden state S-wave potential can give a "strong" 3N binding concerns particularly the strengths of the NN D-wave interactions. To estimate an upper bound for the effect of these interactions on the triton binding energy, we considered the most attractive one, i.e., the $^3D_2$ interaction. We adopted the RSC $^3D_2$ potential and added it to the S-wave interaction model with the KPKS $^3S_1$ potential and to the S-wave interaction model with the RSC $^3S_1$ potential. Thereby we assumed that it acts on all the D-wave states state-independently. As we expected, the inclusion of this D-wave interaction brought about a significant gain in the triton binding energy in the case of the KPKS $^3S_1$ potential (by 40%), while that gain for the RSC $^3S_1$ potential was much smaller (by 5%). However, the resulting triton binding energy for the KPKS $^3S_1$ potential is still below 80% of that for the RSC $^3S_1$ potential, showing that the NN D-wave interactions are much too weak to support a "strong" 3N binding.

To conclude, the present results indicate that a forbidden state NN potential model which fits the 2N data *and* induces a noticeable S-wave short distance node gives a significantly smaller triton binding energy than an on-shell equivalent repulsive core NN potential model gives. In order to explain the experimental triton binding energy with such a forbidden state NN potential, we must expect a significant three-body force contribution of the order of several MeV. In this sense, a noticeable short distance node for the 2N wave function is unlikely.

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