

Elastic scattering of polarized protons from ^3He at intermediate energies

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Differential cross sections and analyzing powers for proton elastic scattering from ^3He have been measured for proton scattering angles between 15° and 150° in the center of mass at incident proton energies of 200, 300, 415, and 515 MeV. These data, together with data from the literature, in the energy range 100–1000 MeV, have been analyzed within the framework of the Glauber multiple scattering theory. Firstly, a simple spin-independent calculation was performed using a parametrized form for the N-N scattering amplitudes. This provided reasonable agreement with the differential cross section data. Secondly, a more detailed calculation was performed incorporating spin dependence and using a complete set of N-N scattering amplitudes as determined from phase shift analyses. The agreement with the experimental data was not improved by the more detailed calculation. Possible reasons for this discrepancy are discussed.

I. INTRODUCTION

The study of few-nucleon systems at intermediate energies can be particularly useful in providing information regarding the nuclear interaction. With only a few nucleons present, the theoretical analysis is relatively simple and can be dealt with in more detail. Yet most aspects of the nuclear interaction are still present in such systems and can be examined with the aim to gain insight into more complex systems and reactions. Considerable data are now available at intermediate energies on the few-nucleon systems of ^2H and ^4He , and to a lesser extent ^3H and ^3He . Ideally, one would like to use the data for the N-N interaction and formalism which already exist to explain the nucleon-nucleus interaction. The multiple scattering theories of Glauber,¹ and Kerman, McManus, and Thaler² provide just such an approach. These theories make use of the free N-N scattering amplitudes and the wave function of the target nucleus. In the past, parametrized forms for the N-N scattering amplitudes and simple analytical forms for the wave functions have been used with encouraging success. However, these analyses have generally been limited to differential cross section data; the parametrization for a spin dependent calculation not being readily apparent.

In order to provide further data on the ^3He system measurements were made of the differential cross section and analyzing powers for elastic scattering of protons from ^3He at incident proton energies of 200, 300, 415, and 515 MeV. The angular distributions measured cover the range 15° to 150° in the center of mass. These data, together

with data available from the literature in the energy range 100–1000 MeV,^{3–10} were analyzed within the framework of the Glauber multiple scattering theory. Firstly, a simple spin independent calculation was performed using a parametrized form for the N-N interaction. This resulted in good agreement with the differential cross section data in general. Secondly, a more rigorous calculation was performed to include spin dependence using the scattering amplitudes derived from N-N phase shift analyses. Qualitative agreement was achieved between the calculated differential cross sections and analyzing powers and the data. However, the supposedly improved calculations generally underestimate the differential cross sections by 20–30% and are out of phase with the oscillations of the analyzing power data. Using N-N scattering amplitudes from different phase shift analyses had negligible effect on this discrepancy. Similarly, different forms for the ^3He wave function could not account for the disagreement. Some qualitative improvement could be made by the choice of off-shell corrections. In the next section a brief description of the experimental setup, data reduction, and error analysis will be presented. Section III will describe the results obtained and compare them with existing data available from the literature. The theoretical model and the analysis will then be discussed in Sec. IV followed by some conclusions.

II. THE EXPERIMENT

The experiment was performed using polarized proton beams accelerated by the TRIUMF cyclotron to energies

of 200, 300, 415, and 515 MeV. The beam position and direction were measured using wire profile monitors and by viewing a fluorescent screen which could be placed at the target position. The diameter of the beam at the target was about 5 mm. The incident proton beam polarization and integrated intensity were determined from the left and right counts derived from a polarimeter located upstream of the ^3He target. A description of the ^3He target system is given in Ref. 11. The polarimeter contained a 5 mg/cm^2 thick CH_2 target and was previously calibrated against a Faraday cup. Typical proton beam polarizations were 65–75% depending on the energy at currents ranging from 1 to 15 nA. During the experiment the polarization of the incident proton beam was automatically changed from “up” to “off” to “down” to “off” cyclically. The ^3He target thickness followed from measurements of the temperature of the target cell using germanium resistors embedded in the body of the cell and from the cell's physical dimensions measured with the same pressure differential. The target thicknesses so determined for the two target cells used in these measurements were 120 ± 5 and $104 \pm 4\text{ mg/cm}^2$. The target cell radii both were 22 mm. The target cell could be raised or lowered remotely, thus permitting the ^3He target cell to be replaced with an identical dummy cell in order to allow appropriate background subtractions. In addition the target cell could be rotated around a central vertical axis in order to ensure that the scattered particles would not strike the side frames.

The quadrupole-dipole magnetic spectrometer MRS was used to detect either the scattered protons or the recoil ^3He particles. The spectrometer detection system consisted of a 0.8 mm thick plastic scintillator plus a 0.128 m by 0.128 m multiwire proportional chamber (MWPC) (with 2 mm wire spacing). The MWPC located before the quadrupole magnet-dipole magnet combination was used to define the solid angle. Typical solid angles were ~ 1 msr. The dipole magnet was followed by a set of 0.256 m by 1.024 m MWPC's (with 2 mm wire spacing) mounted about the focal plane of the magnetic spectrometer and a 25 mm thick plastic scintillator. The flight path through the spectrometer is ~ 11.0 m. Using the MWPC information the track of the detected particle was reconstructed and thus its position in the focal plane of the spectrometer was determined. The momentum acceptance of the spectrometer was $+12\%$ to -10% of the central momentum. The energy resolution was $\sim 0.25\%$ which included effects due to the energy spread of the incident beam, kinematics, and multiple scattering in the target and the various covering foils. Protons were detected from forward angles to angles large enough that the corresponding recoil ^3He particles had sufficient energy to be detected (~ 140 MeV). By detecting the recoil ^3He particles a factor of ~ 3 was gained in the count rate due to the kinematic increase in the effective solid angle. In addition it was possible to perform the measurements at equivalent proton angles larger than the operating range of the magnetic spectrometer. The electronics for the experiment were interfaced via CAMAC to a data acquisition computer. Events satisfying specified limits for the time-of-flight and energy loss were logged on magnet-

ic tape for subsequent analysis. During the experiment an on-line data analysis was performed to check on the performance of the electronics and data acquisition system.

The yields of elastically scattered particles were calculated in the off-line reduction of the data by imposing more stringent tests and defining cuts. Events for which any of the MWPC planes had a missing wire coordinate or events corresponding to firing of nonadjacent wires (with a gap greater than one wire) were rejected together with those outside the solid angle defining cut. The latter was chosen to correspond to the region of flat acceptance of the magnetic spectrometer. The yields were corrected for electronics and computer deadtime, detector inefficiencies, and reaction losses. Typical relative uncertainties of the differential cross section measurements are $\sim 4\text{--}5\%$ and these include uncertainties in the MWPC's efficiency ($\sim 1\%$), in the computer dead time correction ($\sim 1\%$), in the correction for reaction losses (0.4% in the case of protons and 3% for ^3He), and uncertainties in the number of polarimeter counts used for normalization purposes ($\sim 1\%$), in the density of the ^3He ($\sim 3\%$), and finally the statistical error in the number of elastic scattering events observed. The absolute uncertainty in the normalization is $\sim 6\%$ and arises from uncertainties in the number of incident particles ($\sim 5\%$), in the target thickness ($\sim 1\%$), and in the solid angle ($\sim 1\%$). In addition the uncertainties in the detector angle is 0.1° and in the target rotation $\sim 2.5^\circ$. The yields were calculated for each spin orientation and used to determine differential cross sections and analyzing powers.

III. RESULTS

The angular distributions of the differential cross sections and analyzing powers for proton elastic scattering from ^3He at 200, 300, 415, and 515 MeV are shown in Figs. 1 and 2 (numerical values can be obtained in tabulated form from the authors). The differential cross sections of the present work vary smoothly with angle. The first minimum can be clearly seen and there is evidence for the onset of the second minimum also. These minima can be explained by the Glauber theory to be due to interference between the single, double, and triple scattering terms. The analyzing power angular distributions also vary quite smoothly and exhibit structure characteristic of interference of various scattering terms. Figure 3 shows the differential cross section data, as a function of the square of the four momentum transfer, for the present work along with data available from the literature in the energy range 100–1000 MeV. The variation with incident proton energy seems quite regular but at 415 MeV the data from the present experiment are 20–30% lower than the results of previous measurements [which were restricted to $-t < 0.8$ (GeV/c^2)]. This discrepancy is larger than the sum of the uncertainties in the absolute normalization of the two measurements. However, the data from the present work match up quite well with the far backward angle data of Ref 5. Near 600 MeV there are two sets of data available

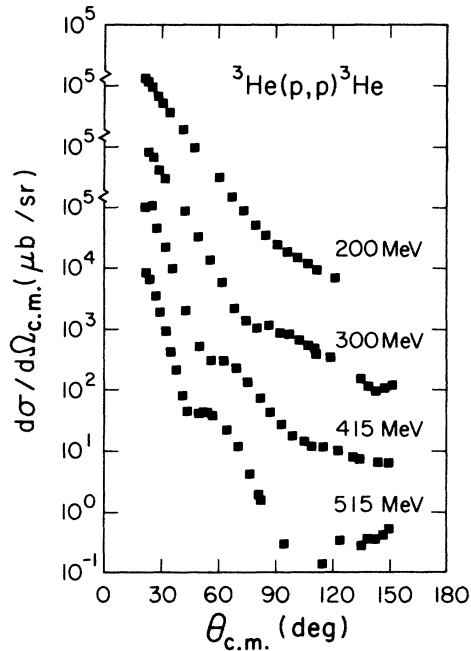


FIG. 1. Differential cross section angular distributions for proton elastic scattering from ${}^3\text{He}$ as measured in the present work. Relative errors are smaller than the size of the symbols.

from the literature. The 582 data of Ref. 6 are generally lower than the data of Ref. 8 at 600 MeV. At 1000 MeV there are also two independent measurements of the differential cross sections which are in fairly good agreement (Refs. 9 and 10).

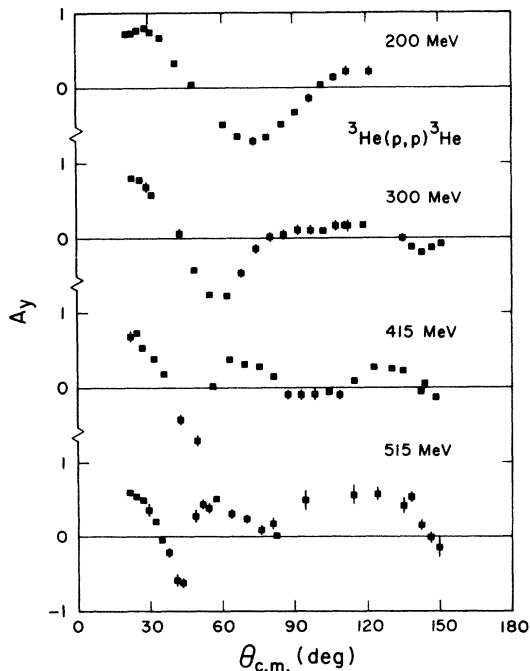


FIG. 2. Analyzing power angular distributions for proton elastic scattering from ${}^3\text{He}$ as measured in the present work. Relative errors are smaller than the size of the symbols unless shown.

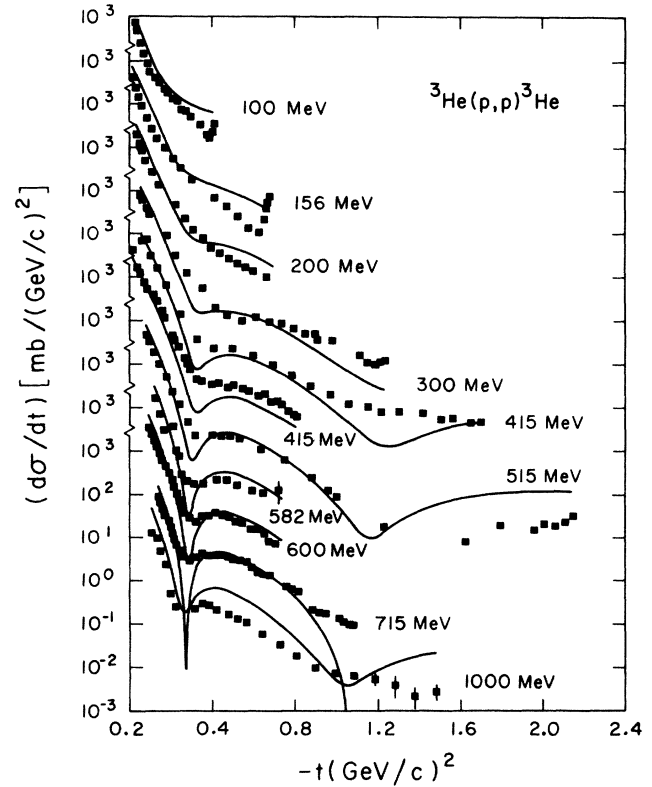


FIG. 3. Simple Glauber model calculations for the differential cross sections obtained in this work and available from the literature at intermediate energies.

IV. THEORETICAL ANALYSIS

The data obtained in the present work, together with the data available from the literature in the energy range (100–1000 MeV), were analyzed within the framework of the Glauber multiple scattering theory. This theory assumes incident energies T sufficiently high so that $T \gg V$, where V is a measure of the strength of the interaction, and $ka \gg 1$, where k is the wave number and a is the effective width of the potential. Then, for small angle scattering such that $\mathbf{k} - \mathbf{k}'$ is effectively perpendicular to \mathbf{k} , the scattering amplitude may be expressed as

$$f(\mathbf{k}', \mathbf{k}) = \frac{-ik}{2\pi} \int e^{i(\mathbf{k}-\mathbf{k}') \cdot \mathbf{b}} (e^{i\chi(\mathbf{b})} - 1) d^2b, \quad (1)$$

where \mathbf{k} and \mathbf{k}' are the incident and outgoing wave vectors, \mathbf{b} is the impact parameter vector perpendicular to \mathbf{k} , and χ is the total phase shift for the interaction. The total phase shift χ is taken to be the sum of the phase shifts due to the incident nucleon scattering from each nucleon of the target individually. Thus for ${}^3\text{He}$:

$$\chi(\mathbf{b}, \mathbf{s}_1, \mathbf{s}_2, \mathbf{s}_3) = \chi_1(\mathbf{b} - \mathbf{s}_1) + \chi_2(\mathbf{b} - \mathbf{s}_2) + \chi_3(\mathbf{b} - \mathbf{s}_3), \quad (2)$$

where \mathbf{s}_1 , \mathbf{s}_2 , and \mathbf{s}_3 are the projections of the target nucleon's coordinates on the impact parameter plane. Expressing

$$\Gamma = 1 - e^{i\chi} \quad (3)$$

(often referred to as the profile function) then for ${}^3\text{He}$ the total profile function can be written as

$$\Gamma = \Gamma_1 + \Gamma_2 + \Gamma_3 - \Gamma_1\Gamma_2 - \Gamma_1\Gamma_3 - \Gamma_2\Gamma_3 + \Gamma_1\Gamma_2\Gamma_3. \quad (4)$$

The terms for single, double, and triple scattering are clearly distinguished. The expression for the individual Γ 's can be obtained by applying Eq. (1) to the case of free N-N scattering.

$$\Gamma = \frac{1}{2\pi ik} \int e^{-iq \cdot (b-s)} f(\mathbf{q}) d^2q, \quad (5)$$

where \mathbf{q} is the momentum transfer, and $f(\mathbf{q})$ is the free N-N scattering amplitude. Substituting the above expression for the individual profile functions, and Eq. (4) into Eq. (1), results in the scattering amplitude for ^3He in terms of the free N-N scattering amplitudes. After substitution the single scattering terms can be expressed as

$$F_i(\mathbf{k}', \mathbf{k}) = \frac{ik}{2\pi} \int e^{i(\mathbf{k}-\mathbf{k}') \cdot \mathbf{b}} \frac{1}{2\pi ik_i} \times \int e^{-iq_i \cdot (b-s_i)} f_i(\mathbf{q}) d^2q_i d^2b. \quad (6)$$

The integration over d^2b results in a delta function which implies conservation of momentum. The integration over d^2q_i can then be performed with the result

$$F_i(\mathbf{k}', \mathbf{k}) = \frac{k}{k_i} f_i(\mathbf{q}) e^{iq \cdot s_i}; \quad \mathbf{q} = \mathbf{k} - \mathbf{k}'. \quad (7)$$

The double scattering terms can be handled in the same manner and after the integration over d^2b and one of the momentum transfers, d^2q_i , the result is

$$F_{ij}(\mathbf{k}', \mathbf{k}) = \frac{k}{2\pi ik_i k_j} \int e^{iq \cdot s_i} e^{iq_j \cdot (s_j - s_i)} \times f_i(\mathbf{q} - \mathbf{q}_j) f_j(\mathbf{q}_j) d^2q_j. \quad (8)$$

Similarly, the expression for the triple scattering term becomes

$$F_{123}(\mathbf{k}', \mathbf{k}) = -\frac{k}{4\pi^2 k_1 k_2 k_3} \int \int e^{iq \cdot s_1} e^{iq_2 \cdot (s_2 - s_1)} e^{iq_3 \cdot (s_3 - s_1)} f_1(\mathbf{q} - \mathbf{q}_2 - \mathbf{q}_3) f_2(\mathbf{q}_2) f_3(\mathbf{q}_3) d^2q_2 d^2q_3. \quad (9)$$

At this point the scattering amplitude for ^3He can in principle be calculated. However, the integrations over the momentum transfers pose a considerable numerical problem. If parametrized forms for the N-N scattering amplitudes are used, the integrations over the momentum transfer in the double and triple scattering terms can be done analytically. However, when this is not the case it is often expedient to remove the scattering amplitudes from the integration. This is usually done by assuming the scattering amplitudes to be constant with an effective value evaluated at one-half or one-third the momentum transfer. This approximation does not effect the single scattering amplitude of Eq. (7) but the double and triple scattering amplitudes become

$$F_{ij}(\mathbf{k}', \mathbf{k}) = -\frac{2\pi ik}{k_i k_j} f_i(\frac{1}{2}\mathbf{q}) f_j(\frac{1}{2}\mathbf{q}) e^{iq \cdot s_i} \delta(s_i - s_j) \quad (10)$$

and

$$F_{123}(\mathbf{k}', \mathbf{k}) = -\frac{4\pi^2 k}{k_1 k_2 k_3} f_1(\frac{1}{3}\mathbf{q}) f_2(\frac{1}{3}\mathbf{q}) f_3(\frac{1}{3}\mathbf{q}) \times e^{iq \cdot s_1} \delta(s_1 - s_2) \delta(s_1 - s_3), \quad (11)$$

respectively.

In the first stage of the analysis the N-N scattering amplitudes were parametrized as

$$f_{p,j} = \frac{\sigma_{p,j}^T}{4\pi} k (i + \alpha_{p,j}) e^{\beta_{p,j} t}; \quad j = p, n, \quad (12)$$

where σ^T is the total cross section, k is the incident momentum, α is the ratio of the real to the imaginary part of the scattering amplitude at 0° , β is the slope parameter, and $-t$ is the square of the four momentum transfer. The values of σ^T , α , and β are given in Table I (Ref. 12). The ground state wave function for ^3He was taken as

TABLE I. Scattering parameters as used in the simple Glauber calculation.

T_p (MeV)	σ_{pp}^T (fm) ²	$1/c = R = 1.50$ fm			α_{pn}	β_{pn} (GeV/c) ⁻²
		α_{pp}	β_{pp} (GeV/c) ⁻²	σ_{pn}^T (fm) ²		
100.0	2.86	0.504	1.897	7.64	8.442	1.036
156.0	2.30	0.175	1.630	4.89	6.935	0.907
200.0	2.19	-0.068	1.323	4.10	5.199	0.686
300.0	2.23	-0.093	0.843	3.48	2.530	0.295
415.0	2.67	0.249	0.577	3.40	1.548	0.047
515.0	3.40	0.753	0.413	3.57	1.560	-0.076
582.0	3.88	1.191	0.302	3.71	1.679	-0.142
600.0	4.00	1.110	0.275	3.75	1.712	-0.158
715.0	4.55	1.949	0.117	3.92	1.945	-0.256
1000.0	4.75	1.990	-0.144	4.02	2.133	-0.518

$$\psi = Ne^{-c^2(r_1^2 + r_2^2 + r_3^2)}, \quad (13)$$

where N is the normalization constant, $1/c = R = 1.50$ fm is the rms radius for ${}^3\text{He}$, and the r_i 's are the nucleon coordinates within the ${}^3\text{He}$ nucleus. Using these expressions the integrations over the momentum transfer and nucleon coordinates can be performed, analytically. A general expression for the differential cross section for nucleon-nucleus scattering is given in Ref. 13.

The results of this rather simple calculation are shown in Fig. 3. The theoretical predictions overestimate the differential cross sections at lower energies and at 1000 MeV but generally agree quite well with the data. The positions of the minima are reproduced but not their depths. It should be noted that this parametrized form for the N-N scattering amplitudes does not contain any spin dependence, thus spin flip processes which might serve to fill in the minima are not considered. Similarly, charge exchange is not included in the calculation. Off-shell corrections were not incorporated at this stage. The same simple approach was previously used for comparison with the experimental data at 580 and 600 MeV (Refs. 6–8) with satisfactory results as well.

In the second stage of the analysis N-N scattering amplitudes¹² were used incorporating their complete spin and isospin dependence. Assuming isotopic invariance the N-N scattering amplitudes can be expressed in terms of two 4×4 matrices in spin space corresponding to isospin singlet and isospin triplet scattering. Each matrix is composed of five independent, complex amplitudes. To simplify the calculation the approximations employed in deriving Eqs. (10) and (11) were used. This has the effect of removing the scattering amplitudes from the integration and replacing them by effective amplitudes dependent only on the momentum transfer, q , and the spin-isospin components of the ${}^3\text{He}$ wave function.

Only the ground state, S wave function for ${}^3\text{He}$ was used in the analysis. This wave function consists of a totally asymmetric spin-isospin component and a symmetric space component. With the N-N scattering amplitudes removed from the integral and using these simple wave functions the integration over the space coordinates (performed in calculating the expectation values) can be done analytically to produce a form factor for each of the single, double, and triple scattering terms. No attempt was made at this point to derive analytic expressions for the differential cross section and analyzing power. Instead two computer programs were used to evaluate the expectation values for the single, double, and triple scattering terms, multiplied by the corresponding form factors, for all the possible initial and final spin-isospin combinations. The differential cross sections and analyzing powers were then calculated. Spin flip and charge exchange processes were allowed in these calculations. Using only the ground state, S wave function for ${}^3\text{He}$, spin flip is allowed for the neutron but the proton spins can only be flipped during double or triple scattering and then only when both change. Charge exchange is allowed within the double and triple scattering processes. As expected, these processes had the effect of filling in the minima of the differential cross sections and reducing the magnitude of the

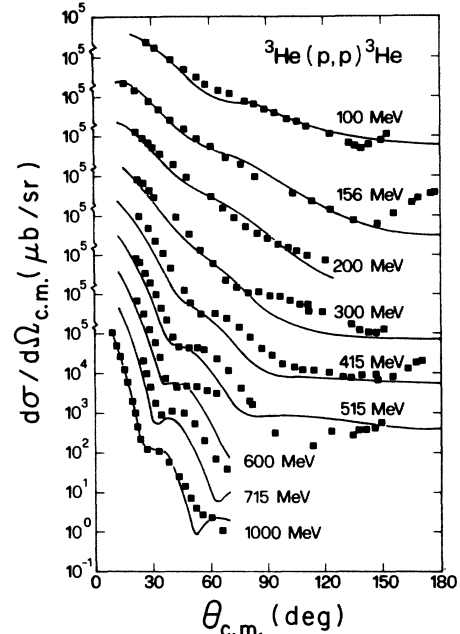


FIG. 4. Complete spin dependent Glauber calculations for the differential cross sections obtained in this work and available from the literature at intermediate energies.

extrema in the analyzing powers.

The results of these calculations are shown in Figs. 4 and 5. The polarization data of Fig. 5 at 156 MeV are from Ref. 14. The agreement with the lower energy differential cross section data is quite good. This is a significant improvement over the simple Glauber calculations which overestimate the experimental differential cross sections. However, as the incident energy increases, the theoretical predictions considerably underestimate the experimental data. The only exceptions are the 1000 MeV

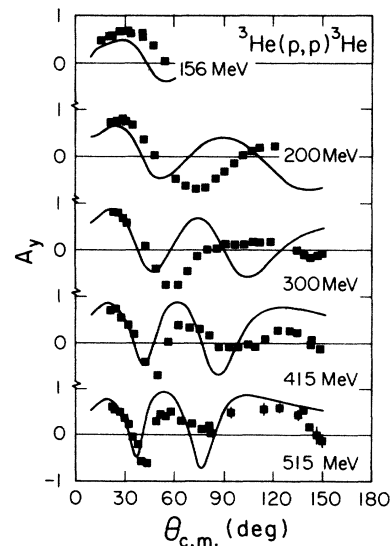


FIG. 5. Complete spin dependent Glauber calculations for the analyzing powers obtained in this work and available from the literature at intermediate energies.

comparisons. The positions of the minima given by the theoretical predictions are at smaller angles than experimentally measured. The calculated analyzing power angular distributions have their forward maxima in good agreement with the experimental data but the calculated values go out of phase with the oscillations in the experimental data at larger angles. On average the results of the theoretically more complete calculations for the differential cross sections are significantly poorer than those of the simple Glauber calculations. The disagreement at forward angles is particularly troublesome as the approximations should hold best in this region.

In order to study these discrepancies the contributions of the various terms which make up the calculations were examined individually. For this investigation an energy of 415 MeV was chosen, being the worst case in terms of agreement with experiment. Also the experimental differential cross section and analyzing power data at this energy cover a broad angular range. Figure 6 illustrates the individual contributions of the single, double, and triple scattering terms. The single scattering contribution agrees well with the forward angle differential cross section data. It appears though that the double scattering contribution, while being significantly smaller, causes considerable interference effects in this region. The triple scattering term similarly interferes in the region of the second maximum and dominates at far backward angles.

The effect of using two different forms of the ^3He wave function was next examined. A comparison was made with the predictions using the ^3He wave function of Ref. 15 which is given by a sum of four Gaussians. Although this wave function better reproduces the ^3He charge form factor than the simple Gaussian wave function, the latter gives Glauber predictions in slightly better agreement with the data. Off-shell corrections in a form given by Chew¹⁶ and as used in Ref. 15 were also calculated: The

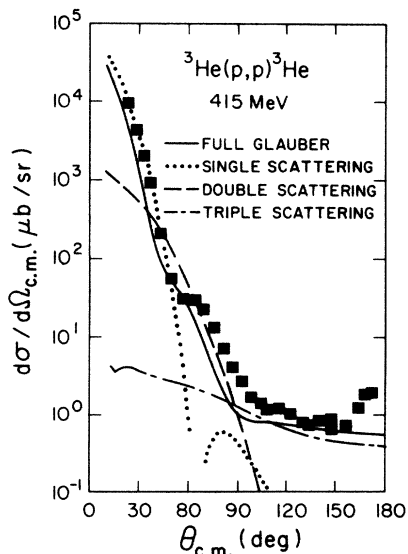


FIG. 6. Single, double, and triple scattering contributions to the differential cross section at 415 MeV.

shoulder in the curve at $\sim 100^\circ$ as well as the back angle data are better reproduced with these corrections, but the region around 60° is better reproduced by the calculation without the off-shell corrections. However, the analyzing power data show that the off-shell corrections enhance the depths of the minima. Spin-flip and charge exchange processes appear to explain the filling in of the minima. This was clearly demonstrated by the 415 MeV calculations when these processes were removed from the full calculations.

Glauber calculations using N-N scattering amplitudes derived from phase shift analyses were performed by Bizard and Osmont¹⁷ for the 415 and 600 MeV data and by Frascaria *et al.*¹⁰ for the 600 and 1000 MeV data. Note that the present data cover a much larger range of the squared momentum transfer variable $-t$. The predictions at 415 and 600 MeV are lower than the experimental data while the 1000 MeV predictions agree with the data. The same features are observed in the present analysis, although more recent N-N scattering amplitudes were used. This appears to be consistent with the observation that the N-N scattering amplitudes derived from free N-N scattering are too small to reproduce nucleon-nucleus scattering observables; an observation earlier made in comparisons of Glauber and optical model analyses of 800 MeV proton elastic scattering from different nuclei.¹⁸ The use of a more realistic ^3He wave function could be considered. Yet a previous analysis made by Narboni¹⁹ with different ^3He wave functions including S' and D state terms did not explain the disparity between theory and experiment. Similarly, improved off-shell corrections could be used. The introduction of intermediate nucleon resonances in the calculations should only be of importance at the higher energies.²⁰ Recently it has been shown²¹ that a relativistic treatment of nucleon-nucleus scattering starting with the Dirac equation gives much better agreement with experiment. However, a formulation for the scattering of nucleons from a spin $\frac{1}{2}$ target nucleus does not exist yet. The Glauber formulation as used here has assumed form factors independent of the spin and isospin of the scattering nucleons. Certainly the neutron distribution within ^3He will be different from that of the protons and also the spin distributions should be considered to be different. Such a formulation would require form factors dependent upon the spin and isospin rather than the simple matter distributions which have been used in the present work. A breakdown of the form factor in the various spin and isospin components has been used by Landau and Páez²² in a microscopic, momentum space optical potential calculation for p - ^3He scattering which incorporates full spin-dependence of the N-N and p - ^3He interactions and which uses antisymmetrized N-N amplitudes. The t matrices have on-shell behavior given by the N-N phase shifts and off-shell behavior from a realistic separable potential model. Qualitative agreement was found with the experimental data in the whole energy range 100–1000 MeV. Indications are that it would be possible to obtain better agreement with the data by varying the various form factors. However, verification of the validity of the form factors by an independent method would be required.

V. CONCLUSION

The success of the simple Glauber model calculations over the more complete calculations for the differential cross sections implies that the parametrized N-N scattering amplitudes act as effective scattering amplitudes in the simple approach. Certainly the constant ratio of the real to the imaginary parts of the scattering amplitudes used in those calculations limits the degree of interference between the first- and second-order scattering which appears to be the problem with the more complete calculations. Improvements to the more complete calculations by explicitly carrying out the integration over the momentum transfer variable q rather than assuming an average value may give better agreement with experiment. Con-

sideration should also be given to formulate a relativistic description of nucleon scattering from a non-spin-zero target nucleus such as has been done with considerable success for nucleon scattering from a spin-zero target nucleus. Effects related to different ${}^3\text{He}$ wave function forms, and excitation of nucleon resonances, would likely be too small to account for the discrepancies observed in comparing the present calculations with the experimental data.

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