Comparison of approximate formalisms for inclusive breakup reactions

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A coupled-channels Born approximation formalism for breakup reactions recently proposed by Udagawa and Tamura is compared with the distorted-wave Born approximation formalism discussed by Ichimura, Austern, and Vincent. Udagawa and Tamura claimed that the distorted-wave Born approximation method erroneously includes pure inelastic scatterings. We show that their argument is invalid and that there is no asymptotically observable inelastic scattering in the distorted-wave Born approximation breakup formalism.

A recent paper by Udagawa and Tamura (UT) (Ref. 1) presents a coupled-channels Born approximation (CCBA) formalism for breakup-fusion reactions. In comparisons with the DWBA formalism discussed by Ichimura, Austern, and Vincent (IAV),[2] UT assert that the IAV approach erroneously includes mere inelastic scattering as part of its breakup-fusion cross section. We again compare the two methods; we show the limitations of the UT method and the absence of inelastic scattering in the IAV method.

Much of the discussion by UT arises from their concern about nonorthogonality of the basis states in a coupled channels (CC) model of a rearrangement reaction. However, this is not a problem of fundamental reaction theory. Although nonorthogonality causes real difficulties in dynamical calculations, it does not affect the structure of formal expressions for cross sections and there is no difficulty in distinguishing breakup from inelastic scattering. The CCBA formalism of UT has no relation to this distinction; they themselves advocate a return to DWBA for actual calculation. (See their subsections II D and II E.)

The definition of *breakup fusion* in Ref. 1 differs from the old one of Udagawa *et al.*,[3] adopted by IAV.[2,4] The old definition actually means nonelastic inclusive breakup, while the new one of UT excludes both the elastic and inelastic parts of the inclusive breakup cross section. To avoid confusion, we avoid these distinctions and discuss only the overall inclusive breakup

$$a+A\rightarrow b+\text{anything} , \tag{1}$$

where b is a directly emitted fragment of a $(=b+x)$ and the remaining system $(x+A)$ is unbound.

Since Refs. 1 and 2 do not use identical notation, and they sometimes introduce different meanings for the same notation, we start with a brief review of the theory and notation.

In the spectator model used by both UT and IAV, the total Hamiltonian is given by

$$H=H_A+T_b+T_x+V_{xA}+V_{xb}+U_b , \tag{2}$$

where H_A is the internal Hamiltonian of the target A, T_b and T_x are the kinetic energy operators of b and x, and V_{ij} is the microscopic interaction between i and j. The microscopic spectator-nucleus interaction V_{bA} has been replaced by an optical potential U_b. It is assumed that A is infinitely massive and b and x are structureless. The wave function and energy of the nth bound state A_n of the target are denoted by Φ_A^n and e_A^n, respectively, and $n=0$ denotes the ground state. The system $(b+x)$ is assumed to have only one bound state whose wave function and energy are written as ϕ_a and e_a, respectively. The eigenstates $\Psi_{xA}^{(-)c}$ of the system $(x+A)$ satisfy the equation

$$H_{xA}\Psi_{xA}^{(-)c}=E^c\Psi_{xA}^{(-)c} , \tag{3}$$

with

$$H_{xA}=H_A+T_x+V_{xA} . \tag{4}$$

The optical model wave functions of relative motion in the $(a+A)$, $(b+A)$, and $(x+A)$ channels satisfy the equations

$$[T_a+U_a-E_a]\chi_a^{(+)}(E_a)=0 ,$$

$$[T_b+U_b^{\dagger}-E_b]\chi_b^{(-)}(E_b)=0 ,$$

$$[T_x+U_x^{\dagger}-E_x]\chi_x^{(-)}(E_x)=0 , \tag{5}$$

respectively. UT introduce the coupled channel wave function

$$\Psi_P=P\Psi_a^{(+)}(E)=\Sigma_n u_n(\mathbf{r}_a)\phi_a\Phi_A^n , \tag{6}$$

where $\Psi_a^{(+)}(E)$ is the eigenstate of the total Hamiltonian (2) with the incident wave in the channel $a+A$ with total energy E. The projection operator P is given by

$$P=|\phi_a\rangle P_A\langle\phi_a|\,,\quad P_A=\Sigma_n|\Phi_A^n\rangle\langle\Phi_A^n|\,. \tag{7}$$

The CC equation for Ψ_P is written as

$$[H_A+T_a+U_P+e_a-E]\Psi_P=0\,, \tag{8}$$

where

$$U_P=P(V_{xA}+U_b)P+PHQ(E-QHQ+i\epsilon)^{-1}QHP\,, \tag{9}$$

with $Q=1-P$.

The DWBA of IAV expresses the differential inclusive breakup cross section in prior form as

$$\frac{d^2\sigma^{\rm IAV}}{d\Omega_b dE_b}=\frac{2\pi}{v_a}\rho(E_b)\Sigma_c|\langle\chi_b^{(-)}\Psi_{xA}^{(-)c}|V_a|\chi_a^{(+)}\phi_a\Phi_A^0\rangle|^2\delta(E-E_b-E^c) \tag{10}$$

$$=-\frac{1}{\pi}\frac{2\pi}{v_a}\rho(E_b)\,{\rm Im}\langle\chi_a^{(+)}\phi_a\Phi_A^0|V_a^\dagger|\chi_b^{(-)}\rangle(E-E_b-H_{xA}+i\epsilon)^{-1}\langle\chi_b^{(-)}|V_a|\chi_a^{(+)}\phi_a\Phi_A^0\rangle\,, \tag{11}$$

where $\rho(E_b)$ is the density of states of particle b and

$$V_a=V_{xA}+U_b-U_a\,. \tag{12}$$

We note that the above U_a is the optical potential of the elastic channel, whereas the U_a of UT is the diagonal element of the CC potential U_P in the elastic channel. The coefficient $(2\pi)^4$ in IAV is replaced by (2π) in (10).

UT prefer to discuss the energy and angle integrated inclusive breakup cross section. For comparison we therefore integrate (11) over the energy and angles of particle b and use the completeness properties of the $\chi_b^{(-)}$ functions. (In this step we follow Sec. III of UT and assume U_b is real and energy independent.) The eigenvalue E_b in the Green's function is replaced by the corresponding operator T_b+U_b, to give

$$\sigma_b^{\rm IAV}=\int\frac{d^2\sigma^{\rm IAV}}{d\Omega_b dE_b}d\Omega_b dE_b$$

$$=-\frac{1}{\pi}\frac{2\pi}{v_a}\,{\rm Im}\langle\chi_a^{(+)}\phi_a\Phi_A^0|V_a^\dagger G_d' V_a|\chi_a^{(+)}\phi_a\Phi_A^0\rangle\,, \tag{13}$$

where

$$G_d'=(E-H_d'+i\epsilon)^{-1}\,, \tag{14}$$

$$H_d'=H_{xA}+T_b+U_b=H_A+T_x+V_{xA}+T_b+U_b\,. \tag{15}$$

On the other hand, UT start with the integrated inclusive breakup cross section [Eq. (2.24) of Ref. 1]

$$\sigma_b^{\rm UT}=-\frac{1}{\pi}\frac{2\pi}{v_a}\,{\rm Im}\langle\Psi_P|V_P^\dagger G_d V_P|\Psi_P\rangle\,, \tag{16}$$

with

$$V_P=V_{xA}+U_b-U_P\,, \tag{17}$$

$$G_d=(E-H_d+i\epsilon)^{-1}\,, \tag{18}$$

$$H_d=H_A+T_x+U_x+T_b+U_b\,, \tag{19}$$

obtained by their starting approximation $(E-H+i\epsilon)^{-1}\approx G_d$ [Eq. (2.22) of Ref. 1]. For consistency of comparison with (13) the present discussion must suppose that U_b in $\sigma_b^{\rm UT}$ is real and energy independent.

Comparing Eqs. (13) and (16), one sees three major differences between the IAV and UT formulas.

1. The initial state is expressed by the optical model wave function $\chi_a^{(+)}\phi_a\Phi_A^0$ in IAV, but by the CC wave function Ψ_P in UT. By using the CC initial state wave function, UT include *two-step* inelastic breakup mechanisms, $a+A\to a+A^*\to b+x+A^*$.

2. The intermediate Green's function is G_d' in IAV, with the full V_{xA} interaction, but it is the optical Green's function G_d in the UT formula. This difference comes from the UT starting approximation, in which the microscopic interaction V_{xA} in the full Green's function for (2) simply is replaced by the optical potential U_x in G_d. This replacement, without derivation, loses the extra terms that IAV (Sec. IV) obtain by their exact optical reduction of (13). These important terms guarantee post-prior symmetry and they help to include breakup that is *simultaneous* with target nucleus excitation.

3. The interaction responsible for breakup is given by V_a of (12) in IAV but by V_P of (17) in UT. Namely, the potential subtracted is the optical potential U_a in IAV, but the CC potential U_P in UT. This is an automatic result of the different choice of the unperturbed Hamiltonian in the initial channel.

We comment first regarding the second difference in the above list: The exact optical reduction of IAV, Sec. IV can be applied to the CCBA formula of UT as easily as to the DWBA formula of IAV. The optical reduction

$$(\Phi_A|(E^+-E_b-H_{xA})^{-1}|\Phi_A)=(E_x-T_x-U_x)^{-1}$$

is generalized in the form

$$P_A(E^+-E_b-H_{xA})^{-1}P_A=(E^+-E_b-H_A-T_x-\hat{U}_x)^{-1}$$
$$\equiv\hat{G}_x\,, \tag{20}$$

where $\hat{U}_x$ is the effective interaction in CC calculations for the $x+A$ system within the P_A space. The U_x of UT is understood as the diagonal part of $\hat{U}_x$. The nonorthogonality term and the source function are now given by

$$|\tilde{n}\rangle=(\chi_b^{(-)}|\Psi_P\rangle=\Sigma_n n_n(\mathbf{r}_x)\Phi_A^n\,,$$
$$|\tilde{\rho}_a\rangle=(\chi_b^{(-)}|U_b+\hat{U}_x-U_P|\Psi_P\rangle=\Sigma_n\rho_n(\mathbf{r}_x)\Phi_A^n\,. \tag{21}$$

The missing terms of the CCBA formalism of UT are then

$$2\,\mathrm{Re}\langle\tilde{n}\,|\,\hat{W}_x\hat{G}_x\,|\,\tilde{\rho}_a\rangle+\langle\tilde{n}\,|\,\hat{W}_x\,|\,\tilde{n}\rangle\ , \tag{22}$$

where $\hat{W}_x=(\hat{U}_x-\hat{U}_x^\dagger)/(2i)$. These correspond to the second and third terms of Eq. (4.14) of IAV, respectively.

By the step from DWBA to CCBA, UT include in principle a part of the breakup that is simultaneous with target nucleus excitation. The major difference between use of the Green's functions G_d' and G_d is, finally, that IAV treat the final state wave functions of the $(x+A)$ system exactly, but UT deal with them implicitly through the approximate optical potential Green's function G_d.

UT consider the third of the above differences to be of major importance. They claim that the use of V_a includes mere inelastic scatterings in the inclusive breakup formula, but that this does not happen if V_P is used. Their argument is the following: They apply the projection operator P to select a component of the integrated IAV cross section (13),

$$\sigma_P^{\mathrm{IAV}}\equiv-\frac{1}{\pi}\frac{2\pi}{v_a}\,\mathrm{Im}\langle\chi_a^{(+)}\phi_a\Phi_A^0\,|\,V_a^\dagger PG_d'PV_a\,|\,\chi_a^{(+)}\phi_a\Phi_A^0\rangle\ . \tag{23}$$

The projected Green's function is converted to the form

$$PG_d'P\equiv G_a^{\mathrm{eff}}=[E_a+\langle\phi_a\,|\,V_{xb}\,|\,\phi_a\rangle-H_A-T_a-U_a^{\mathrm{eff}}+i\epsilon]^{-1}\ , \tag{24}$$

with $E_a=E-e_a$, and with

$$U_a^{\mathrm{eff}}=P(V_{xA}+U_b)P+PH_d'Q(E-QH_d'Q+i\epsilon)^{-1}QH_d'P\ . \tag{25}$$

From the identity

$$\mathrm{Im}\,G=(1+G^\dagger U^\dagger)\,\mathrm{Im}G_0(1+UG)+G^\dagger(\,\mathrm{Im}U)G$$

for $G=G_0+G_0UG$, Eq. (23) is written as

$$\sigma_P^{\mathrm{IAV}}=\sigma_{P,0}^{\mathrm{IAV}}+\sigma_{P,ab}^{\mathrm{IAV}}\ , \tag{26}$$

where

$$\sigma_{P,0}^{\mathrm{IAV}}=\Sigma_n\frac{2\pi}{v_a}\rho(E_a')\,|\,\langle\Psi_P^{(-)}(A_n,E_a')\,|\,V_a\,|\,\chi_a^{(+)}\phi_a\Phi_A^0\rangle\,|^2\ , \tag{27}$$

$$\sigma_{P,ab}^{\mathrm{IAV}}=-\frac{1}{\pi}\frac{2\pi}{v_a}\langle\chi_a^{(+)}\phi_a\Phi_A^0\,|\,V_a^\dagger G_a^{\mathrm{eff}^\dagger}(\,\mathrm{Im}U_a^{\mathrm{eff}})G_a^{\mathrm{eff}}V_a\,|\,\chi_a^{(+)}\phi_a\Phi_a^0\rangle\ , \tag{28}$$

and

$$E_a'=E_a-(e_A^n-e_A^0)+\langle\phi_a\,|\,V_{xb}\,|\,\phi_a\rangle\ . \tag{29}$$

The wave function $\Psi_P^{(-)}(A_n,E_a')$ satisfies the CC equation

$$(H_A+T_a+U_a^{\mathrm{eff}}-E_a'-e_A^n)\Psi_P^{(-)}(A_n,E_a')=0\ , \tag{30}$$

and has its inhomogeneous term in the channel $a+A_n$ with the relative energy E_a'. [We think that in Ref. 1, H_d in Eq. (3.12) and $\Phi_{a'A}^{(-)*}$ in Eq. (3.13) were meant to be H_d' and $\Psi_P^{(-)}(A_n,E_a')$, respectively. We also correct the folded potential term of U_a^{eff} in Eq. (25).] Interpreting $\sigma_{P,0}^{\mathrm{IAV}}$ of Eq. (27) as the sum of inelastic cross sections and arguing that

$$\langle\Psi_P^{(-)}(A_n,E_a')\,|\,V_a\,|\,\chi_a^{(+)}\phi_a\Phi_A^0\rangle\neq0\ ,$$

but

$$\langle\Psi_P^{(-)}(A_n,E_a')\,|\,V_P\,|\,\chi_a^{(+)}\phi_a\Phi_A^0\rangle\approx0$$

(with suitable changes due to the difference of H_d and H_d'), UT conclude that σ_b^{IAV} erroneously includes mere inelastic scatterings but σ_b^{UT} does not.

We criticize (23) and the subsequent discussion on three levels.

1. As already noted in UT Appendix B, the final state asymptotic kinetic energy of inelastic scattering, $a+A\to a+A_n$, is $E_a-(e_A^n-e_A^0)$, which differs from E_a' of (29) by the expectation of the potential energy of a. This means that although expression (27) looks like a sum of "inelastic" cross sections, these are not the cross sections of actually observable inelastic processes. The UT interpretation of (27) does not hold.

2. The UT derivation of (27) is invalid. The interaction $PH_d'Q$ in the second term of (25) is of infinitely long range, because it includes a term $PT_{xb}Q$ $(=-PV_{xb}Q)$. This pathology is inherited by U_a^{eff}, with the result that no solutions of (30) exist that satisfy the boundary conditions appropriate to an inelastic distorted wave.

3. One can directly confirm that the result (26)–(28) obtained by the UT analysis of the projected Green's function is invalid, without examining the misleading transformation (24). To have a relation to a sum of inelastic scattering cross sections, the intermediate states in the Green's function $PG_d'P$ in (23) should have asymptotic channels proportional to the bound state $\phi_a(\mathbf{r}_{bx})$. We therefore examine the function

$$\Xi=G_d'PV_a\,|\,\chi_a^{(+)}\phi_a\Phi_A^0\rangle\ . \tag{31}$$

To contain inelastic scattering the asymptotic behavior of $P\Xi$ would have to be

$$P\Xi\to\Sigma_n r_a^{-1}f_n(\hat{\mathbf{r}}_a)\exp(ik_nr_a)\phi_a\Phi_A^n\ , \tag{32}$$

with the appropriate inelastic momenta k_n. However, because G_d' in (31) operates on a source function that is short ranged in all variables, the solution Ξ actually reduces asymptotically to a sum of terms of the familiar three-body breakup form $e^{ik\mathscr{R}}/\mathscr{R}^{5/2}$, with

$$\mathscr{R}=[(m_xr_b^2+m_br_x^2)/(m_x+m_b)]^{1/2}\ ,$$

using various values of the breakup momentum k for the various excited states of A. Because the quantity $P\Xi$ is proportional to $(\phi_a \mid \Xi\rangle$, and because the range of $\phi_a(\mathbf{r}_{bx})$ is small compared to r_b or r_x, $P\Xi$ is asymptotically proportional to a sum of terms of the form $r_a^{-5/2}\exp(ikr_a)$. Obviously, $(\phi_a \mid \Xi\rangle$ decreases faster than the right-hand side of Eq. (32) as $r_a \to \infty$. Therefore, the function $P\Xi$ has vanishing asymptotic flux in the inelastic channels. It is not surprising that the actual $P\Xi$ falls off faster than the UT analysis claims. Because G_d' has no interaction between b and x, as seen in (14) and (15), (31) has no channels in which those particles are bound asymptotically. We again conclude that the UT analysis of the projected Green's function is erroneous, and the IAV formalism is free from the inelastic scatterings that UT criticized.

Finally we note that UT object to the use of a Huby-Mines[5] convergence factor $\exp(-\epsilon r_x)$ in the Appendix of IAV. They warn that the possibility of taking the limit $\epsilon \to 0^+$ before evaluating the nonuniformly convergent integral leads to an "α-dependent ambiguity" in the IAV result. However, this convergence factor is already discussed in Sec. II (not Sec. III) of IAV, where it is used to provide a mathematically unambiguous definition of the post-form sum rule. There it is emphasized that the limit $\epsilon \to 0^+$ must be taken *after* the radial integration is performed; the identities derived (including the equivalence of the post and prior sum rules), hold if and only if the limit and integration are performed in this order. It is only by departing from this canonical prescription that one can produce an "α-dependent ambiguity."

Note added in proof. In response to comments, we wish to emphasize that the DWBA approximation in Eq. (10) consists in the use of product wave functions in the entrance channel, not in approximating some other Green's function by G_d'. The steps leading from Eq. (10) to Eq. (13) are exact.

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