Kemmer-Duffin-Petiau equation for pionic atoms and anomalous strong interaction effects

E. Friedman and G. Kälbermann
Racah Institute of Physics, Hebrew University of Jerusalem, Jerusalem, Israel

C. J. Batty
Rutherford Appleton Laboratory, Chilton, United Kingdom
(Received 15 July 1986)

Fits to data on strong interaction level shifts and widths in pionic atoms, over the whole of the periodic table, are made with the Kemmer-Duffin-Petiau equation. Comparisons are made with the conventional Klein-Gordon equation and emphasis is placed on the problem of anomalously small shifts and widths. The Kemmer-Duffin-Petiau equation yields fits which are marginally better than those obtained with the Klein-Gordon equation.

I. INTRODUCTION

The preceding paper[1] discussed the current interest in relativistic effects in the interaction between hadrons and nuclei with particular emphasis placed on the meson-nucleus interaction, in the context of pionic atoms. This system provides a unique opportunity for testing relativistic effects at essentially zero kinetic energy.

The interaction of zero energy pions with nuclei has been extensively studied[2-6] within the framework of the Klein-Gordon (KG) equation, using the Ericson-Ericson[2] (EE) potential. In the present paper we study the Kemmer-Duffin-Petiau (KDP) equation as a possible alternative for describing experimental results on strong interaction level shifts and widths in pionic atoms. The theoretical aspects of this problem have been developed in the accompanying paper.[1] The present paper is confined to numerical evaluations and comparisons with experimental data.

In Sec. II we discuss the selection of data. This is quite an important point as one of the tests of the formalism is its ability to describe data over the whole of the periodic table. The results are given in Sec. III where special attention is paid to the long-standing problem of states whose experimental strong interaction level shifts and particularly widths are anomalously small.[7-14] The conclusions are presented in Sec. IV together with a short discussion of the applicability of the KDP equation to pion scattering.

II. SELECTION OF EXPERIMENTAL DATA

To provide a basis for comparison we used the same set of experimental measurements as in Ref. 6 together with more recent results in order both to extend the range of nuclei used for the analysis and to include improved measurements for "anomalous" nuclei. In the earlier paper[6] a "standard" and an "extended" data set were used to give a total of 11 nuclei with "normal" shift and width measurements. To these a further three cases were added in the present work to give a new "normal" data set consisting of 14 cases (i.e., a total of 28 shift and width measurements). Also used previously[6] was a set of three nuclei comprising an "anomalous" data set. A further three cases were added to give a new "anomalous" data set consisting of six cases in all.

The additional experimental measurements used in this analysis are listed in Table I, where the energy shifts quoted are those relative to a full quantum electrodynamics (QED) calculation including finite size effects. The range of nuclei covered in the new, more complete data set are discussed below for each of the pionic atom states, together with some comments on the additional measurements.

For the $1s$ states ^{16}O and ^{20}Ne were chosen as representing typical isospin zero nuclei while ^{18}O and ^{19}F were included so as to determine the isospin dependent terms. An attempt was made to extend the range of data fitted for $1s$ states by including the recent measurements[15] for ^{10}B and ^{11}B. Unfortunately $1/A$ corrections[16] to the standard optical potential become quite significant for these two cases and so these nuclei were excluded from the final fits presented in this paper. In the original analysis[6] Na was chosen as showing "anomalous" results for $1s$ states. In the present work the recent experimental results of Taal *et al.*[12] for Mg, which give anomalously small width values when compared to standard optical model predictions, were also used.

For $2p$ states, the earlier work included Si, Ca, ^{44}Ca, Fe, and Ge as examples of "normal" nuclei and As as a nucleus showing anomalous results. The range of data was extended by including the recent accurate measurements[17] made for ^{16}O using a crystal spectrometer. The very high precision of these results compared to those for the other nuclei used in the analysis caused the overall fit to the wide range of data to be distorted, due to the large weight given to the ^{16}O data in the χ^2 fits. It was therefore necessary to increase the quoted errors by a factor of three in order to avoid this problem. It should be noted that both the $1s$ and $2p$ state measurements for ^{16}O are now included in this new "normal" data set.

The $3d$ state has so far shown the greatest range of "anomalous" results. In the earlier work the Ta results of Konijn *et al.*[9] were used as a typical example; to this case we added the recent experimental measurements for ^{168}Er and Pb. The case of ^{168}Er is interesting since it is on the

TABLE I. Additional[a] experimental results used in analysis (2pF denotes two-parameter Fermi distribution and 3pF1 denotes three-parameter Fermi distribution with $\omega=-0.051$).

Level	Nucleus	Shift (keV)	Width (keV)	Ref.	Distribution	R_p (fm)	a_p (fm)	R_n (fm)	a_c (fm)
$1s$	Mg	-61.5 ± 0.6	17.2 ± 1.6	12	2pF	3.046	0.474	3.046	0.523
$2p$	^{16}O[b]	0.0151 ± 0.0008	0.0068 ± 0.0011	17	3pF1	2.608	0.461	2.608	0.513
$3d$	^{168}Er	16.3 ± 0.3	19.4 ± 1.0	11	2pF	6.17	0.548	6.27	0.591
$3d$	Pb	26.1 ± 2.4	47.1 ± 3.0	13,14	2pF	6.689	0.474	6.839	0.523
$4f$	^{168}Er	0.351 ± 0.020	0.220 ± 0.030	18,11	2pF	6.17	0.548	6.27	0.591
$4f$	U	5.04 ± 0.20	3.85 ± 0.65	4	2pF	6.805	0.562	6.955	0.605

[a]In addition to the results used in Ref. 6.
[b]Experimental errors increased (see the text).

lower edge of the region where the "anomalous" effect seems to develop in $3d$ states. The recent measurements of Tanaka *et al.*[11] were used. For Pb two sets of measurements by de Laat *et al.*[13] and by Olin *et al.*[14] are available. The width measurements are in good agreement and a weighted mean value of $\Gamma(3d)=47.1\pm3.0$ keV was used. However, the measured energies for the $4f\rightarrow3d$ transition of 1279.4 ± 1.2 keV by de Laat *et al.*[13] and of 1274.7 ± 1.2 keV by Olin *et al.*[14] differ significantly. A mean value of 1277.1 ± 2.4 keV has been adopted, which gives an energy shift for the $3d$ level $\epsilon(3d)=26.1\pm2.4$ keV when the calculated QED energy of Olin *et al.*[14] is used. Measurements for Ag were included as being representative of those for a "normal" $3d$ level.

A comment about the measurements for Ta should also be made. In the earlier work an energy shift of $\epsilon(3d)=14.4\pm1.0$ keV was used as measured by Konijn *et al.*[9] However, this result differs from that of Batty *et al.*,[18] who obtained $\epsilon(3d)=18.1\pm1.2$ keV. For some cases this latter result is in better agreement with the predictions of standard potentials.

For $4f$ states, the earlier analysis used the results available for Bi. In the present work ^{168}Er was added since shift and width measurements[12] are available in this case for both $3d$ and $4f$ levels. The weighted mean of the results for $4f$ states of Tanaka *et al.*[11] and of Batty *et al.*,[18] which are in good agreement, was used. The range of nuclei was extended further by also including measurements[4] for U. There is no evidence of any anomalous effects for pionic $4f$ levels for any target nucleus.

Also given in Table I are the parameters of the density distributions used for each nucleus. For most nuclei this was a simple two-parameter Fermi distribution except for ^{16}O and ^{40}Ca (see Ref. 6), where a three-parameter Fermi form was used. The values R_c and a_c refer to the charge distributions. For the proton distribution $R_p=R_c$ was used and a_p adjusted to give the correct rms radius after unfolding the finite proton size. For the neutron distribution $a_n=a_p$ in all cases and the value of R_n was chosen to give reasonable values for $\langle r_n^2\rangle^{1/2}-\langle r_p^2\rangle^{1/2}$ in line with recent theoretical predictions and experimental measurements. Varying R_n values for individual cases can affect the fit to a specific nucleus but does not affect the overall picture.

III. RESULTS

Fits to the pionic atom data discussed in Sec. II were made using the KG equation with the EE potential[6] and using the KDP equation with the vector potential added. In the latter case the EE-equivalent form was first used [Eqs. (45)—(47) of Ref. 1] and then the vector term was added and a search on all parameters was made. We focus attention on two data sets—the first consists of 14 "normal" cases covering the range of nuclei from ^{16}O to U, where no problems were encountered when comparing calculation with experiment. The second is a wider data set consisting of the above normal set with six cases added where the experimental values of the level shift and width are too small when compared to predictions made with the EE potential within the KG equation. Table II summarizes the results of these fits. χ^2/F is the χ^2 divided by the number of degrees of freedom. In order to get some idea on the quality of fits to the level shift and width values separately, the table includes the χ^2 per point for the shifts (χ^2_ϵ) and widths (χ^2_Γ).

Starting with the KG equation containing the EE potential, very good fits are obtained for the normal data set, in full agreement with Ref. 6, where a somewhat less extensive normal data set was used. When fits were made to the full data set, which also contains the six "anomalous" cases, the quality of the fits deteriorated significantly and it is seen that a fit to this broad data set is not achieved. With the KDP equation we first repeated fits with the EE equivalent version of the potential, obtaining the same fits as with the KG equation. Table II shows results for the KDP equation with the vector potential added. It is seen that the fits for the normal data set are somewhat better than the fits achieved with the KG equation. The improvement is more noticeable in the case of the full data set. It should be remembered that there are 12 parameters in this version of the KDP equation as compared to eight parameters in the KG equation (for 40 experimental values), but the reduction in the value of χ^2/F is significant. However, the fit to the full data set is still poor, showing that the KDP equation with the extra vector term[1] is unable to simultaneously fit both normal and anomalous pionic atom data. A better perspective on the problem of anomalous cases is obtained with the help of

TABLE II. Results of χ^2 fits to pionic atoms.

		KG equation EE potential	KDP equation EE + vector potential
Normal data set (14 cases)	χ^2/F	2.0	1.4
	χ^2_ϵ/N	1.6	0.7
	χ^2_Γ/N	1.2	1.0
Full data set (20 cases)	χ^2/F	11.9	8.0
	χ^2_ϵ/N	6.5	3.2
	χ^2_Γ/N	5.4	2.4

Fig. 1. The figure shows the differences between calculation and experiment, in units of the relevant experimental error, separately for level shifts ϵ and widths Γ and for the two wave equations considered. The calculations use the best fit potentials obtained from fits to the normal data set, thus displaying the departure of the anomalous cases from the normal behavior. It is interesting to note that the anomaly is equally pronounced in the shifts and widths. Variations of the radial parameters of the neutron density distributions for the anomalous cases did not significantly alter the picture. Comparing the results for the KG and the KDP equation, the improvement with the latter is evident, particularly for the shifts. The shifts of the $2p$ states seem to be much better reproduced with the KDP equation.

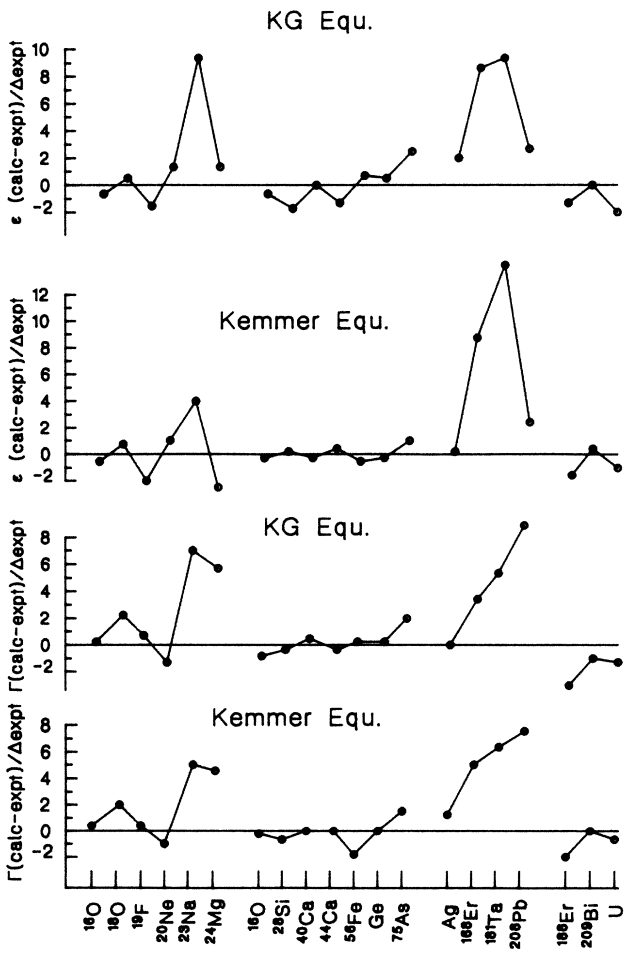

FIG. 1. Differences between calculated and experimental shifts and widths normalized to the corresponding experimental error. Calculations use the parameters obtained from fits to the normal data set.

IV. DISCUSSION

The ability of the KDP equation to describe strong interaction effects in pionic atoms in general and its performance in connection with the anomalous states in particular has been studied in the present work. It was shown in Ref. 1 that the KDP equation could be written in a form equivalent to the KG equation with the successful EE potential. It was, therefore, expected that the KDP equation will be at least as successful as the KG one and this expectation is fulfilled. It was also hoped that the KDP equation would shed some light on the puzzling problem of anomalous pionic atom states. The fits to these states are indeed improved with the KDP equation but the anomaly essentially remains. Some remarks on the experimental evidence for the anomaly are in order.

In general the principal evidence for these anomalous shifts and widths is given by measurements for atomic $3d$ states. Only two cases (Na,Mg) are clearly seen for $1s$ states and only one (As) for $2p$ states. For $3d$ states the "anomalous" effect seems to start at about $Z=68$ (Er) and then appears to become larger as Z increases. However, even for this latter range of nuclei, some qualifications about the experimental evidence are necessary. For example, it has been shown[18] that in the case of Ta it was possible to fit the measured x-ray spectrum to give strong interaction shift and width values which are in agreement with those calculated using standard optical model potentials. Further, some of the first strong evidence for anomalous effects in $3d$ states was obtained by Konijn *et al.*[9] for Bi. More recently, de Laat *et al.*[13] have obtained values for Bi more in accord with predictions from the standard potential, although there still seems to be some residual discrepancy.

The problem is aggravated by the fact that the anomalous effects are only seen in very broad and strongly shifted levels, where the measurement and interpretation of the x-ray spectra is technically difficult. A further problem for $3d$ states is that many of the nuclei are deformed and it is necessary to include[18] the effects of hyperfine splitting for those nuclei with spin $I>1$.

Finally, a comment on the use of the KDP equation for scattering of low energy pions. We found that at 30 MeV the KDP equation with the vector term included fits the elastic scattering data as well as does the KG equation. In both approaches there is a sufficient number of parameters to produce excellent fits to the data and it is hard to see any numerical evidence which shows preference for the KDP equation.

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