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Kemmer-Duffin-Petiau equation approach to pionic atoms

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We lay down the formalism for the treatment of pionic atoms level widths and shifts using the Kemmer-Duffin-Petiau equation. Interactions are introduced in a Lorentz invariant way. An analytical transformation is shown that connects between different sets of interactions. The Kisslinger potential is obtained as a special case of a scalar-tensor optical potential. Results are shown in the accompanying paper.

I. INTRODUCTION

The study of relativistic effects in nucleon-nucleus reactions[1–3] has proved to be a valuable tool in increasing our understanding of the nucleus. The investigation of relativistic effects for meson nucleus reactions has recently been approached using a first order relativistic equation[4] in close spirit to the Dirac equation, namely the Kemmer-Duffin-Petiau equation (KDP).[5] Impulse approximation calculations were found in the KDP equation to be as good as or better than the relativistically corrected Schrödinger equation results. The richness of possibilities and the simplicity of the equation has prompted us to undertake the investigation of low energy pionic-atom widths and shifts using the KDP equation. We deal here with various theoretical aspects essential for the phenomenological treatment of the problem. We expect the present approach to be different from the Klein-Gordon (KG) approach on the following grounds: the KG equation is a second order equation in which interactions can be introduced either by minimal substitution, as in the case of conserved currents or by scalar operators, local or nonlocal;[6] the KDP equation, on the other hand, being a first order relativistic equation, has a clear prescription as to how interactions are to be introduced by observing their Lorentz character.

In Sec. II we develop the basic tools concerning the KDP equation. Section III deals with parametrizations of the optical potential for pionic atoms and transformations among sets of interactions. Section IV connects to the usual Kisslinger potential and provides us with a starting point for the parameter search. Results are shown in the accompanying paper.

II. THE KDP EQUATION

The free particle KDP equation is[5]

$$(i\beta^\mu\partial_\mu - m)\psi = 0 , \tag{1}$$

where m is the meson mass. The matrices β^μ obey[5]

$$\beta^\mu\beta^\nu\beta^\lambda + \beta^\lambda\beta^\nu\beta^\mu = g^{\mu\nu}\beta^\lambda + g^{\nu\lambda}\beta^\mu . \tag{2}$$

The commutation relations of Eq. (2) can be found from the algebra generated by the matrices

$$\beta^\mu = \tfrac{1}{2}(\gamma_1^\mu I_2 + \gamma_2^\mu I_1) , \tag{3}$$

where γ^μ is a Dirac matrix in the space of particle 1 and I_2 is the identity operator in the space of particle 2. This scheme of building first order equations using an algebra generated by the direct sum of Dirac matrices in different spaces can be generalized to higher spins.[7] Equation (3) implies that the irreducible representations of the β^μ's have to describe particles of spin 0 and 1. Kemmer[5] found the irreducible representations to be of dimension 5 for spin 0 and 10 for spin 1. He also showed that the generators of the Poincaré group can be constructed in close analogy to the Dirac case.

By applying spatial rotations we find a spin operator

$$S_i = \epsilon_{ijk}\beta_j\beta_k \tag{4}$$

obeying

$$S_i^3 = S_i . \tag{5}$$

Therefore, the eigenstates of Eq. (4) are particles of rest frame spin 0 and 1. We can also build the boost operator as in the Dirac case to be

$$B = e^{-\omega\boldsymbol{\delta}\cdot\hat{\mathbf{v}}} = 1 - \sinh(\omega)\boldsymbol{\delta}\cdot\hat{\mathbf{v}} + [\cosh(\omega) - 1](\boldsymbol{\delta}\cdot\hat{\mathbf{v}})^2 , \tag{6}$$

where

$$\cosh(\omega) = E/m , \tag{7a}$$

$$\sinh(\omega) = -p/m . \tag{7b}$$

$\hat{\mathbf{v}}$ is a unit velocity vector and

$$\boldsymbol{\delta} = \beta_0\boldsymbol{\beta} - \boldsymbol{\beta}\beta_0 . \tag{8}$$

To obtain the KG equation obeyed by ψ we multiply (1) by $i\beta^\nu\partial_\nu i\beta^\lambda\partial_\lambda$ and use Eq. (2), obtaining

$$m(\Box + m^2)\psi = 0 , \tag{9}$$

as long as $m \neq 0$ ψ obeys the KG equation. To include the massless case we write a KDP equation in the form

$$[i\beta^\mu\partial_\mu - a(1-P)]\psi = 0 , \tag{10}$$

where P is the matrix which is a scalar operator upon Lorentz transformation:

$$P=\tfrac{1}{2}(\beta_\mu\beta^\mu-2) \tag{11}$$

and a is a parameter of mass dimension. In the five dimensional representation, it takes the form of a projection operator onto the first component of ψ:

$$P_{ij}=\delta_{i1}\delta_{j1},\quad 1\le i,j\le 5\ . \tag{12}$$

This component is easily identifiable with KG wave function; the other four components are, in the free case, the four gradient. In the ten dimensional representation the KDP wave function consists of a four vector (first four components) and of a six-component antisymmetric tensor. P projects onto the first four components in this case:

$$P_{ij}=\begin{cases}\delta_{ij}, & 1\le i,j\le 4\\ 0, & 5\le i,j\le 10\ .\end{cases} \tag{13}$$

It is noteworthy that the KDP equation of Eq. (10) for the spin 1 case is exactly Maxwell's source free set of equations and Eq. (1) becomes its massive counterpart. The latter is, in the free case, Proca's equation, differing from it even for the simple case of minimal substitution (work in this direction is currently underway).[4]

We can generate the Hamiltonian equation corresponding to Eq. (1) in the following way: Multiply it by $i\beta^\nu\partial_\nu\beta^0$ and use Eq. (3) to obtain

$$m(i\partial_0-i\beta^\nu\partial_\nu\beta^0)\psi=0\ . \tag{14}$$

Multiplying Eq. (1) by β^0 and adding to Eq. (14) we obtain

$$(i\partial_0+i\boldsymbol{\delta}\cdot\nabla-m\beta^0)\psi=0\ , \tag{15}$$

where $\boldsymbol{\delta}$ is defined in Eq. (8). Equation (15) has to be supplemented with a constraint equation independent of time obtained from multiplying Eq. (1) by $(1-\beta_0^2)$

$$i\boldsymbol{\beta}\cdot\nabla(\beta_0^2\psi)-(1-\beta_0^2)m\psi=0\ . \tag{16}$$

From Eq. (1) we can obtain the propagator of the theory

$$G_0=\frac{(\beta^\mu p_\mu+m)+m^{-1}[(\beta_\mu p^\mu)^2-p^\mu p_\mu]}{(p_\mu p^\mu-m^2)}\ . \tag{17}$$

If we replace β^μ in Eq. (17) with γ^μ we get the Dirac propagator because $(\gamma_\mu p^\mu)^2=p_\mu^2$, but here the KDP algebra is of the third order in β^μ [Eq. (3)] and therefore $(\beta_\mu p^\mu)^2$ does not reduce to any simpler expression. Equation (17) can be used together with a meson-nucleon interaction (pseudoscalar or pseudovector) to generate an optical potential for the meson in the nuclear medium. A field theory can be developed using the approach of Akhiezer and Berestetskii.[8] We now proceed in a phenomenological way and postpone this task for the future.

III. INTERACTIONS IN THE KDP EQUATION

We concentrate hereforth on the spin 0 five dimensional representation. The operators generated by the β^μ's are

$$S_1=I\quad\text{(scalar)}\ , \tag{18a}$$

$$S_2=P\quad\text{(scalar)}\ , \tag{18b}$$

$$V_\mu^1=\beta_\mu\quad\text{(vector)}\ , \tag{18c}$$

$$V_\mu^2=P\beta_\mu\quad\text{(vector)}\ , \tag{18d}$$

$$T_{\mu\nu}^1=\beta_\mu\beta_\nu+\beta_\nu\beta_\mu-1\quad\text{(symmetric tensor)}\ , \tag{18e}$$

$$T_{\mu\nu}^2=\beta_\mu\beta_\nu-\beta_\nu\beta_\mu\quad\text{(antisymmetric tensor)}\ . \tag{18f}$$

We can accordingly introduce interactions in the KDP equation

$$(i\beta^\mu\partial_\mu-m-U_sS_1-U_s'S_2-U_\mu V_1^\mu - U_\mu' V_2^\mu-U_{\mu\nu}^1T_1^{\mu\nu}-U_{\mu\nu}^2T_2^{\mu\nu})\psi=0\ . \tag{19}$$

In the relativistic impulse approximation for meson-nucleus scattering[4] a scalar-vector combination was used with considerable success. Tensors were avoided because of noncausal propagation.[9] Nevertheless, at the phenomenological level these should not be excluded. Moreover, for a certain potential set, a unitary transformation exists that takes it to a different one. We will show an explicit example by transforming a scalar-vector interaction into either a scalar-tensor or a vector-tensor set.

We choose to start with only U_s and U_μ different from zero in Eq. (19), i.e.,

$$(i\beta_\mu\partial^\mu-m-U_\mu\beta^\mu-U_s)\psi=0\ . \tag{20}$$

First we can eliminate the spatial components of U_μ by the following gauge transformation:

$$\psi=e^{i\alpha}\phi\ . \tag{21}$$

Inserting in Eq. (20) we get

$$e^{i\alpha}(i\beta^\mu\partial_\mu-m-U_s-U_\mu\beta^\mu-\boldsymbol{\beta}\cdot\nabla)\phi=0\ , \tag{22}$$

equating

$$\nabla\alpha=\mathbf{U}\ , \tag{23}$$

and multiplying Eq. (21) by $e^{-i\alpha}$ we obtain

$$(i\beta^\mu\partial_\mu-mU_s-U_0\beta^0)\phi=0\ . \tag{24}$$

We have managed to eliminate the spatial part of U_μ. The spectrum of ϕ is identical to the spectrum of ψ because the transformation is unitary.

We now proceed to mix between the different sets of potentials. As in the Dirac case[10] we can do it with the aid of operators of the form

$$X=e^{iBA}\ , \tag{25}$$

where B is a KDP matrix and A, an unknown function of coordinates.

One such choice that enables us to eliminate either the vector or the scalar interaction is

$$O(X)=e^{i\eta_0 A}=\cos(A)+i\eta_0\sin(A)\ , \tag{26}$$

where

$$\eta_0=T_{00}=2\beta_0^2-1\ . \tag{27}$$

Introducing Eq. (26) into Eq. (24) for KDP states,

$$\phi=e^{-iEt}\psi\ , \tag{28}$$

where E can be taken as a complex quantity as usual for metastable states, we obtain

$$[i\boldsymbol{\beta}\cdot\nabla+\eta_0\boldsymbol{\beta}\cdot\nabla A+e^{2i\eta_0 A}\beta_0(E-U_0) -e^{2i\eta_0 A}(m+U_s)]\phi=0\ . \tag{29}$$

Suppose now we want to eliminate the vector interaction while leaving the same spectrum of states; we then demand

$$e^{2iA}(E-U_0)=E\ . \tag{30}$$

We usually choose to leave explicitly the Coulomb interaction, i.e.,

$$e^{2iA}(E-U_0)=E-V_C\ . \tag{31}$$

Equation (31) implies two separate equations for the real and imaginary parts of A. The equation is always solvable if we allow A to be complex. The new scalar potential becomes

$$\overline{U}_s=\cos(2A)(m+U_s)-m \tag{32}$$

at the same time a tensor potential arises, namely

$$T_{00}U_T=i\eta_0\sin(2A)(m+U_s)\ . \tag{33}$$

Equation (30) then becomes

$$[(E-V_C)\beta_0+i\boldsymbol{\beta}\cdot\nabla+(m+\overline{U}_s) +\boldsymbol{\beta}\cdot\nabla A-2\beta\cdot\nabla AP-T_{00}U_T]\phi=0\ . \tag{34}$$

The term $\boldsymbol{\beta}\cdot\nabla A$ can be further eliminated by a transformation

$$\chi=e^{-i\tilde{A}}\phi\ , \tag{35}$$

leaving only an extra term $2\boldsymbol{\beta}\cdot\nabla AP$ that cannot be eliminated. If instead we wish to eliminate the scalar interaction, we demand

$$\cos(2A)(m+U_s)=m\ , \tag{36}$$

and the new vector potential becomes

$$e^{2iA}(E-U_0)=E-\widetilde{U}_0 \tag{37}$$

and a new tensor potential arises,

$$T_{00}\widetilde{U}_T=i\sin(2A)(m+U_s)\ . \tag{38}$$

IV. THE KISSLINGER POTENTIAL

The success of the Kisslinger potential in the phenomenological treatment of pionic atom level shifts and widths[11] provides us with a starting basis for the present approach. We will now show which choice among the interactions in Eq. (19) yields a KG equation with a Kisslinger potential. The Kisslinger potential generates a KG equation of the form

$$\{(E-V_C)^2+\nabla^2-m^2-q(\mathbf{r})-\nabla[\alpha(\mathbf{r})\nabla]\}\psi_{\rm KG}=0\ , \tag{39}$$

where V_C is the Coulomb potential. Equivalently,

$$\{(E-V_C)^2+[1-\alpha(r)]\nabla^2-q(r)-\nabla\alpha(r)\nabla\}\psi_{\rm KG}=0\ . \tag{40}$$

Introducing the Ericson-Ericson-Lorentz-Lorenz (EELL)[6] effect we obtain

$$\left[(E-V_C)^2+\left[1-\frac{\alpha(\mathbf{r})}{1+\xi\frac{\alpha(\mathbf{r})}{3}}\right]\nabla^2-q(r) -\nabla\left[\frac{\alpha(\mathbf{r})}{1+\xi\frac{\alpha(\mathbf{r})}{3}}\right]\nabla\right]\psi_{\rm KG}=0\ , \tag{41}$$

ξ being the EELL parameter.

In order to connect to Eq. (41) we make the following choices: $U_\mu=0$, $U'_\mu=0$, $U^2_{\mu\nu}=0$, and

$$U_{\mu\nu}=-\delta_{\mu 0}\delta_{\nu 0}U_T\ . \tag{42}$$

Equation (19) then becomes

$$[i\beta^\mu\partial_\mu-\beta^0V_C-(m+U_s)-U_s'P+\eta_0U_T]\psi=0\ . \tag{43}$$

We therefore get for the first component ψ_1 an equation

$$\left[(E-V_C)^2-(m+U_s-U_T)(m+U_s-U_T+U_{s'}) +(m+U_s-U_T)\nabla\left[\frac{1}{m+U_s+U_T}\right]\nabla\right]\psi=0\ . \tag{44}$$

Demanding $U_s=U_T$, we obtain

$$\left[(E-V_C)^2-m^2-mU_s'+\nabla\frac{1}{\left[1+\frac{2U_s}{m}\right]}\nabla\right]\psi_1=0\ . \tag{45}$$

Identifying

$$\frac{2U_s}{m}=\frac{\alpha(\mathbf{r})}{1+(\xi/3-1)\alpha(\mathbf{r})} \tag{46}$$

and

$$mU_s'=q(r)\ , \tag{47}$$

we recover Eq. (41).

We note that symmetric tensor interactions cannot arise in a first order relativistic impulse approximation (RIA) potential due to the absence of a symmetric tensor in the Dirac space of the (target) nucleon to contract with the meson tensor (like $\eta_0=T^1_{00}$) in building a relativistically invariant meson nucleon elementary amplitude. Tensor interactions nevertheless do arise in higher order corrections to the meson-nucleus optical potential. These interactions appear also in the case where the meson is considered a composite object made of fermions (quarks) interacting with the nucleon.[12]

We start our parameter search with Eq. (45) constrained by Eqs. (46) and (47); we then add vector potentials U_V or U_V'. Using the transformation of Krell and Ericson[13] we obtain

$$\left\{(E^2-M^2)+\left[1-\frac{\alpha}{(1+\xi\alpha/3)}\right]\nabla^2-U\right\}\psi_1=0\ , \quad (48)$$

where

$$U=E(2U_V+2V_C+U_V')-(U_V+V_C)(U_V+V_C+U_V')$$
$$+mU_s'+\frac{3}{4}\left[\frac{F'}{F}\right]^2-\frac{F''}{2F}-\frac{F'}{rF} \quad (49)$$

and

$$F=1+\frac{2U_s}{m}=(1+\xi\alpha/3)/[1+(\xi/3-1)\alpha]\ . \quad (50)$$

In the accompanying paper we show the results for pionic atoms widths and shifts. The best fits are obtained when we chose $U_V=0$ ($U_V'\neq 0$). We therefore select and display the results for that case only.

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