$^{90,91}Zr$ (n,α) $^{87,88}Sr$ reactions at 14.3 and 18.15 MeV incident neutron energy

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Measurements of alpha spectra in the (n, α) reactions induced on $^{90,91}Zr$ at 14.3 and 18.15 MeV incident neutron energy are presented. A microscopic calculation of these spectra has been made using both pick-up and knock-on theories, and in both cases only one overall normalizing factor, which is the same for the two target nuclei and incident energies and all the considered transitions, appears as a free parameter in the calculation. Pick-up calculations provide a very satisfactory reproduction of the data. Knock-on calculations reproduce many qualitative features of the measured spectra, but do not allow a fully satisfactory reproduction of them. While the results obtained do not exclude knock-on contributions to these reactions, their presence is not established.

I. INTRODUCTION

During a systematic study of (n,α) reactions on medium heavy and heavy nuclides, at neutron energies of about 20 MeV, we have measured the cross section of the processes induced on ^{90}Zr and ^{91}Zr at 14.3 and 18.15 MeV.

Cross section measurements of neutron induced reactions at incident energies greater than 15 MeV are still scarce.

The systematic study of (n,α) reaction cross sections at these energies is important for applied physics research, e.g., for investigating bulk radiation damage caused by high fluxes of neutrons. Helium production may alter the mechanical properties of materials since it can migrate to grain boundaries, creating strains or voids leading to swelling.

The study of this process on the magic and near magic Zr isotopes might also prove important for investigating the reaction mechanism. It has been shown that a slightly bound unpaired nucleon outside a magic shell in most cases does not contribute to a (N,α) process, acting as a "spectator." This spectator effect manifests itself in two ways:

(a) Through the weak population of low energy states in the residual nucleus, since the lowest energy excited states have a 1p-1h nature and an excitation energy of 2–4 MeV.[1,2]

(b) Through the excitation of homologous states in the residuals from a magic target (with a completed magic shell) and a near magic target with one nucleon outside the magic shell.[2] For instance, one may excite 1h states in the magic nucleus residual and states corresponding to the coupling of the 1h states with the nucleon outside the shell in the residual from the near magic nucleus.[2]

This effect is valuable for investigating the structure of states at some MeV excitation energy in near-magic nuclei, and it also affects the continuous part of the alpha spectrum. This has been clearly demonstrated by the comparison of alpha particle spectra in the (p,α) reactions induced on ^{90}Zr and ^{91}Zr at about 25 MeV incident proton energy.[2] When the two spectra are superimposed as a function of the alpha particle energy, one finds that, in spite of the widely different Q values, the spectrum of alphas from ^{91}Zr looks like an energy average of the more structured spectrum from ^{90}Zr, thus showing that the total strength for transitions to states in a given energy interval is nearly the same when the excitation energy of the odd-odd ^{88}Y exceeds by about 2.16 MeV that of the odd-even ^{87}Y. This result is convincingly attributed to the spectator role of the unpaired neutron of ^{91}Zr.

A theoretical analysis of data of this kind may prove useful for understanding the reaction mechanism. In fact, if the calculations explicitly consider the structure of excited residual nucleus states, they can differentiate between mechanisms leading to a different population of these states.

In Sec. II of this paper the experimental results are presented. In Sec. III the α spectra are compared with data from other reactions to try to understand the nature of the states excited in the (n,α) process. In Sec. IV the alpha energy distributions are analyzed using the pick-up and knock-on theories and the comparison with experimental data is discussed. Section V is devoted to the conclusions.

II. EXPERIMENTAL RESULTS

The experimental setup and procedure were as described in the previous studies of (n,α) reactions,[3] so only the details specific to the present experiment are given here.

Fast neutrons were produced by the ^{3}H(d,n)^{4}He reaction on a thin, water cooled, titanium tritide target. The deuteron beam energy was 2 MeV, the beam current ~ 25 μA. The tritium content of the ^{3}H-Ti target, 0.9 mg/cm^2 thick, on a copper support, was ~ 4 Ci. To produce neutrons of the chosen energy, a suitable neutron emission angle was selected. The neutron energy spread [full width at half maximum (FWHM)] was calculated by a Monte Carlo method, taking into account the experimental geometry, the emission angle dependence of the neutron energy, and the deuteron energy loss in the ^{3}H-Ti target, and this gave results of 140 and 200 keV for 14.3 and 18.15 MeV neutrons, respectively. The neutron flux was monitored by counting the recoil protons from a polyethylene foil 66.6 mg/cm^2 thick with a thin CsI(Tl) scintillator counter. The accuracy in determining neutron fluence is estimated to be about 6%. The targets, with thicknesses between 1.5 and 4 mg/cm^2, were prepared by sedimentation of Zr oxides onto thick aluminum backings. The isotopic enrichment was equal to 91.1 and 97.7%, respectively, for ^{91}Zr and ^{90}Zr. The alpha energy was measured by silicon surface barrier detectors. The alpha particles of smaller energy, produced by the (n,α) reaction in the detector by the incident neutrons, were used for the energy calibration. The energy resolution due to neutron energy spread, alpha energy loss, geometrical conditions, and counter energy resolution was estimated to vary from 400 to 650 keV (FWHM). The alpha particle spectra were measured at seven angles varying from 24.5° to 157° with an angular resolution of about $\pm 16°$.

The angle integrated spectra of alpha particles from ^{90}Zr and ^{91}Zr at 18.15 MeV incident neutron energy are shown in Fig. 1; the angle integrated spectra of alpha particles, from the same nuclei, at 14.3 MeV, are shown in Fig. 2.

Due to the negative Q values for (n,α) reactions on ^{16}O, ^{27}Al, and ^{28}Si, α particles from the backing, the oxide part of the target, and the detector were prevented from contributing to the background in the energy regions of interest in present work.

The angular distributions of the alpha particles display a forward peaking and decrease smoothly with increasing the observation angle. In Fig. 3, as an example, the angular distribution of alpha particles from ^{90}Zr at 18.15 MeV is reported.

III. COMPARISON WITH RESULTS FROM OTHER REACTIONS

The dominant processes contributing to the (n,α) cross section may be found from the results of one and two nucleon transfer reactions leading to the same residual nuclei. This section is devoted to a summary of these arguments.

The most energetic alphas from the ^{90}Zr(n,α)^{87}Sr reaction at 18.15 MeV display, as shown in Fig. 1, a structured spectrum for excitation energies of ^{87}Sr varying from 0 to 2.2 MeV, with a centroid at approximately 1 MeV and a pronounced minimum at 2.2 MeV. The nucleus ^{87}Sr has only ten excited states from 0 to 2 MeV of excitation energy. Their dominant configurations may be found by considering the results of other transfer reactions.

The sequence of proton and neutron states for $A \approx 90$ nuclei, as deduced from neutron and proton transfer reactions, is shown in Fig. 4. For both ^{90}Zr and ^{91}Zr all proton orbits up to $2p_{1/2}$ are filled. In ^{90}Zr the magic $28 < N \leq 50$ shell is completed; in ^{91}Zr the 51st neutron is in the $2d_{5/2}$ orbit.

Single neutron hole states in ^{87}Sr may be excited in the (n,α) process if the two protons of the α were on the $2p_{1/2}$ orbit in ^{90}Zr. The same states are also strongly excited in the ^{88}Sr(p,d)^{87}Sr reaction.[4,5] In Fig. 1(a) the excitation energies of these states and the spectroscopic factors for neutron pick-up, as reported by Blok et al.,[5] are shown.

It is also possible for the (n,α) reaction to take place by the removal of one neutron and two protons from different orbits and, among the possibilities, the transfer of a $2p_{1/2}2p_{3/2}$ proton pair is the most energetically favored. The final states which are excited by this process can be found from the ^{89}Y(d,α)^{87}Sr reaction because the ground state of ^{89}Y has one $2p_{1/2}$ proton and filled lower orbitals. The study of this reaction[6] indeed provides evidence that the states of ^{87}Sr at 1.229 and 1.739 MeV are the $\frac{5}{2}^+$ and $\frac{13}{2}^+$ members of the multiplet arising from the coupling of a $\pi(2p_{1/2}^{-1}2p_{3/2}^{-1})_{2+}$ proton core excitation with a $1g_{9/2}^{-1}$ neutron hole. The excitation energies of these states are shown in Fig. 1(b). Below about 2.2 MeV of excitation energy, there are no states of ^{87}Sr excited in the (n,α) process by removal of other proton pairs.

If the reaction proceeds by the knock-on mechanism, then single neutron states of ^{87}Sr could also be excited. At the excitation energies we are considering, this happens by removal of a $2p_{1/2}^2$ proton pair and a $1g_{9/2}^2$ neutron pair, followed by capture of the incident neutron in the $2d_{5/2}$ or $3s_{1/2}$ orbits. These states are excited in the ^{86}Sr(d,p)^{87}Sr process, and in Fig. 1(c) the excitation energies and spectroscopic factors for their excitation, in that reaction,[7] are shown. Only two states of ^{87}Sr below 2 MeV, at 1.921 and 1.997 MeV, have not been considered. The first of these has been observed to decay to the g.s. of ^{87}Sr with appreciable gamma intensity in the ^{87}Rb(p,nγ)^{87}Sr (Ref. 8) and ^{84}Kr(α,nγ)^{87}Sr (Ref. 9) reactions. It is very weakly excited, both in the ^{88}Sr(p,d)^{87}Sr (Ref. 5) and ^{89}Y(d,α)^{87}Sr (Ref. 6) reactions. The level at 1.997 MeV has been observed only in the ^{86}Sr(n,γ)^{87}Sr reaction.[10] Most likely, these states have a configuration that cannot be excited in a one step (n,α) process.

The yield of alphas from ^{91}Zr is appreciable only for excitation energies of ^{88}Sr above 3.5—4 MeV. Above 4 MeV the density of states of ^{88}Sr is exceedingly high and one cannot hope to identify with certainty all the possible excited states. However, on the basis of the evidence collect-

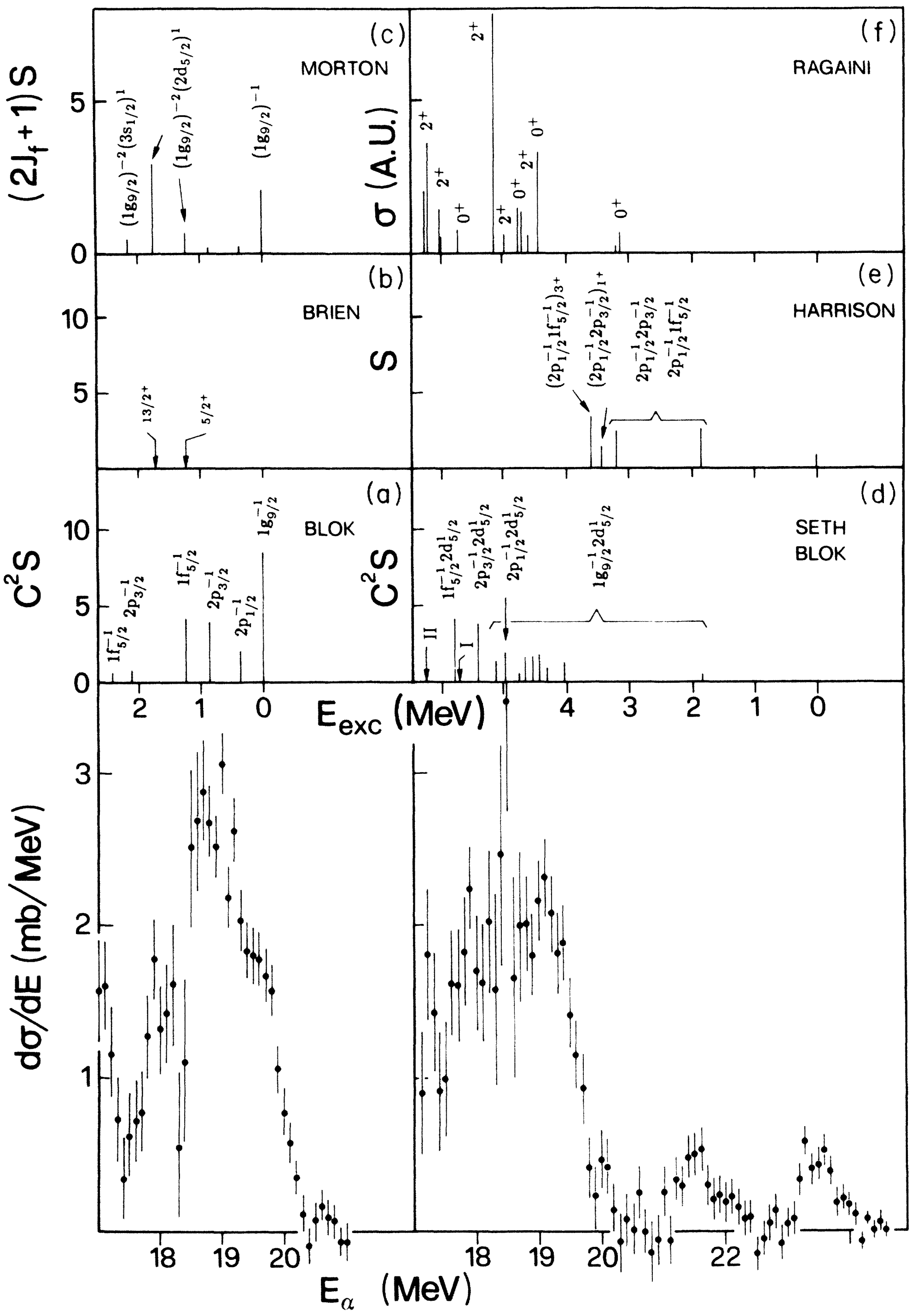

FIG. 1. Angle integrated spectra of alpha particles emitted from ^{90}Zr and ^{91}Zr at 18.1 MeV incident neutron energy. The energies or centroid energies of levels likely to contribute to the two spectra are reported in panels (a)—(f) of this figure. The height of the bars indicating these energies are, in most cases, proportional to the spectroscopic factors for excitation of these levels in the reactions indicated. (a) Excitation energy and spectroscopic factors for excitation of neutron hole states of ^{87}Sr in the ^{88}Sr(p,d)^{87}Sr reaction (Ref. 5). (b) Excitation energies of $[\pi(2p_{1/2}^{-1}2p_{3/2}^{-1})_{2^+}\nu(1g_{9/2})^{-1}]_{5/2^+,13/2^+}$ two proton—one neutron hole states of ^{87}Sr (Ref. 6). (c) Excitation energy and spectroscopic factors for exciting single neutron states of ^{87}Sr in neutron stripping reactions (Ref. 7). (d) Excitation energies and spectroscopic factors for excitation of $1g_{9/2}^{-1}2d_{5/2}^{1}$ neutron particle-hole states of ^{88}Sr in neutron stripping reactions (Refs. 11 and 12); estimated centroid energies and spectroscopic factors for $2p_{1/2}^{-1}2d_{5/2}^{1}$, $2p_{3/2}^{-1}2d_{5/2}^{1}$, and $1f_{5/2}^{-1}$-$2d_{5/2}^{1}$ neutron particle-hole multiplets of ^{88}Sr in neutron pick-up processes and estimated excitation energies (I and II) of centroids of $[\pi(2p_{1/2}^{-1}2p_{3/2}^{-1})_{2^+}\nu(1g_{9/2})^{-1}]_{5/2^+,13/2^+}\nu(2d_{5/2})^1$ multiplets. (e) Spectroscopic factor for pick-up of the $2d_{5/2}$ neutron and excitation energies and spectroscopic factors for excitation of $2p_{1/2}^{-1}2p_{3/2}^{-1}$ and $2p_{1/2}^{-1}1f_{5/2}^{-1}$ two proton hole states of ^{88}Sr in the ^{89}Y(d,^{3}He)^{88}Sr reaction (Ref. 13). (f) Excitation energies and spectroscopic factors for excitation of $(2d_{5/2})^2_{0^+,2^+}$ two neutron states of ^{88}Sr in the ^{86}Sr(t,p)^{88}Sr reaction (Ref. 14).

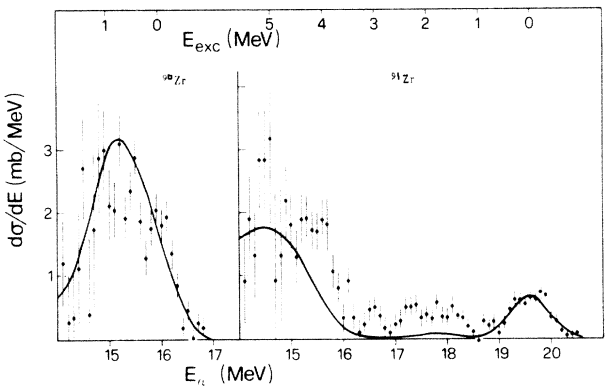

FIG. 2. Comparison of the experimental angle integrated spectra from Zr isotopes at 14.3 MeV incident neutron energy with those calculated by pick-up theory.

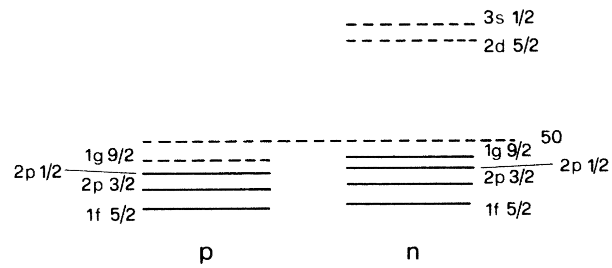

FIG. 4. Sequence of proton and neutron single particle states in $A \sim 90$ nuclei.

ed in the study of (p,α) reactions in Ref. 2, we assume that most of the measured alpha yield corresponds to transitions to states that are *homologous* to the low energy ^{87}Sr states resulting from the weak coupling of the excited states of the ^{87}Sr core with the spectator $2d_{5/2}$ neutron outside the $N \leq 50$ neutron shell. The correctness of this conjecture will be confirmed or disproved by the comparison of the experimental alpha yields with those calculated on the basis of this assumption. Obviously, the pick-up of the $2d_{5/2}$ neutron leads to the excitation of states that are not homologous to the ^{87}Sr states. However, as discussed later, these processes contribute in a minor way to the measured spectrum.

The excitation energies and spectroscopic factors for exciting, in the ^{87}Sr(d,p)^{88}Sr reaction, $1g_{9/2}^{-1}2d_{5/2}^{1}$ neutron particle-hole states of ^{88}Sr (which correspond to the $1g_{9/2}^{-1}$ ground state of ^{87}Sr) are shown in Fig. 1(d). In this part of the figure are also reported the excitation energies and spectroscopic factors for exciting the $2p_{1/2}^{-1}2d_{5/2}^{1}$, $2p_{3/2}^{-1}2d_{5/2}^{1}$, and $1f_{5/2}^{-1}2d_{5/2}^{1}$ neutron particle-hole states of ^{88}Sr, in a neutron pick-up process. These are not directly measured, but are estimated on the hypothesis of a weak coupling of the neutron hole states of ^{87}Sr(4,5) with the $2d_{5/2}$ neutron outside the magic shell. By this procedure only the centroid energy of the multiplet of states due to the coupling and the sum of the spectroscopic factors corresponding to these states may be estimated. The centroid energy is estimated by adding to the excitation energy of the neutron hole states of ^{87}Sr the centroid energy of the $1g_{9/2}^{-1}2d_{5/2}^{1}$ states of ^{88}Sr, estimated to be about 4.57 MeV.[11,12] This tentative procedure does not take accurately into account pairing energy effects. The sum of the spectroscopic factors for excitation of the states of these multiplets is assumed to be equal to that for exciting the corresponding neutron hole states of ^{87}Sr.

The estimated centroid energy of the

$$[\pi(2p_{1/2}^{-1}2p_{3/2}^{-1})_{2^+}\nu(1g_{9/2})^{-1}]_{5/2^+,\,13/2^+}\nu(2d_{5/2})^1$$

multiplets is also given in this part of the figure (I and II). States excited via pick-up of the $2d_{5/2}$ neutron include the g.s. of ^{88}Sr, excited when the two removed protons were on the $2p_{1/2}$ orbit, and the states strongly populated in the ^{89}Y(d,^{3}He)^{88}Sr reaction[13] which, in the ^{91}Zr(n,α)^{88}Sr process, may be reached by pick-up of the $2d_{5/2}$ neutron and two protons, one of which is the $2p_{1/2}$, while the other is one on a deeper orbit in the $28 < Z \leq 50$ shell. Their excitation energies and the spectroscopic factors, for their excitation in the (d,^{3}He) process, are shown in Fig. 1(e). The contribution of these processes to the (n,α) reaction should be relatively unimportant, as shown by the fact that a small alpha yield is measured at energies corresponding to these states. States populated via pick-up of the $2d_{5/2}$ neutron and two protons coupled to 0^+ on a deep orbit might also be excited. However, we could not identify these states from the sequence of known levels of ^{88}Sr.

Finally, the knock-out from ^{91}Zr of the two $2p_{1/2}$ protons and two $1g_{9/2}$ neutrons, each pair coupled to 0^+, with subsequent capture of the incident neutron in the $2d_{5/2}$ orbit, might excite a number of 0^+ and 2^+ levels also populated in the ^{86}Sr(t,p)^{88}Sr reaction.[14] The excitation energies of these states and the peak cross sections for their population in the (t,p) reaction on ^{86}Sr are shown in Fig. 1(f). Some 4^+ states might also be populated by this

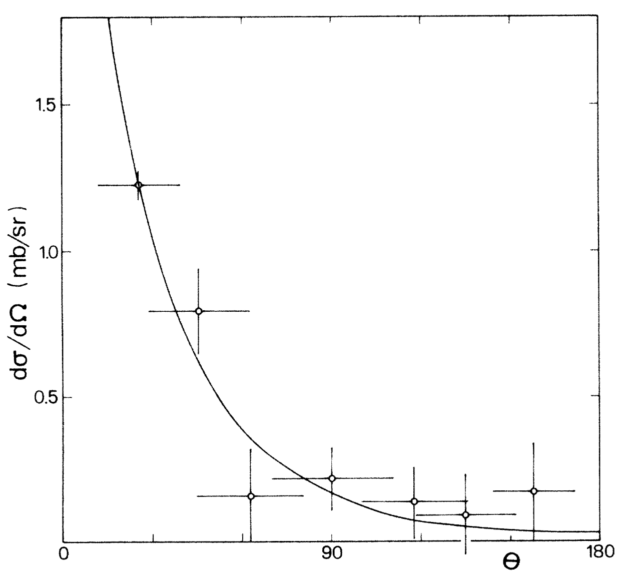

FIG. 3. Comparison of the experimental angular distribution of alpha particles from ^{90}Zr(n,α)^{87}Sr reaction at 18.15 MeV incident neutron energy, corresponding to the residual nucleus excitation energy interval 0—1.75 MeV, with that calculated by pick-up theory.

mechanism, but from the results of Ragaini *et al.*[14] we estimate that most of them are located at excitation energies above 6 MeV.

Comparison of the energies of the states expected—from the above considerations—to contribute to the (n,α) reaction with the observed α-particle energy spectra, shows that these states are indeed in the energy region where we observe the greatest yield of emitted α particles, strongly supporting the validity of the above arguments. More detailed comparisons can be made with microscopic calculations and these are described in the next section.

IV. THEORETICAL INTERPRETATION

The alpha spectra have been analyzed using both pick-up and knock-on distorted wave Born approximation (DWBA) theories. In the following we discuss the results and the corresponding conclusions.

A. Pick-up theory

Our discussion is based on the semimicroscopic approach of Smits and Siemssen.[15] In the case of an even-even target nucleus, like ^{90}Zr, in absence of coherent contributions from different configurations of the residual nucleus B, the following relation holds,

$$\sigma_{\rm exp}(\theta)=(A^{LJ}_{\alpha_B J_B})^2\sigma_{\rm PU,DW}(\theta) , \quad (1)$$

where $\sigma_{\rm exp}(\theta)$ is the measured angular distribution, $\sigma_{\rm PU,DW}(\theta)$ is the angular distribution evaluated in the DWBA, and the spectroscopic amplitude $A^{LJ}_{\alpha_B J_B}$ for the excitation of a given state is given by

$$A^{LJ}_{\alpha_B J_B}=N(A)\left[\frac{A}{A-3}\right]^{N+L/2}\left[\frac{A-2}{A}\right]^{\bar{N}+\bar{J}/2}$$

$$\times G_{\rm n}G_{\bar{N}\bar{J}}\{S^{NLJ}_{j_{\rm n},\bar{N}\bar{J}}\}^{1/2} , \quad (2)$$

where α_B denotes the internal coordinates of B. N, L, and J are, respectively, the radial quantum number and the orbital and total momenta of the transferred three-nucleon cluster, A is the target nucleus mass, $N(A)$ a mass dependent normalization factor, and $G_{\rm n}$ and $G_{\bar{N}\bar{J}}$ the spectroscopic amplitudes for, respectively, the pick-up of the neutron and of the two protons.

To excite the neutron hole states of ^{87}Sr, shown in Fig. 1(a), the $(2p_{1/2})^2_{0^+}$ proton pair and a neutron from one of the gpf orbits of the $28<N\leq 50$ shell must be picked up. The number of nodes and the angular momentum of the proton pair are equal to $\bar{N}=3$ and $\bar{J}=0$ and the total momentum $j_{\rm n}$ of the transferred neutron is that of the corresponding orbit. The two proton spectroscopic amplitudes are those of Glendenning,[16] and the neutron spectroscopic amplitudes are those obtained by Blok *et al.*[5] by comparison of experimental and calculated ^{88}Sr(p,d)^{87}Sr cross sections. For each transition only one value of N, L, and J is allowed. The algebraic factors $\{S^{NLJ}_{j_{\rm n},\bar{N}\bar{J}}\}^{1/2}$ are tabulated by Smits.[17] In Table I the excitation energy, the configuration of the neutron hole states, the values of N, L, and J, and the calculated spectroscopic amplitudes are given. In the case of the excitation of the $\frac{5}{2}^+$ and $\frac{13}{2}^+$ states of ^{87}Sr, at 1.230 and 1.739 MeV, assumed to be members of a

$$[\pi(2p^{-1}_{1/2}2p^{-1}_{3/2})_{2^+}\nu(1g_{9/2})^{-1}]$$

multiplet, $\bar{N}$ and $\bar{J}$ are both equal to 2.

For the $\frac{5}{2}^+$ state, N,L,J are, respectively, equal to $4,2,\frac{5}{2}$, and for the $\frac{13}{2}^+$ state they are equal to $2,6,\frac{13}{2}$. In these cases the two proton spectroscopic amplitude has also been taken from Glendenning, while the neutron spectroscopic amplitude has been set equal to $\sqrt{2j_{\rm n}+1}=\sqrt{10}$. The corresponding estimated spectroscopic amplitudes are also given in Table I.

To evaluate the cross section using the TWOFNR code[18] in the *finite range* approximation, the optical model elas-

TABLE I. Excitation energy and configuration of states of ^{87}Sr assumed to be populated in a pick-up process on ^{90}Zr. Values of N, L, and J, of the estimated spectroscopic amplitudes and of the normalized angle integrated DWBA cross sections are also reported.

$E_{\rm exc}$ (MeV)	Configuration	N	L	J	$A^{LJ}_{\alpha_B J_B}/N(A)$	$\sigma_{\rm PU,DW}$ (mb)
0.0	$\nu(1g_{9/2})^{-1}$	3	4	$\frac{9}{2}$	0.1411	44.79
0.388	$\nu(2p_{1/2})^{-1}$	4	1	$\frac{1}{2}$	0.1333	48.68
0.873	$\nu(2p_{3/2})^{-1}$	4	1	$\frac{3}{2}$	0.1864	51.83
1.23	$[\pi(2p^{-1}_{1/2}2p^{-1}_{3/2})_{2^+}\nu(1g_{9/2})^{-1}]_{5/2^+}$	4	2	$\frac{5}{2}$	0.0537	56.94
1.257	$\nu(1f_{5/2})^{-1}$	3	3	$\frac{5}{2}$	0.1314	31.35
1.739	$[\pi(2p^{-1}_{1/2}2p^{-1}_{3/2})_{2^+}\nu(1g_{9/2})^{-1}]_{13/2^+}$	2	6	$\frac{13}{2}$	0.1537	14.5
2.11	$\nu(2p_{3/2})^{-1}$	4	1	$\frac{3}{2}$	0.0766	44.02
2.236	$\nu(1g_{9/2})^{-1}$	3	4	$\frac{9}{2}$	0.0464	27.58
2.422	$\nu(1f_{5/2})^{-1}$	3	3	$\frac{5}{2}$	0.0542	24.76
2.66	$\nu(2p_{3/2})^{-1}$	4	1	$\frac{3}{2}$	0.0442	44.02

tic scattering wave functions of the incident neutron and the outgoing alpha particle have been calculated with the parameters given in Table II and taken, respectively, from Refs. 19 and 20. In Table II the geometrical factors for evaluating the helion bound state wave function are also given. These are the values suggested by Peterson and Rudolph for the triton bound state, in the analysis of (p,α) reactions on Zr isotopes at about 23 MeV incident proton energy.[21] The neutron-helion interaction potential was assumed to be of Gaussian form,

$$V(r_{\rm nh})=V_0\exp\left[-\left[\frac{r_{\rm nh}}{\xi}\right]^2\right], \tag{3}$$

with ξ equal to 2 fm. For V_0 we assumed a value of 40 MeV. However, this numerical value is not very significant since, as it is well known, the theory is unable to predict correct absolute values and the calculated cross sections must be multiplied by an overall normalizing factor which, however, is independent on the nuclei and the particular transition considered and the incident energy. The calculated angle integrated cross sections, for each transition, multiplied by the square of the corresponding spectroscopic amplitudes, were spread using a Gaussian distribution with $\sigma=0.3$ MeV to simulate the effect of the experimental energy resolution. Finally, all the yields were added to give the final calculated spectrum that was then normalized to the experimental one.

The products of the angle integrated cross section evaluated by the TWOFNR code, at 18.1 MeV incident neutron energy, times the normalizing factor $[N(A)]^2$, are given in last column of Table I.

The shape of the measured alpha particle spectrum is satisfactorily reproduced as shown in Fig. 5. Also, the shape of the calculated angular distributions is in fair agreement with the experimental findings, as shown, as a typical example, in Fig. 3.

The use of the same normalizing coefficient allows one to obtain a satisfactory reproduction also of the measured alpha spectrum at 14.3 MeV, as shown in Fig. 2.

The analysis of the alpha spectrum from ^{91}Zr is much more tentative. As previously stressed, we expect in this case to excite predominantly states which are homologous to the excited states of ^{87}Sr. We first consider the states corresponding to the coupling of the $2d_{5/2}$ neutron with the neutron hole states in ^{87}Sr. As discussed in Sec. III, one directly knows the energies and the neutron spectroscopic factors for exciting the $1g_{9/2}^{-1}$-$2d_{5/2}^{1}$ neutron particle-hole states. On the contrary, the centroid energies and the sum of neutron pick-up spectroscopic factors of the states of the $2p_{1/2}^{-1}2d_{5/2}^{1}$, $2p_{3/2}^{-1}2d_{5/2}^{1}$, and $1f_{5/2}^{-1}2d_{5/2}^{1}$ multiplets can be only estimated. The same procedure is adopted for evaluating the centroid energy of the multiplet of states homologous to the 1.230 MeV ($\frac{5}{2}^+$) and 1.739 MeV ($\frac{13}{2}^+$) states of ^{87}Sr.

Assuming the spectator role of the $2d_{5/2}$ neutron, the same formalism as for even even targets may be applied to calculate these cross sections. In addition to the states considered above, a number of states of ^{88}Sr that are not homologous of the states of ^{87}Sr are expected to be excited, as discussed in Sec. III. These are the states populated via pick-up of the $2d_{5/2}$ neutron and include the g.s. of ^{88}Sr and the $2p_{1/2}^{-1}2p_{3/2}^{-1}$ and $2p_{1/2}^{-1}1f_{5/2}^{-1}$ two proton hole states investigated with the ^{89}Y(d,^{3}He)^{88}Sr reaction.[13] While the spectroscopic amplitude for the g.s. transition may be easily evaluated because the transferred N, L, and J are uniquely determined, several different values of N, L, and J coherently contribute to the other transitions considered. Taking into account the uncertainty affecting the calculations of form factors, it is unlikely that one can obtain reliable estimates of the cross sections for these transitions.

States populated via pick-up of the $2d_{5/2}$ neutron and pairs of protons of the same orbits coupled to 0^+ are also expected to be excited. The excitation energies of these 0^+ states, whose spectroscopic amplitudes could be easily evaluated because they have fixed N, L, and J values, are unknown, so their contribution to the alpha particle spectrum cannot be estimated.

For these reasons only the ground state and the states of ^{88}Sr homologous to the ^{87}Sr states contribute to our calculated spectrum. On the basis of experimental data discussed in Ref. 2, we expect that this is indeed the most important contribution to the experimental spectrum; however, we have, *a priori*, to expect that, using the same overall normalizing factor as in the case of the ^{90}Zr(n,α)^{87}Sr reaction, the α spectrum from ^{91}Zr will be somewhat underestimated.

In Table III the excitation energy, the assumed configuration of the residual nucleus states considered, and the

TABLE II. Optical model parameters utilized for the evaluation of the elastic scattering wave functions of the incident neutron and outgoing alpha particle, and geometrical factors utilized in pick-up calculations for the helion bound state and in knock-on calculations for the neutron and alpha particle bound states. The notation and the potential forms are the ones adopted by Perey and Perey (Ref. 22) (index d means derivative, index so means spin orbit).

	V (MeV)	r (fm)	a (fm)	W (MeV)	r_W (fm)	a_W (fm)	W_d (MeV)	r_d (fm)	a_d (fm)	$V_{\rm so}$ (MeV)	$r_{\rm so}$ (fm)	$a_{\rm so}$ (fm)	r_c (fm)
n	41.59	1.28	0.66				8.56	1.25	0.48	6.2	1.01	0.75	
α	187.3	1.44	0.52	22.3	1.44	0.52							1.3
Pick-up													
$h_{\rm BS}$		1.38	0.35										
Knock-on													
$\alpha_{\rm BS}$		1.6	0.67										
$n_{\rm BS}$		1.35	0.75										

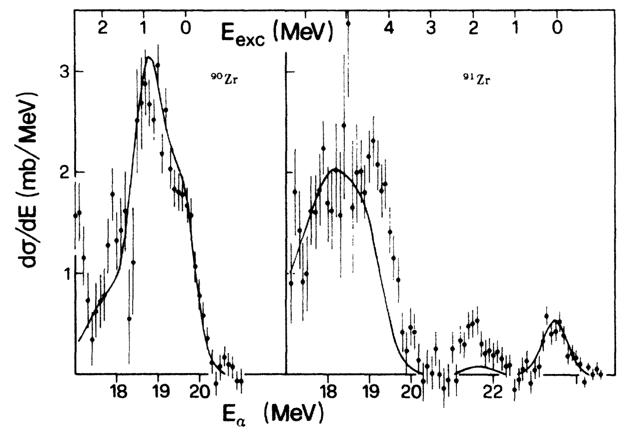

FIG. 5. Comparison of the experimental angle integrated alpha particle spectra at 18.15 MeV incident neutron energy with those calculated by pick-up theory.

corresponding values of N, L, and J are given, together with the spectroscopic amplitudes and the product of the angle integrated cross sections evaluated by the TWOFNR code times the normalizing factor used for reproducing the alpha spectrum in the case of ^{90}Zr(n,α)^{87}Sr reaction.

Also, in this case the alpha spectrum is calculated by adding all the cross sections reported in Table III, for 18.1 MeV incident neutron energy, multiplied by the squares of the calculated spectroscopic amplitudes and spread with a Gaussian distribution of standard deviation equal to 0.3 MeV for transitions to single states and equal to 0.6 MeV for transitions to a multiplet of states, of which only the centroid energy is estimated. The spectrum thus obtained is compared in Fig. 5 to the experimental one for the incident neutron energy 18.15 MeV. As expected, the calculation underestimates the measured yield in the case of the 1.836 MeV level and for excitation energies between 3.5 and 4.5 MeV, where there are transitions that we did not take into account. The excellent reproduction of the absolute cross section for populating the ^{88}Sr ground state is remarkable.

Also in this case, as shown in Fig. 2, the measured spectrum at 14.3 MeV is reproduced with the same accuracy.

The comparison of the theoretical and experimental results clearly indicates that a good reproduction of the data may be achieved on the hypothesis of a pick-up mechanism. The energy dependence of the measured spectra is reasonably well reproduced in all the cases considered. A unique normalizing factor (the only free parameter in all the calculation) is used to reproduce the absolute cross sections. The underestimate of the alpha particle spectrum from ^{91}Zr, at excitation energies lower than 4.5 MeV, is to be expected because part of the contributions has not been evaluated for the reasons already discussed.

TABLE III. Configuration and excitation energy [or centroid energy (CE) for unresolved multiplets] of states that have been considered for evaluating the spectrum of alpha particles in the ^{91}Zr(n,α)^{88}Sr reaction in the hypothesis of a pick-up mechanism. Values of N, L, and J, of the spectroscopic amplitudes and of the normalized angle integrated DWBA cross sections are also reported.

E_{exc} (MeV)	Configuration	N	L	J	$A^{LJ}_{\alpha_B J_B}/N(A)$	$\sigma_{PU,DW}$ (mb)
0.0	$\pi(2p_{3/2})^4\nu(1g_{9/2})^{10}$	4	2	$\frac{5}{2}$	0.0753	70.10
1.838					0.0315	60.35
4.032					0.0528	
4.294					0.0481	
4.431					0.0648	
4.45	$\nu(1g_{9/2})^{-1}(2d_{5/2})^1$	3	4	$\frac{9}{2}$	0.0206	38.9
4.514					0.0628	
4.633					0.0619	
4.744					0.0333	
5.094					0.0591	
4.958	$\nu(2p_{1/2})^{-1}(2d_{5/2})^1$	4	1	$\frac{1}{2}$	0.1331	45.54
5.443 (CE)					0.1861	
6.681 (CE)	$\nu(2p_{3/2})^{-1}(2d_{5/2})^1$	4	1	$\frac{3}{2}$	0.0764	48.07
7.23 (CE)					0.0441	
5.827 (CE)					0.1313	
6.992 (CE)	$\nu(1f_{5/2})^{-1}(2d_{5/2})^1$	3	3	$\frac{5}{2}$	0.0542	23.84
7.68 (CE)					0.044	
7.95 (CE)					0.0476	
5.799 (CE)	$[\pi(2p_{1/2}^{-1}2p_{3/2}^{-1})_{2^+}\nu(1g_{9/2})^{-1}]_{5/2^+}\nu(2d_{5/2})^1$	4	2	$\frac{5}{2}$	0.0536	51.78
6.309 (CE)	$[\pi(2p_{1/2}^{-1}2p_{3/2}^{-1})_{2^+}\nu(1g_{9/2})^{-1}]_{13/2^+}\nu(2d_{5/2})^1$	2	6	$\frac{13}{2}$	0.1536	12.57

B. Knock-on theory

The knock-on calculations are made with the cluster approximation. The target nucleus A is considered to consist of a core C and an alpha particle in a bound state, and the residual nucleus B of the core C and the incident neutron captured in a bound state after the interaction.[18,23]

We assume that the alpha particle comprises pairs of neutrons and protons coupled to 0^+. After the interaction the neutron may be captured in a state either vacated by the two neutrons of the pair knocked out or in a state previously empty. In the first case, the states already considered in the pick-up reactions (with the notable exception of states populated via the pick-up of two protons from different orbits) will be populated; in the second case, single neutron states of ^{87}Sr, or two neutron states of ^{88}Sr, will be excited, as discussed in Sec. III.

With these assumptions, in the absence of configuration mixing, both in the case of ^{90}Zr and ^{91}Zr (if one makes the hypothesis that only the 0^+ ^{90}Zr core participates to the process) only one value of the transferred orbital, spin, and total angular momenta (l,s,j) and of the orbital and total angular momenta of the alpha and the neutron bound states, is allowed in a given transition. The form factor for the transition matrix can be factorized into the product of a spectroscopic amplitude A_{lsj} times a dynamical term.

The spectroscopic amplitude A_{lsj} is the product of the alpha particle spectroscopic amplitude $A^{\alpha}_{n_\alpha l_\alpha}$ times the neutron spectroscopic amplitude $A^{\mathrm{n}}_{l_\mathrm{n} j_\mathrm{n}}$. The cross section for a given transition is then

$$\sigma_{\exp}(\theta)=(A^{\alpha}_{n_\alpha l_\alpha}A^{\mathrm{n}}_{l_\mathrm{n} j_\mathrm{n}})^2\sigma_{\mathrm{KO,DW}}(\theta)\ , \tag{4}$$

where $\sigma_{\mathrm{KO,DW}}(\theta)$ is the DWBA cross section evaluated with the TWOFNR code,[18] as for the pick-up calculations.

In the case of both ^{90}Zr and ^{91}Zr, the experimental data discussed in Sec. III indicate that only the $(2p_{1/2})^2_{0^+}$ proton pair is involved in the transitions contributing to the measured spectra. The $(1g_{9/2})^2_{0^+}$, $(2p_{1/2})^2_{0^+}$, $(2p_{3/2})^2_{0^+}$, and $(1f_{5/2})^2_{0^+}$ neutron pairs may be knocked out, depending on the final states considered. Now we make the additional hypothesis that the alpha particle spectroscopic amplitude $A^{\alpha}_{n_\alpha l_\alpha}$ is, in all cases, proportional to the corresponding two neutron spectroscopic amplitude $\mathscr{A}^{2\mathrm{n}}_{\bar{N},0}$ that has been taken from Glendenning.[16] This tentative assumption is based on the idea that the alpha particle spectroscopic amplitude can be factorized into the product of two nucleon spectroscopic amplitudes by a proper generalization of the methods of Smits and Siemssen.[15] The neutron spectroscopic amplitude $A^{\mathrm{n}}_{l_\mathrm{n} j_\mathrm{n}}$ is given by the square root of the ratio of the number of vacancies and the total number of states in the considered orbit. If a residual nucleus configuration is fragmented over several states, the strength of the transitions to each one of these states is evaluated by partitioning the total strength proportionately to the measured spectroscopic factors for population of these states in the corresponding single neutron transfer reactions considered in Sec. III. The corresponding weight factor is indicated by W. Under these hypotheses, the final expression for the cross section in a knock-on process is

$$\sigma_{\exp}(\theta)=N(\mathscr{A}^{2\mathrm{n}}_{\bar{N},0}A^{\mathrm{n}}_{l_\mathrm{n} j_\mathrm{n}})^2 W\sigma_{\mathrm{KO,DW}}(\theta)\ , \tag{5}$$

where N is a normalizing factor that is assumed to be the same for all the transitions and the two target nuclei considered.

In Tables IV and V the residual nucleus levels, the corresponding configurations, the calculated spectroscopic amplitudes, the square roots of weighting coefficients for evaluating the partition of a given cross section among fragmented residual nucleus states, and the product of the normalizing factor N times the calculated angle integrated DWBA cross section at 18.1 MeV are reported for ^{90}Zr and ^{91}Zr.

In the evaluation of $\sigma_{\mathrm{KO,DW}}(\theta)$, the elastic scattering wave functions of the incident neutron and the outgoing alpha are the same as in pick-up calculations. The geometrical factors relative to the bound state wave functions are also reported in Table II. Their value is close to that used in previous knock-on calculations.[24] The neutron-alpha particle interaction potential was assumed to be of the Woods-Saxon type

$$W(r_{\mathrm{n}\alpha})=V_0\left[1+\exp\left[\frac{r_{\mathrm{n}\alpha}-R_{\mathrm{WS}}}{a_{\mathrm{WS}}}\right]\right]^{-1}\ , \tag{6}$$

with $V_0=70$ MeV, $R_{\mathrm{WS}}=1.3$ fm, and $a_{\mathrm{WS}}=0.2$ fm. Also in this case, the V_0 value is not very significant since the calculated cross sections must be normalized to the experimental ones to get the correct absolute value. As in the case of pick-up calculations, the normalizing factor is independent of the nucleus and the transition and the incident energy considered. Variations of R_{WS} and a_{WS} by, respectively, ±0.2 and ±0.1 fm, do not produce appreciable modifications in the calculated results.

The calculated cross sections, multiplied by the square of the spectroscopic amplitudes and the weight factors W, were spread using Gaussian distributions, as in the case of the pick-up calculations, and added to give the final calculated spectra that were finally normalized to the experimental ones using the same factor for both targets.

The comparison with the experiment, for 18.15 MeV incident neutron energy, is shown in Fig. 6, and it is notable that several qualitative features are correctly reproduced and also that the calculated relative intensities of the two alpha particle spectra are in agreement with the experimental findings. However, a detailed reproduction of the data is not obtained. In the case of ^{90}Zr the cross section for population of the g.s. of ^{87}Sr is greatly underestimated. An *ad hoc* variation of the spectroscopic amplitude for this transition, to match the measured yield, would also lead to a great overestimate of the measured spectrum at 1.779 MeV of excitation, corresponding to the excitation of the single neutron $\frac{5}{2}^+$ state at 1.779 MeV of ^{87}Sr. The same conclusions essentially hold in the case of ^{91}Zr where, in addition, the measured yield below $\sim$4 MeV excitation energy necessarily implies a pick-up contribution to the reaction.

Due to the somewhat tentative procedure we adopted, it

TABLE IV. Excitation energy and configuration of states that have been considered for evaluating the spectrum of alpha particles in the ^{90}Zr(n,α)^{87}Sr reaction in the hypothesis of a knock-on mechanism. Spectroscopic amplitudes, weighting factors, and normalized angle integrated DWBA cross sections are also reported.

E_{exc} (MeV)	Configuration	$\mathscr{A}_{\bar{N},0}$	$A^{n}_{l_n j_n}$	$\sqrt{W}$	$\sigma_{\text{KO,DW}}$ (mb)
0.0	$\nu(1g_{9/2})^{-1}$	0.0889	0.447	0.917	70.83
0.388	$\nu(2p_{1/2})^{-1}$	0.2953	1.0	1.0	11.32
0.873	$\nu(2p_{3/2})^{-1}$	0.4177	0.707	0.989	22.02
1.231	$\nu(1g_{9/2})^{-2}(2d_{5/2})^{1}$	0.0889	1.0	0.351	120.36
1.257	$\nu(1f_{5/2})^{-1}$	0.1461	0.577	0.836	10.56
1.779	$\nu(1g_{9/2})^{-2}(2d_{5/2})^{1}$	0.0889	1.0	0.7	120.36
2.11	$\nu(2p_{3/2})^{-1}$	0.4177	0.707	0.406	20.25
2.175	$\nu(1g_{9/2})^{-2}(3s_{1/2})^{1}$	0.0889	1.0	5.29	61.50
2.236	$\nu(1g_{9/2})^{-1}$	0.0889	0.447	0.302	62.97
2.422	$\nu(1f_{5/2})^{-1}$	0.1461	0.577	0.345	10.56
2.66	$\nu(2p_{3/2})^{-1}$	0.4177	0.707	0.235	20.25
2.681	$\nu(1g_{9/2})^{-2}(2d_{5/2})^{1}$	0.0889	1.0	0.241	120.36

TABLE V. Configuration and excitation energy [or centroid energy (CE) for unresolved multiplets] of states that have been considered for evaluating the spectrum of alpha particles in the ^{91}Zr(n,α)^{88}Sr reaction in the hypothesis of a knock-on mechanism. Spectroscopic amplitudes, weighting factors, and normalized angle integrated DWBA cross sections are also reported.

E_{exc} (MeV)	Configuration	$\mathscr{A}_{\bar{N},0}$	$A^{n}_{l_n j_n}$	$\sqrt{W}$	$\sigma_{\text{KO,DW}}$ (mb)
1.838				0.205	61.96
4.032				0.344	
4.294				0.313	
4.431				0.422	
4.45	$\nu(1g_{9/2})^{-1}(2d_{5/2})^{1}$	0.0889	0.447	0.134	70.35
4.514				0.409	
4.633				0.402	
4.744				0.217	
5.094				0.385	
4.958 (CE)	$\nu(2p_{1/2})^{-1}(2d_{5/2})^{1}$	0.2953	1.0	1.0	11.31
5.443 (CE)				0.989	
6.681 (CE)	$\nu(2p_{3/2})^{-1}(2d_{5/2})^{1}$	0.4177	0.707	0.406	22.70
7.230 (CE)				0.235	
5.827 (CE)				0.836	
6.992 (CE)	$\nu(1f_{5/2})^{-1}(2d_{5/2})^{1}$	0.1461	0.577	0.345	8.16
7.680 (CE)				0.278	
7.950 (CE)				0.303	
3.151				0.086	96.74
4.484	$\nu(2d_{5/2})^{2}_{0^+}$	0.0889	0.913	0.189	104.54
4.794				0.125	106.07
5.766				0.090	110.09
3.22				0.056	97.17
4.619				0.102	
4.763				0.150	
4.983				0.102	107.4
5.165	$\nu(d_{5/2})^{2}_{2^+}$	0.0889	0.913	0.361	
6.005				0.095	
6.048				0.150	
6.270				0.245	111.32
6.307				0.190	
6.512				0.171	

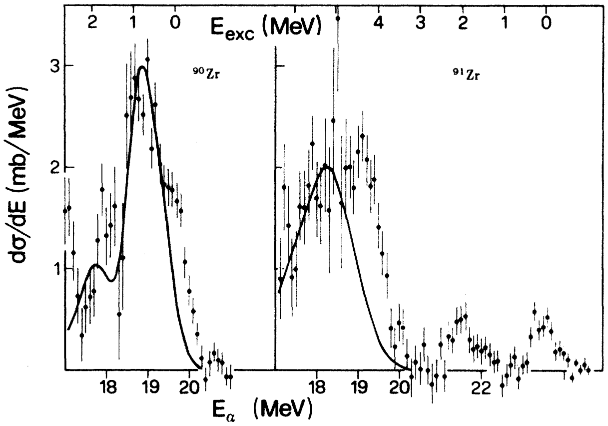

FIG. 6. Comparison of the experimental angle integrated alpha particle spectra at 18.15 MeV incident neutron energy with those calculated by knock-on theory.

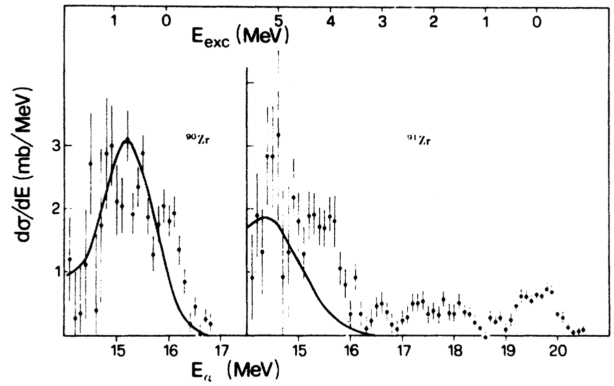

FIG. 7. Comparison of the experimental angle integrated alpha spectra at 14.3 MeV incident neutron energy with those calculated by knock-on theory.

appears premature to draw definite conclusions from such a comparison; however, the presence of a knock-on contribution to these reactions does not seem to be necessary, although it cannot be excluded.

Essentially the same results are obtained from the comparison of experimental and calculated alpha particle spectra at 14.3 MeV, as shown in Fig. 7.

V. CONCLUSIONS

A microscopic calculation of the alpha spectra in the (n,α) reactions induced on the ^{90,91}Zr isotopes at 18.1 and 14.3 MeV incident neutron energy has been made.

Both pick-up and knock-on theories have been considered and in both cases only one overall normalizing factor, which is the same for the two target nuclei and incident energies, appears as a free parameter in the calculation.

Pick-up calculations provide a very satisfactory reproduction of the data, and a discrepancy between experimental results and calculations appears only in the case of the ^{91}Zr(n,α)^{88}Sr reactions, where there are contributions that have not been explicitly considered, requiring the coherent superpositions of many terms in the transition matrix element.

Knock-on calculations reproduce many qualitative features of the measured spectra, but do not allow a fully satisfactory reproduction of them. While the results obtained do not exclude knock-on contributions to these reactions, their presence is not established.

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