Interacting boson model calculation of octupole states in deformed nuclei

A. F. Barfield*
Department of Physics, University of Arizona, Tucson, Arizona 85721

J. L. Wood
School of Physics, Georgia Institute of Technology, Atlanta, Georgia 30332

B. R. Barrett†
Physics Division, National Science Foundation, Washington, D.C. 20550

(Received 17 July 1986)

An interacting boson model study of octupole states in deformed rare-earth nuclei is discussed. The parameters and octupole bandhead energies are given for ^{154}Sm, ^{158}Gd, ^{168}Er, ^{172}Yb, and ^{178}Hf. Detailed spectra and $B(E3)$ transition rates are presented for ^{168}Er. Reasonable agreement with experiment is obtained for eight of the nine nuclei studied.

In the geometrical model, octupole states are associated with collective vibrational degrees of freedom of the nucleus,[1] leading to a simple picture for the $K^{\pi}=0^-$, 1^-, 2^-, and 3^- bands commonly seen in deformed nuclei. Octupole bands are identified experimentally by enhanced $E3$ transition rates between the ground state and the $I^{\pi}=3^-$ member of the band. It is often found that a known octupole state is also strongly populated in single-nucleon transfer reactions.[2-5] Thus, such collective states have a complex nature, with one or more large 2qp components. Octupole states in deformed nuclei have been described microscopically (see, e.g., Neergard and Vogel[6]) and have also been treated macroscopically (see, e.g., Ref. 7).

In this paper we report on a study of octupole bands in deformed rare-earth nuclei, carried out within the interacting boson model (IBM).[8-10] While this model has been widely applied to positive parity collective states in nuclei, previous applications to octupole states have been limited in scope. Octupole bands in ^{152}Sm (Ref. 11), ^{156}Gd (Ref. 12), and ^{168}Er (Refs. 13 and 14) have been studied in some detail. The transition from spherical to deformed nuclei has been studied in the samarium isotopes.[15] Several other vibrational and translational nuclei have also been considered (see, e.g., Ref. 16). In the present study, the IBM phenomenology is applied systematically to a range of nuclei within the deformed region, in an attempt to obtain a global fit with smoothly varying parameters.

Negative parity states are described in the IBM octupole model by adding a single angular momentum $L=3$ boson with intrinsic negative parity (an f boson) to the usual s-d model space.[8,10] This is done in the IBM-1 framework, in which neutron and proton degrees of freedom are not separately distinguished. The total number of bosons is conserved,

$$N=n_s+n_d+n_f \;, \tag{1}$$

where N is the total number of bosons, n_s, n_d, and n_f are the numbers of s, d, and f bosons, respectively, and $n_f=0$ or 1. The Hamiltonian for the combined system is

$$H=H_{sd}+H_f+V_{sdf} \;, \tag{2}$$

where H_{sd} describes the positive parity core, H_f is the f-boson Hamiltonian, and V_{sdf} describes the f-sd interaction. The sd core Hamiltonian employed in this study is the so-called "consistent-Q" Hamiltonian,[17] which can be written

$$H_{sd}=a_1L_d\cdot L_d+a_2Q_d\cdot Q_d \;, \tag{3}$$

where the dipole and quadrupole operators are defined by

$$L_d=\sqrt{10}(d^\dagger\times\tilde{d})^{(1)} \tag{4}$$

and

$$Q_d=s^\dagger\times\tilde{d}+d^\dagger\times s+\chi(d^\dagger\times\tilde{d})^{(2)} \;. \tag{5}$$

The f-boson Hamiltonian is given by

$$H_f=\epsilon_f\hat{n}_f \;, \tag{6}$$

where ϵ_f is the f-boson energy. The interaction between the f boson and the sd core is given by[18]

$$V_{sdf}=A_1L_d\cdot L_f+A_2Q_d\cdot Q_f+A_3{:}E^\dagger_{df}\cdot E_{df}{:} \;, \tag{7}$$

where

$$L_f=2\sqrt{7}(f^\dagger\times\tilde{f})^{(1)} \;, \tag{8}$$

$$Q_f=-2\sqrt{7}(f^\dagger\times\tilde{f})^{(2)} \;, \tag{9}$$

and

$$E^\dagger_{df}=\sqrt{5}(d^\dagger\times\tilde{f})^{(3)} \;. \tag{10}$$

The first term in Eq. (7) is of minor importance; most of the structure comes from the quadrupole-quadrupole and exchange terms, which have strengths A_2 and A_3, respectively.[12,18] The normal-ordered exchange term can be shown to arise naturally from an octupole-octupole interaction.[18,19]

The same $E2$ transition operator is utilized for both positive and negative parity states, since it is assumed that the $E2$ collectivity is carried by the d boson. This operator is given by[17]

$$T^{(E2)}=eQ_d \ . \tag{11}$$

The $E3$ transition operator can be written[18,19]

$$T^{(E3)}=e_3[(s^\dagger\times\tilde{f}+f^\dagger\times s)+\chi_3(d^\dagger\times\tilde{f}+f^\dagger\times\tilde{d})]^{(3)} \ . \tag{12}$$

Finally, reduced transition rates are defined by

$$B(EL;J_i\to J_f)=[1/(2J_i+1)]\,|\langle J_f||T^{(EL)}||J_i\rangle|^2 \ . \tag{13}$$

The parameters a_1 and a_2 in Eq. (3) are determined by fitting to the ground state (g.s.) and gamma bands, without any consideration of excited 0^+ states. This choice is made because it is not always clear that excited 0^+ states in the rare-earth region belong to the s-d boson model space.[20] The quadrupole parameter χ [Eq. (5)] is determined from the experimental ratio

$$B(E2;2_2^+\to 0_1^+)/B(E2;2_1^+\to 0_1^+) \ ,$$

in the usual consistent-Q manner.[17] It should be noted that all previous IBM octupole studies, except that of Refs. 13 and 19, have used the SU(3) form of the quadrupole operator, in which the parameter χ has the value $(-\sqrt{7})/2$, rather than the more general form given in Eq. (5). The parameters ϵ_f, A_2, and A_3 depend on the energy ordering of the experimental octupole bands. Phenomenological details can be found in Refs. 18 and 21.

The selection of octupole bands from among the low-lying experimental negative parity bands is made on the basis of known $B(E3)$ transition rates. No attempt is made to fit low-lying 2qp bands; these are manifestly outside of the IBM model space. Band assignments for the calculated negative parity states are determined from the $B(E2)$ transition rates, with intraband transition matrix elements assumed to be much larger than corresponding interband matrix elements.

The nuclei considered in the study are ^{154}Sm, ^{156}Gd, ^{158}Gd, ^{158}Dy, ^{162}Dy, ^{168}Er, ^{172}Yb, ^{178}Hf, and ^{182}W. They were selected because of the availability of extensive data and because they represent a broad cross section within the deformed rare-earth region. The IBM parameters and energies for low-lying octupole bandheads are presented in Table I for five of the nuclei studied. (Detailed results for all nine nuclei will be published later[21] and are available in Ref. 18.) The octupole effective charge, e_3, is constant for all of the nuclei studied. Thus, the calculated $B(E3)$'s

TABLE I. IBM parameters and low-lying octupole bandhead energies for five nuclei. Energies and all parameters, except χ, e_3, and χ_3 are given in keV.

		^{154}Sm	^{158}Gd	^{168}Er	^{172}Yb	^{178}Hf
Positive parity parameters						
a_1		0.0	3.0	6.5	4.5	5.0
a_2		−36.0	−27.5	−17.5	−22.5	−27.5
$\sqrt{5}\chi$		−1.6	−1.4	−1.1	−1.8	−1.1
Negative parity parameters						
ϵ_f		1430	1260	1340	1250	750
A_1		10	10	0	0	0
A_2		−50	−60	−42	−57	−37
A_3		0	−120	−135	−145	−150
e_3 (e b$^{3/2}$)[a]		0.07	0.07	0.07	0.07	0.07
χ_3[a]		−0.5	−0.3	4.1	3.5	1.7
Bandhead energies						
$K^\pi=0^-$	Calc.	996	1207	1798	1582	
	Expt.	992[b]	1263[c]	1786[d]	1600[e]	f
$K^\pi=1^-$	Calc.	1373	1026	1344	1142	1308
	Expt.	1476[b]	977[c]	1358[d]	1155[e]	1310[g]
$K^\pi=2^-$	Calc.		1726	1577	1753	1259
	Expt.	f	1793[c]	1569[d]	1757[e]	1260[g]

[a]The input parameters for the computer code FBEM (Ref. 22) are $E3=e_3$ and $E3DF=\sqrt{7}e_3\chi_3$.
[b]Reference 23.
[c]Reference 7.
[d]Reference 28.
[e]Reference 24.
[f]Not known experimentally.
[g]Reference 25.

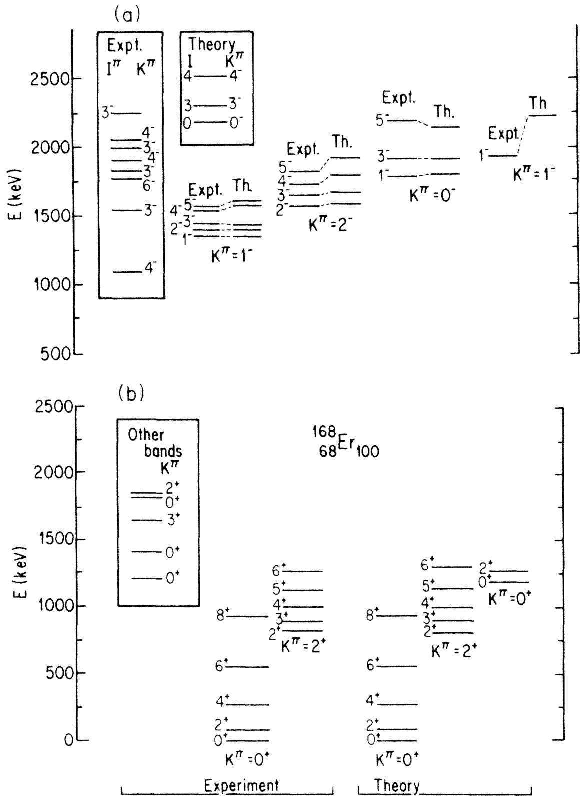

FIG. 1. Energy spectra for ^{168}Er. Negative parity spectra are shown in (a). Experimental 2qp bandheads between ~ 1 and 2 MeV are shown in an inset, along with the collective 3^- level at 2.27 MeV. Three calculated bandheads between 2 and 2.5 MeV are also shown in an inset. Positive parity spectra are shown in (b). The g.s. and gamma bands are shown, as well as the first two levels in the calculated beta band. Several excited K^+ bandheads are shown in the insets. The data are from Refs. 4, 5, 26, 28, and 31.

are effectively determined by one parameter, χ_3. The variation in this parameter reflects different ways in which the $E3$ strength is distributed among the low-lying experimental octupole bands. The f-boson energy, ϵ_f, and the f-sd quadrupole and exchange strengths, A_2 and A_3, are found to be similar but do not vary smoothly with mass number and, thus, cannot be predicted for neighboring nuclei by simple extrapolation or interpolation. In view of the known dominance of many octupole states by 2qp components, this is not surprising (although disappointing).

More detailed results are given for the nucleus ^{168}Er, as an example. This nucleus has been particularly well characterized experimentally. Studies of this nucleus include (d,d$'$),[26] (α,α') Coulomb excitation,[27] (n,γ) and (n,e$^-$),[28] (p,p$'$),[29] (n,n$'\gamma$),[30] (d,p) and (t,d),[31] ($\vec{t}$,α),[4] (t,p) and (p,t),[5] and (d,d$'$) and (α,α') inelastic scattering.[14] From such studies, it is known that the lowest $K^\pi=0^-$, 1^-, and 2^- bands have octupole collectivity, along with several higher-lying bands, and that the $K^\pi=3^-$, 4^-, and 6^- bands between ~ 1 and 2 MeV are 2qp in nature. The bandhead energies for these 2qp bands are shown in Fig. 1(a), along with a comparison between theory and experiment for the octupole bands. The theory predicts a second relatively low-lying $K^\pi=1^-$ band. The bandhead is shown in Fig. 1(a) and compared with the second experimental $K^\pi=1^-$ bandhead. The nature of this latter band has not been determined experimentally. The 3^- level at 2269 keV is collective and presumably corresponds to the calculated 3^- bandhead at 2297 keV. Two other higher-lying calculated bandheads are also shown in Fig. 1(a). The calculated and experimental g.s. and gamma bands are shown in Fig. 1(b), as well as the two lowest calculated levels in the beta band and excited K^+ experimental bandheads between ~ 1 and 1.9 MeV.

Table II compares the calculated $B(E3)$'s for ^{168}Er

TABLE II. $B(E3;0_1^+\rightarrow 3^-)$ for ^{168}Er. Reduced $E3$ transition rates (in e^2b^3) are given from the g.s. to the $I=3$ member of the K^- bands. The excitation energies for the transitions are also given (in MeV).

	IBM		RPA[a]		Expt.[b]	
K^π	$E(3^-)$	$B(E3)$	$E(3^-)$	$B(E3)$	$E(3^-)$	$B(E3)$
1^-	1.43	0.046[c]	1.6	0.041	1.43	0.046(5)
3^-	d		1.4	0.002	1.54	0.003(1)
2^-	1.67	0.066	1.7	0.055	1.63	0.058(6)
3^-	d				1.83	0.007(2)
0^-	1.91	0.038	2.2	0.016	1.91	0.023(3)
3^-	d				2.00	0.005(1)
1^-	2.40	0.009			2.02	
3^-	2.30	0.048			2.27	0.055(6)[e]
0^-	2.65	0.028			2.32	0.018(2)[e]
2^-	2.72	0.003				
1^-	2.78	0.022			2.49	0.020(2)[e]
2^-	2.86	0.016				
0^-	3.06	0.001				

[a]Reference 6; results after Coriolis coupling.
[b]Reference 14.
[c]Normalized to experiment.
[d]2qp band, not in IBM space.
[e]K value not experimentally determined.

with experiment and also with the microscopic random-phase-approximation (RPA) calculations of Neergard and Vogel.[6] The energies of the 3^- states are given also, for comparison. The distribution of $E3$ strength among the experimental bands is well reproduced by the theory, even though the 3^- energies are too high, in general, for the higher-lying bands. (Comparison at higher energies should be made with caution because of the limited fitting to the experimental positive parity states.) The 2qp $K^\pi=3^-$ bands are within the domain of the RPA calculations but outside of the IBM model space. We note that our calculated $B(E3)$'s in Table II and spectra in Fig. 1(a) do not agree with the IBM calculations reported by Govil *et al.*[14] Using the parameters given in Ref. 14 and our method of assigning states to K bands, we do not reproduce their band structure. It appears that the authors of Ref. 14 did not consider the intraband $B(E2)$'s in making band assignments.

The detailed ^{168}Er results are consistent with the available experimental data. Seven of the other eight nuclei studied are also reasonably well described by the theory, although the data base is not always extensive enough to tie down the parameters.[18] In contrast, the octupole states for the nucleus ^{162}Dy cannot be reproduced by the present model.[18,21] Thus, it is clear that the picture is incomplete.

Nucleus by nucleus, with the exception of ^{162}Dy, the model gives a reasonable description of the states fitted. In addition, it predicts higher-lying band structure. On a global scale, the parameter sets are similar but cannot be predicted for neighboring nuclei. More experimental data are needed, both to test the predictions of the model and to extend the scope of the calculations. The present study illustrates the power of the IBM in correlating data and providing a simple view of nuclear structure.

ACKNOWLEDGMENTS

The authors are indebted to O. Scholten for many helpful discussions. One of us (A.F.B.) would like to thank G. Leander and UNISOR, and also O. Scholten and the National Superconducting Cyclotron Laboratory, for their hospitality at Oak Ridge National Laboratory and Michigan State University, respectively, where many of the calculations were done. This work was supported in part by the National Science Foundation under Grant No. PHY-84-05172 and by the Department of Energy under Contract Nos. DE-AS05-80ER10599 and DE-AC05-76OR00033.

---

*Present address: II Physikalisches Institut, Universität Göttingen, D-3400 Göttingen, Federal Republic of Germany.
†Permanent address: Department of Physics, University of Arizona, Tucson, AZ 85721.

[1]A. Bohr and B. R. Mottelson, *Nuclear Structure* (Benjamin, Reading, Mass., 1975), Vol. II.
[2]D. G. Burke, D. E. Nelson, and C. W. Reich, Nucl. Phys. **A124**, 683 (1969).
[3]R. A. O'Neil and D. G. Burke, Nucl. Phys. **A182**, 342 (1972).
[4]D. G. Burke, W. F. Davidson, J. A. Cizewski, R. E. Brown, and J. W. Sunier, Nucl. Phys. **A445**, 70 (1985).
[5]D. G. Burke, W. F. Davidson, J. A. Cizewski, R. E. Brown, E. R. Flynn, and J. W. Sunier, Can. J. Phys. **63**, 1309 (1985).
[6]K. Neergard and P. Vogel, Nucl. Phys. **A145**, 33 (1970).
[7]R. C. Greenwood, C. W. Reich, H. A. Baader, H. R. Koch, D. Breitag, O. W. B. Schult, B. Fogelberg, A. Bäcklin, W. Mampe, T. von Egidy, and K. Schreckenbach, Nucl. Phys. **A304**, 3327 (1978).
[8]A. Arima and F. Iachello, Phys. Lett. **57B**, 39 (1975).
[9]A. Arima and F. Iachello, Ann. Phys. (N.Y.) **99**, 253 (1976).
[10]A. Arima and F. Iachello, Ann. Phys. (N.Y.) **111**, 201 (1978).
[11]J. Konijn, J. B. R. Berkhout, W. H. A. Hesselink, J. J. van Ruijven, P. van Nes, and H. Verheul, Nucl. Phys. **A373**, 397 (1982).
[12]J. Konijn, F. W. N. deBoer, A. van Poelgeest, W. H. A. Hesselink, M. J. A. deVoigt, H. Verheul, and O. Scholten, Nucl. Phys. **A352**, 191 (1981).
[13]A. F. Barfield, B. R. Barrett, O. Scholten, and J. L. Wood, Bull. Am. Phys. Soc. **30**, 1276 (1985).
[14]I. M. Govil, H. W. Fulbright, D. Cline, E. Wesolowski, B. Kotlinski, A. Bäcklin, and K. Gridnev, Phys. Rev. C **33**, 793 (1986).
[15]O. Scholten, F. Iachello, and A. Arima, Ann. Phys. (N.Y.) **115**, 325 (1978).
[16]R. De Leo, M. Pignanelli, W. T. A. Borghols, S. Brandenburg, M. N. Harakeh, H. J. Lu, and S. Y. van der Werf, Phys. Lett. **165B**, 30 (1985), and references therein.
[17]D. D. Warner and R. F. Casten, Phys. Rev. Lett. **48**, 1385 (1982).
[18]A. F. Barfield, Ph.D. thesis, University of Arizona, 1986.
[19]A. F. Barfield and O. Scholten (unpublished).
[20]J. L. Wood, Nucl. Phys. **A421**, 43c (1984).
[21]A. F. Barfield, J. L. Wood, B. R. Barrett, and O. Scholten (unpublished).
[22]O. Scholten, The Program-Package PHINT, Kernfysisch Versneller Instituut Internal Report No. 63, 1979, and private communication.
[23]B. Harmatz, Nucl. Data Sheets **26**, 281 (1979).
[24]J. R. Cresswell, P. D. Forsyth, D. G. E. Martin, and R. C. Morgan, J. Phys. G **7**, 235 (1981).
[25]B. Fogelberg and A. Bäcklin, Nucl. Phys. **A171**, 353 (1971).
[26]P. O. Tjøm and B. Elbek, Nucl. Phys. **A107**, 385 (1968).
[27]F. K. McGowan, W. T. Milner, R. L. Robinson, P. H. Stelson, and Z. W. Grabowski, Nucl. Phys. **A297**, 51 (1978).
[28]W. F. Davidson, D. D. Warner, R. F. Casten, K. Schreckenbach, H. G. Börner, J. Simić, M. Stojanovic, M. Bogdanović, S. Koički, W. Gelletly, G. B. Orr, and M. L. Stelts, J. Phys. G **7**, 455 (1981).
[29]T. Ichihara, H. Sakaguchi, M. Nakamura, T. Noro, F. Ohtani, H. Sakamoto, H. Ogawa, M. Yosoi, M. Ieiri, N. Isshiki, Y. Takeuchi, and S. Kobayashi, Phys. Lett. **149B**, 55 (1984).
[30]S. W. Yates, E. D. Kleppinger, and E. W. Kleppinger, J. Phys. G **11**, 877 (1985).
[31]D. G. Burke, B. L. W. Maddock, and W. F. Davidson, Nucl. Phys. **A442**, 424 (1985).