Bose-Fermi symmetry chain for the description of odd-odd nuclei

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A Bose-Fermi dynamical symmetry scheme to describe odd-odd nuclei is proposed. A better agreement with the experimental spectra of ^{196}Au and ^{198}Au is obtained as compared to previous studies.

Recently introduced dynamical supersymmetry schemes[1,2] have been very successful in studying correlations between the energy spectra and the electromagnetic transition probabilities of even-even and odd-even nuclei. In particular, the U(6/4) supersymmetry[1] was applied to the nuclei in the supermultiplet (^{194}Pt, ^{195}Au, ^{196}Hg*,...) and the U(6/12) supersymmetry[2] to the nuclei in the supermultiplet (^{194}Pt, ^{195}Pt, ^{196}Pt*,...). The existence of these two schemes together implies the possibility of having a larger supersymmetry describing the odd-odd nucleus ^{196}Au as well.[3] However, the simplest such scheme[3-5] incorporating both the U(6/4) and U(6/12) chains, could not account for the correct ground state angular momentum of this nucleus. Since very little data are available for ^{196}Au, the theoretical predictions of this scheme were instead compared[3,4] with the experimental spectra for ^{198}Au. It was found that[4] most of the low-lying levels of ^{198}Au could not be accounted for either. In this article we present an alternative symmetry chain which yields the correct ground state angular momentum and greatly improves the description of the low-lying states.

In general, the group structure of the interacting boson-fermion Hamiltonian[6] is $U_B(6)\times U_F(m_\pi+m_\nu)$, where m_π (m_ν) is the dimension of the unpaired proton (neutron) subspace. The group chain studied in Refs. 3 and 4 starts with the decomposition

$$U_B(6)\times U_F(m_\pi+m_\nu)\supset U_B(6)\times U_F(m_\pi)\times U_F(m_\nu) \ . \quad (1)$$

A new possibility arises when $m_\pi=m_\nu=m$ and when the odd-proton is holelike and the odd-neutron is particlelike or *vice versa*. When many orbitals are involved, the average particle or hole character of the nucleons could be considered. The odd fermions should be placed in m-dimensional representations of the group $U_F(m)$. There are two m-dimensional representations of this group: the fundamental representation (denoted by □ in the Young tableau notation) and its conjugate representation (denoted[7] by ⊡). We place the particlelike fermions in the former and holelike fermions in the latter representation. A similar distinction for bosons was previously considered in Ref. 8; here the distinction is made for fermions. Such a scheme naturally leads[9] to the parabolic rule[10] for odd-odd nuclei.

A detailed study of this Bose-Fermi symmetry scheme will be given elsewhere.[11] In this article we concentrate on the description of the excitation spectra of the odd-odd nuclei. It follows from the above discussion that we have a Bose-Fermi symmetry[12] with a group structure

$$U_B(6)\times U_F(m)\times \overline{U}_F(m)\supset U_B(6)\times U_F(m) \ , \quad (2)$$

where the overbar on $\overline{U}_F(m)$ reminds us that the conjugate representation (⊡) is employed and the last $U_F(m)$ is the group obtained by taking the direct product of the first $U_F(m)$ and $\overline{U}_F(m)$.

In the Pt-Au region, the odd-neutron occupies mostly the levels $j^\nu=\frac{1}{2}^-,\frac{3}{2}^-,\frac{5}{2}^-$ ($p_{1/2},p_{3/2},f_{5/2}$), and the odd-proton occupies the levels $j^\pi=\frac{1}{2}^+,\frac{3}{2}^+,\frac{5}{2}^+$ ($s_{1/2},d_{3/2},d_{5/2}$). For this region $m_\pi=m_\nu=m=12$. We assume that for ^{196}Au and ^{198}Au nuclei, the neutron orbitals enumerated above can be considered as particlelike.[13] Since the protons are holelike, unpaired protons and neutrons can be placed in conjugate representations and the scheme described in Eq. (2) may be applicable. Accordingly, we form the group chain

$$\begin{aligned} &U_B(6)\times U_F(12)\times \overline{U}_F(12)\supset U_B(6)\times U_F(12)\supset U_B(6)\times U_F(6)\times SU_F(2)\\ &\supset SO_B(6)\times SO_F(6)\times SU_F(2)\supset \mathrm{Spin}_{B+F}(6)\times SU_F(2)\\ &\supset \mathrm{Spin}_{B+F}(5)\times SU_F(2)\supset \mathrm{Spin}_{B+F}(3)\times SU_F(2)\supset \mathrm{Spin}_{B+F}(3) \ , \end{aligned} \quad (3)$$

where we used the same notation as in Ref. 2. In general, the corresponding Hamiltonian would contain a large number of parameters. We choose the appropriate parameters of the Hamiltonian such that the representation with the $SU_F(6)$ adjoint and the $SU_F(2)$ singlet is the lowest one in energy for the odd-odd nuclei. Furthermore, a third parameter will be chosen such that the $(n,0,0)$ representation of the $SO_B(6)$ is the lowest one in energy for the even-even core which is the standard SO(6) choice.[14] Here n is the number of bosons in the even-even core. Under these assumptions, the low-lying spectra of the odd-odd nucleus is given by the following simplified energy formula:

$$E=E_0'+\delta[\sigma_1^F(\sigma_1^F+4)+\sigma_2^F(\sigma_2^F+2)]-\frac{A}{4}[\sigma_1(\sigma_1+4)+\sigma_2(\sigma_2+2)+\sigma_3^2]+\frac{B}{6}[\tau_1(\tau_1+3)+\tau_2(\tau_2+1)]+CL(L+1)\ , \tag{4}$$

where $(\sigma_1^F,\sigma_2^F,0)$ are the $SO_F(6)$ quantum numbers, $(\sigma_1,\sigma_2,\sigma_3)$ are the $\mathrm{Spin}_{B+F}(6)$ quantum numbers, (τ_1,τ_2) are the $\mathrm{Spin}_{B+F}(5)$ quantum numbers, and J is the quantum number of the final $\mathrm{Spin}_{B+F}(3)$. [The $SU_F(2)$ in Eq. (3) is in the singlet representation.]

The adjoint representation of $SU_F(6)$ contains two $SO_F(6)$ representations; one with $(\sigma_1^F=2,\sigma_2^F=0,\sigma_3^F=0)$ and another one with $(\sigma_1^F=1,\sigma_2^F=1,\sigma_3^F=0)$. Since, as discussed above, the contribution of the even-even core is expressed by the $(n,0,0)$ representation of $SO_B(6)$, the values of σ_1, σ_2, and σ_3 can be obtained from the multiplication rules

$$(n,0,0)\otimes(1,1,0)=(n,1,1)\oplus(n,1,-1)\oplus(n,0,0)\oplus(n+1,1,0)\oplus(n-1,1,0)\ , \tag{5a}$$

$$(n,0,0)\otimes(2,0,0)=(n+2,0,0)\oplus(n,0,0)\oplus(n-2,0,0)\oplus(n+1,1,0)\oplus(n-1,1,0)\oplus(n,2,0)\ . \tag{5b}$$

The decomposition of the representation $(\sigma_1,1,0)$ of $\mathrm{Spin}_{B+F}(6)$ into $\mathrm{Spin}_{B+F}(5)$ representations is given in Ref. 2. For the representation $(\sigma_1,1,1)$ the decomposition is

$$(\sigma_1,1,1)=(\sigma_1,1)\oplus(\sigma_1-2,1)\oplus(\sigma_1-2,1)\oplus\cdots\ . \tag{6}$$

Finally, the $\mathrm{Spin}_{B+F}(3)$ representations included in the above $\mathrm{Spin}_{B+F}(5)$ representations are given in Refs. 2 and 14.

In Eq. (4) $\delta>0$ and the representation $(1,1,0)$ of $SO_F(6)$ is lower in energy. Consequently, since $A>0$, the $(n+1,1,0)$ multiplet of $\mathrm{Spin}_{B+F}(6)$ represents the low-lying states of the odd-odd nuclei. Choosing $B>0$, the $(1,0)$ representation of $SO_{B+F}(5)$ is the ground state. Its angular momentum content is $L=2$. Our scheme thus predicts the correct ground state of ^{196}Au and ^{198}Au, as opposed to the earlier treatments[3,4] which yielded a wrong value $(L=1)$. A typical spectrum predicted by our scheme is shown in Fig. 1, where the boson number is taken to be $n=5$ (the same as the ^{196}Au core). Unfortunately, there are very little data available on the level scheme of ^{196}Au. If one assumes that the structure of the low-lying levels would not considerably change from ^{198}Au, one might expect to get a rough idea about the applicability of this scheme by examining the level scheme of ^{198}Au. A comparison of Fig. 1 with the experimental level scheme[15] for ^{198}Au is encouraging. In this figure a third band corresponding to the $(n,1,1)$ representation in Eq. (5a) is not shown. The two lowest levels of this band have $L=1$ and 3 [cf. Eq. (6) and Ref. 2], and the bandhead $(L=1)$ state can be placed at $\sim$200 keV by choosing the parameter A appropriately. Except for a low-lying 3^- state, there is a reasonable correspondence between the experimental spectra and the levels predicted by Eq. (4) for $E<300$ keV. In particular, the ground state spin is correct, and the experimentally observed three 1^- states and the 4^- state are accounted for. On the contrary, the scheme presented in Refs. 3 and 4 predicts the wrong ground state spin, and an additional low-lying 0^- state which is not experimentally seen. Furthermore, it cannot account for the experimentally observed 4^- state. These states cannot originate from the coupling of the positive-parity $i_{13/2}$ neutron orbit and the negative-parity $h_{11/2}$ proton orbit either, as was also pointed out in Ref. 4. Further experimental exploration of a low-lying 3^- state in ^{198}Au and a study of the low-lying negative parity states in ^{196}Au is requisite to establish the validity of our scheme in this region. Obviously, a study of the energy spectrum alone is not sufficient in assessing the significance of a new symmetry, especially since the energy

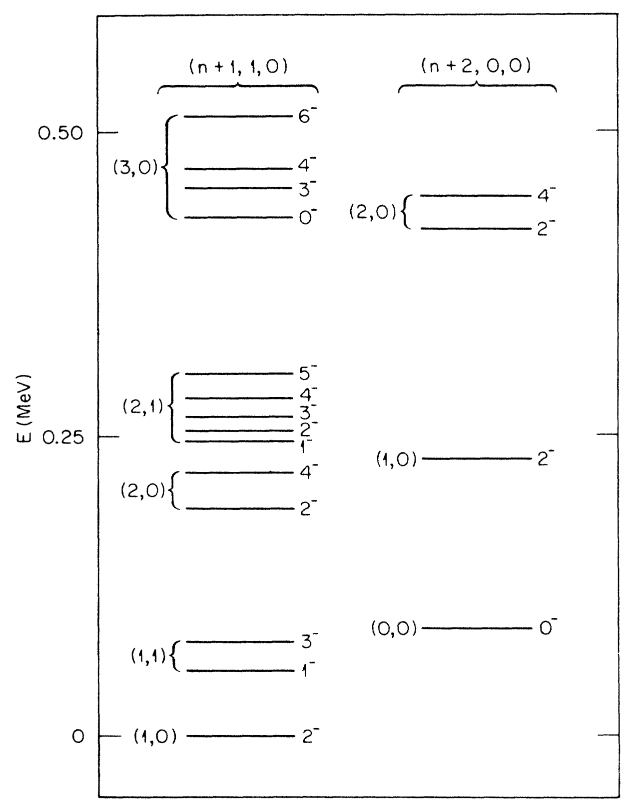

FIG. 1. A typical spectrum with the $SU_B(6)\times U_F(12)\times\bar{U}_F(12)$ symmetry. The energy levels are calculated using Eq. (8) where $4\delta-7A/2=230$ keV, $B=190$ keV, and $C=2$ keV. The labels for $\mathrm{Spin}_{B+F}(6)$ representations (at the top) and for $\mathrm{Spin}_{B+F}(5)$ representations (next to the levels) are also given. A third band starting at $\sim$300 keV is not shown.

differences are very small (~ 75 keV). It is also essential to study experimentally the electromagnetic transition rates, since such a study provides the best test of the wave functions of the model.

We also would like to point out that energies of the lowest-lying 0^-, 1^-, 2^-, and 3^- states lie on a parabola as a function of $L(L+1)$. A more complete discussion of the relation between the parabolic rule and the supersymmetry schemes for odd-odd nuclei is presented elsewhere.[9]

We have presented a new class of Bose-Fermi symmetries applicable to odd-odd nuclei and applied it to the odd-odd Au isotopes. We should indicate, however, a potential difficulty if one wants to extend the present scheme to a dynamical supersymmetry describing the neighboring even-even, odd-even, and even-odd nuclei in this region. Namely, the odd-proton isotopes ^{195}Au and ^{197}Au would not be satisfactorily described in such an extended scheme. In particular, the appropriate Spin(6) limit, which was shown[1] to be successful in describing these isotopes, cannot be obtained when the odd proton occupies three orbitals with $j=\frac{1}{2},\frac{3}{2},\frac{5}{2}$. Hence in this article no attempt has been made to extend the dynamical Bose-Fermi symmetry for odd-odd nuclei to a supersymmetry.

We have shown that for Au isotopes ($Z=79$) with $N=117$ and 119, taking the unpaired neutron to be particlelike yields a better description of the excitation spectra, and hence of the neutron-proton residual interaction. For negative parity odd-neutron orbitals this assumption may be reasonable, since there is a large gap between $3p_{3/2}$ and $1h_{9/2}$ orbits. In contrast, the treatments in Refs. 3 and 4 consider the unpaired neutrons to be holelike. In a more rigorous treatment, one could assume particle character for neutron bosons as well. Doing so does not significantly alter the results discussed above, since, as described in Ref. 9, the lowest-lying (<250 keV) states of odd-odd nuclei are not very sensitive to the description of the bosonic core.

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