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Three-pion correlations in relativistic heavy ion collisions

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We report the results of three-pion interferometry for two streamer chamber data samples: 1.8 GeV/nucleon Ar on Pb and 1.5 GeV/nucleon Ar on KCl. The fitted pion source parameters are consistent with previously reported results of two-pion interferometry. The effect of three-pion interference on the pion source parameters obtained from two-pion interferometry is also studied. The distortion becomes a serious problem in the data analysis only when the pion source has a small radius and low temperature.

The space-time structure of a pion emitting source can be studied through the correlation of the emitted like-pion pairs.[1,2] Such interferometric studies have been fruitfully utilized in relativistic heavy ion central collisions.[3–6] Many effects, such as final state interactions, nuclear shadowing, and averaging over different impact parameters and event topologies, can affect the interpretation of the pion interferometry results. It is therefore important to test the assumption that the enhancement of the correlation function in the low relative momentum region comes solely from Bose-Einstein statistics, and to verify the adequacy of the phenomenological Gaussian model[7] used to describe the pion emitting source in relativistic heavy ion collisions. With increasing statistics, the measurement of multipion correlations, especially three pion correlations,[8] becomes possible. If the above-mentioned assumptions underlying two-pion interferometry analyses are not valid, it is unlikely that completely independent three-pion analyses will yield consistent results.

The data samples for this investigation comes from two streamer chamber experiments at the Lawrence Berkeley Laboratory Bevalac, in which the systems 1.8 GeV/nucleon Ar on Pb_3O_4, and 1.5 GeV/nucleon Ar on KCl were studied. In the case of Ar on Pb_3O_4, a central collision trigger was used, resulting in the suppression of events corresponding to collisions of Ar on oxygen.[9] Table I provides details of the available statistics for each data sample. A momentum cut $p_{\mathrm{lab}} \geq 100$ MeV/c has been imposed to remove the effects of multiple scattering in the target, and possible electron contamination of the π^- sample.[9]

TABLE I. The number of events, average multiplicity $\langle M_{\pi^-} \rangle$, and number of three-pion correlated entries, $N_{3\pi^-}$, for a kinematic selection $p_{\mathrm{lab}} \geq 100$ MeV/c.

	1.8 GeV/nucleon Ar on Pb	1.5 GeV/nucleon Ar on KCl
Total events	3463	3845
$\langle M_{\pi^-} \rangle$	9.28	4.52
$N_{3\pi^-}$	259 192	30 243

For multiparticle interferometric studies, it is convenient to use a graphical technique to obtain the correlation function.[10] Biyajima[11] derived the general form of the three-pion correlation function and introduced both coherent and incoherent emitters to describe partially coherent sources. He showed that Bose-Einstein statistics for two-pion and three-pion correlations may be conveniently represented by Mueller-like diagrams. Figure 1 shows the two-pion correlation function $C(p_1,p_2)$ for a chaotic source. The diagram in the denominator represents the background. The numerator in Fig. 1 is composed of the background and the interference term of the two negative pions. If a wavy line is introduced to represent a pion emitted by a coherent emitter, it is easy to extend the Mueller-like diagrams to describe the two-pion and multipion correlations for a partially coherent source. For the two-pion correlation, Fig. 2(a) represents the background. Figure 2(b) represents the interference terms required by Bose-Einstein statistics for two negative pions. Here we neglect correlations from the simultaneous emission of both quanta by a single emitter.[12] Because there is no Bose-Einstein interference between coherent quanta,[13,14] no permutations between coherent lines are performed in constructing the interference diagrams in Fig. 2(b).

The contribution of a given diagram is the product of all the factors corresponding to all the lines in that diagram. The factor corresponding to a straight line is the sum of the Fourier transforms of the incoherent pion emitter characteristic functions,[12] and the factor corresponding to a wavy line is the Fourier transform of the coherent

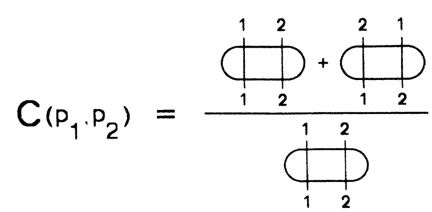

FIG. 1. The two-pion correlation function for a chaotic pion emitting source.

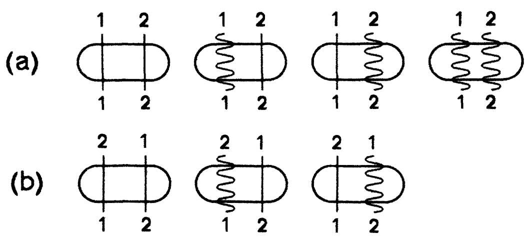

FIG. 2. (a) The background for two negative pions. (b) The interference effect of Bose-Einstein statistics for two negative pions.

emitter characteristic function.[11] In the Gaussian model, these factors are

$$n_{\rm in}\exp(-q_{ij0}^2\tau^2/4-q_{ij}^2R^2/4)$$

and

$$n_c\exp(-q_{ij0}^2\tau^2/4-q_{ij}^2R^2/4)\ ,$$

respectively, where $n_{\rm in}$ (n_c) is the average pion multiplicity from the incoherent (coherent) emitters, $q_{ij}=p_i-p_j$ and $q_{ij0}=E_i-E_j$ are the momentum and energy difference between the pions, and i and j are the starting and ending labels on the line.

The correlation function, i.e., the ratio of the correlated to the uncorrelated density distributions, is obtained by dividing the sum of all diagrams by the background. For a partially coherent Gaussian source, the two-pion correlation function is then given by

$$C(p_1,p_2)=1+\lambda\exp(-q_{120}^2\tau^2/2-q_{12}^2R^2/2)\ , \tag{1}$$

where

$$\lambda=(1+2\gamma)/(1+\gamma)^2 \tag{2}$$

and

$$\gamma=n_c/n_{\rm in}\ .$$

Similarly, Fig. 3 gives the Mueller-like diagrams for the three-pion correlations for a partially coherent source. The three-pion correlation function can be written as

$$C(p_1,p_2,p_3)=1+\lambda\exp[-(q_{120}^2\tau^2+q_{12}^2R^2)/2]+(2-3)+(3-1) \\ +2\xi\exp\{-[(q_{120}^2+q_{230}^2+q_{310}^2)\tau^2+(q_{12}^2+q_{23}^2+q_{31}^2)R^2]/4\}\ , \tag{3}$$

where

$$\xi=(1+3\gamma)/(1+\gamma)^3\ , \tag{4}$$

and $(i-j)$ represents the cyclically permuted form of the second term.

It is impractical to experimentally observe three pion interference in the six-dimensional manifold

$$q=\{q_{12},q_{23},q_{31},q_{120},q_{230},q_{310}\}\ .$$

In the following analysis, we will use the variables

$$Q_0^2=q_{120}^2+q_{230}^2+q_{310}^2$$

and

$$Q^2=M^2(3\pi)-9m_\pi^2\ ,$$

where $M(3\pi)$ is the invariant mass for the three-pion system. The variable Q was first suggested by Goldhaber.[15] Using Eq. (3), the correlated density distribution for three like pions, integrated over all variables other than Q and Q_0, is given by

$$\begin{aligned}\mathrm{Cor}(Q,Q_0)=&\{1+2\xi\exp[-Q_0^2\tau^2/4-(Q^2+Q_0^2)R^2/4]\}\mathrm{Uncor}(Q,Q_0)\\ &+\lambda\sum\exp(-q_{120}^2\tau^2/2-q_{12}^2R^2/2)\mathrm{Uncor}(q_{12},q_{120},Q,Q_0)\\ &+\lambda\sum\exp(-q_{230}^2\tau^2/2-q_{23}^2R^2/2)\mathrm{Uncor}(q_{23},q_{230},Q,Q_0)\\ &+\lambda\sum\exp(-q_{310}^2\tau^2/2-q_{31}^2R^2/2)\mathrm{Uncor}(q_{31},q_{310},Q,Q_0)\ ,\end{aligned} \tag{5}$$

where $\mathrm{Uncor}(Q,Q_0)$ is the background density distribution for three uncorrelated pions, and

$$\mathrm{Uncor}(q_{12},q_{120},Q,Q_0)\ ,$$

$$\mathrm{Uncor}(q_{23},q_{230},Q,Q_0)\ ,$$

and

$$\mathrm{Uncor}(q_{31},q_{310},Q,Q_0)$$

are subsets of the uncorrelated density distribution for a given set of values of their arguments. Experimentally, the correlated density distribution is constructed by selecting three like pions from the same event, while the three pions for the uncorrelated density distribution are each

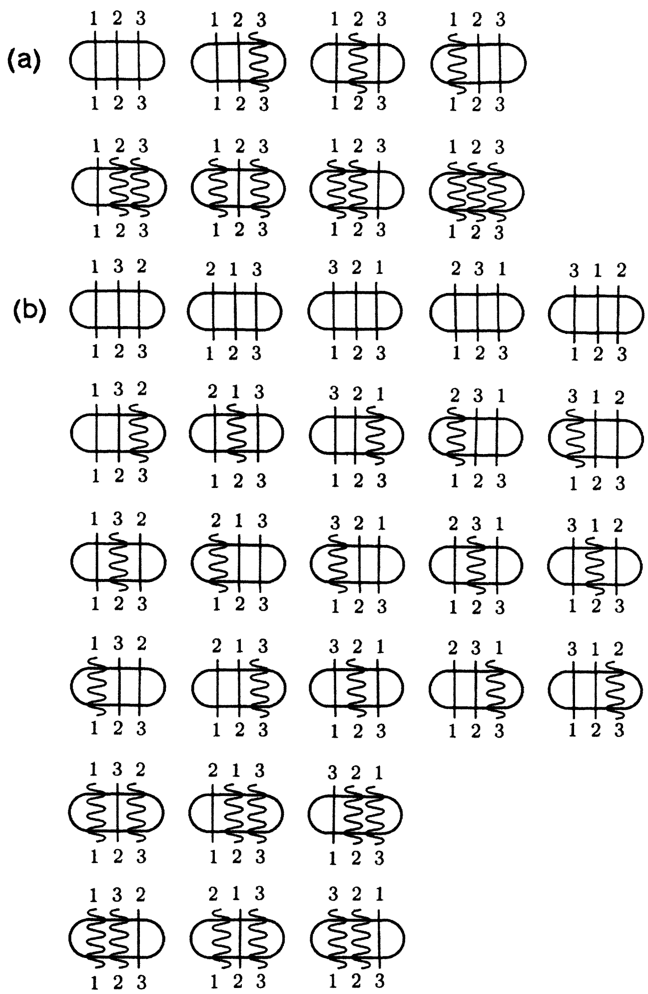

FIG. 3. (a) The background for three negative pions. (b) The interference effect of Bose-Einstein statistics for three negative pions.

selected from a different event with the same multiplicity. The integrated three-pion correlation function is

$$C(Q,Q_0)=\mathrm{Cor}(Q,Q_0)/\mathrm{Uncor}(Q,Q_0)\ .$$

In general, the correlated density distribution for three pions contains contributions associated with pion-pair interference. To isolate the contribution from the pure pion-triplet interference, $\mathrm{Tri}(p_1,p_2,p_3)$, we use the general formula[11]

$$\frac{\mathrm{Tri}(p_1,p_2,p_3)}{\mathrm{Uncor}(p_1,p_2,p_3)}=C(p_1,p_2,p_3)-[C(p_1,p_2)+C(p_1,p_3)+C(p_2,p_3)]+2\ .$$

$\mathrm{Tri}(Q,Q_0)$, the pure pion-triplet correlated density distribution integrated over all the variables other than Q and Q_0, is given by

$$\mathrm{Tri}(Q,Q_0)=\mathrm{Cor}(Q,Q_0)-[\mathrm{Two}_{12}(Q,Q_0)+\mathrm{Two}_{23}(Q,Q_0)+\mathrm{Two}_{31}(Q,Q_0)]+2\,\mathrm{Uncor}(Q,Q_0)\ ,$$

where $\mathrm{Two}_{12}(Q,Q_0)$, $\mathrm{Two}_{23}(Q,Q_0)$, and $\mathrm{Two}_{31}(Q,Q_0)$ are the correlated density distributions for the three pion pairs. To construct the distributions $\mathrm{Two}_{ij}(Q,Q_0)$, we select a correlated pion pair from the same event, then combine it with another pion from a different event with the same multiplicity. In the Gaussian model, from Eq. (5),

$$\mathrm{Tri}(Q,Q_0)/\mathrm{Uncor}(Q,Q_0)=2\xi\exp[-Q_0^2\tau^2/4-(Q^2+Q_0^2)R^2/4]\ .$$

The Coulomb correction of the final state is very important for extracting meaningful estimates of pion source parameters from the data.[4,5] The exact form of this correction for three pions requires the exact solution of the three body problem. As an approximation, we propose that the Gamow factor G for three pions can be expressed as the product of three Gamow factors, one for each pion pair:

$$G=G(\eta_{12})G(\eta_{23})G(\eta_{31})\ ,$$

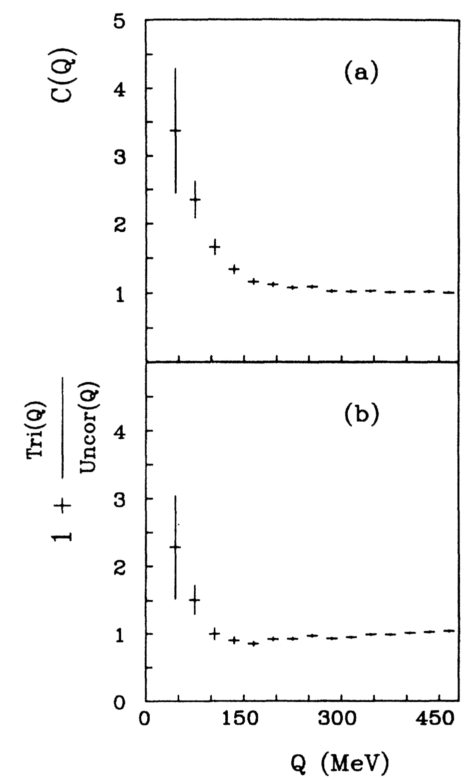

FIG. 4. The observed correlation function for 1.8 GeV/nucleon Ar on Pb, integrated over Q_0: (a) for three correlated pions; (b) for pure pion triplets.

TABLE II. The fitted results, uncorrected and corrected by a Gamow factor, for 1.8 GeV/nucleon Ar on Pb and 1.5 GeV/nucleon Ar on KCl. The values of λ and ξ are calculated from the fitted value of γ, using Eqs. (2) and (4).

	Source parameters	1.8 GeV/nucleon Ar + Pb Uncorrected	1.8 GeV/nucleon Ar + Pb Gamow corrected	1.5 GeV/nucleon Ar + KCl Uncorrected	1.5 GeV/nucleon Ar + KCl Gamow corrected
Three-pion analysis	R (fm)	6.08 ± 0.45	5.65 ± 0.49	5.00 ± 0.97	5.51 ± 0.86
	λ	$0.82^{+0.14}_{-0.12}$	$0.98^{+0.02}_{-0.26}$	$0.76^{+0.24}_{-0.27}$	$1.00^{+0.00}_{-0.15}$
	ξ	$0.61^{+0.27}_{-0.18}$	$0.95^{+0.05}_{-0.49}$	$0.51^{+0.49}_{-0.31}$	$1.00^{+0.00}_{-0.33}$
Pure pion-triplet analysis	R (fm)	5.14 ± 0.80	5.80 ± 0.75	4.02 ± 0.46	4.06 ± 0.49
	ξ	$0.55^{+0.39}_{-0.22}$	$1.00^{+0.00}_{-0.25}$	$0.98^{+0.02}_{-0.48}$	$1.00^{+0.00}_{-0.36}$
Two-pion analysis	R (fm)	5.58 ± 0.42	5.53 ± 0.45	5.10 ± 0.35	4.72 ± 0.30
	λ	$0.76^{+0.12}_{-0.10}$	$0.99^{+0.01}_{-0.24}$	$1.00^{+0.00}_{-0.25}$	$1.00^{+0.00}_{-0.34}$

where

$$G(\eta_{ij})=2\pi n_{ij}/[\exp(2\pi\eta_{ij})-1]\ ,$$

$$\eta_{ij}=e_ie_j/\hbar c\beta_{ij}=\alpha/\beta_{ij}\ ,$$

and α is the fine structure constant; β is the magnitude of the relative velocity of the two pions. This expression for the Gamow factor is symmetric under the exchange of any two of the three pions, and reduces to the appropriate two-pion Gamow factor when one of the pion charges is switched off. It is correct to first order of α. The Gamow factor G represents the effect of the relative Coulomb repulsion between the three pions in the final state, which leads to a suppression of events with small relative Q. Weighting the background events to account for this suppression, we obtain the Gamow corrected results for three-pion analysis.

Figure 4 shows the observed correlation function, integrated over Q_0, for correlated sets of three pions and pure pion triplets, for 1.8 GeV/nucleon Ar on Pb. Table II shows the fitted pion source parameters for the two data samples under consideration. The results of two-pion and three-pion analyses are quite consistent with each other within statistical errors. This fact indicates that at the present level of accuracy, the Gaussian model is a self-consistent phenomenological description of a pion emitting source in relativistic heavy ion collisions. The agreement is also consistent with the assumption that the enhancement solely comes from Bose-Einstein statistics.

When we carry out two-pion interferometry, the correlated pions are selected from each event in which pion multiplicity ≥ 2. When we apply two-pion analysis to like-pion pairs selected from events with multiplicity 3, we actually exclude the last term in Eq. (3), which represents the correlation contribution of pure pion triplets. This exclusion might distort the results of the two-pion analysis. To study this effect, we used a Monte Carlo calculation to generate 500 000 correlated pion pairs for events with multiplicity 2 and an equal number for events with multiplicity 3, for various combinations of values of source temperature T and radius R. The fit parameters are shown in Table III. The data and the fits for the case

TABLE III. Results of Monte Carlo simulations for two-pion analysis in events of multiplicity 2 and 3.

Monte Carlo parameters ($\lambda=1, \xi=1$)	M_{π^-}	Two-pion interferometry results R (fm)	λ	$\Delta\lambda$ [Eq. (6)]
$R=3$ fm	3	2.53 ± 0.02	1.25 ± 0.02	0.17
$T=30$ MeV	2	2.96 ± 0.03	0.99 ± 0.01	
$R=3$ fm	3	2.74 ± 0.04	1.07 ± 0.03	0.09
$T=60$ MeV	2	2.96 ± 0.04	0.96 ± 0.02	
$R=5$ fm	3	4.49 ± 0.06	1.03 ± 0.03	0.06
$T=30$ MeV	2	4.97 ± 0.06	0.96 ± 0.03	
$R=5$ fm	3	4.74 ± 0.14	1.02 ± 0.05	0.02
$T=60$ MeV	2	5.01 ± 0.15	0.96 ± 0.05	

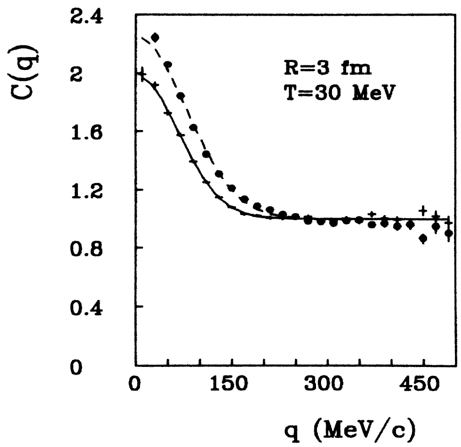

FIG. 5. Comparison of the simulated two-pion correlation function for events of multiplicity 3 with that for events of multiplicity 2 ($\lambda=1.0$, $\xi=1.0$). The solid circles and dashed lines are for events of multiplicity 3.

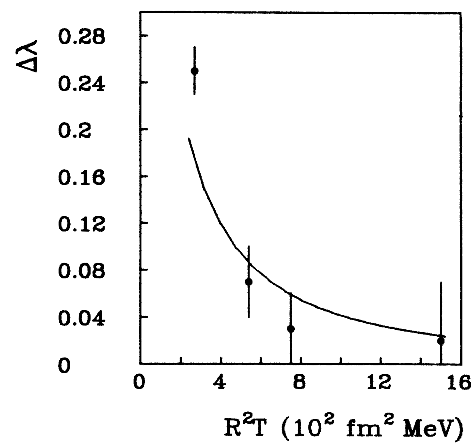

FIG. 6. Distortion of the two-pion coherence factor λ vs R^2T. The four data points correspond to $R^2T=270$ fm^2 MeV ($T=30$ MeV, $R=3$ fm), $R^2T=540$ fm^2 MeV ($T=60$ MeV, $R=3$ fm), $R^2T=750$ fm^2 MeV ($T=30$ MeV, $R=5$ fm), and $R^2T=1500$ fm^2 MeV ($T=60$ MeV, $R=5$ fm).

of maximum distortion, $R=3$ fm and $T=30$ MeV, have been integrated over q_0 and displayed in Fig. 5.

We can obtain an analytic expression for the distortion of the coherence factor, $\Delta\lambda$, if we assume that the single pion inclusive cross section in heavy ion collisions is isotropic, and may be approximated by an exponential distribution

$$\frac{d\sigma}{dp} \propto \exp(-E/T) .$$

Using Eq. (3) the analytic expression for $\Delta\lambda$ is

$$\Delta\lambda=(4\sqrt{2}/3)\xi/(3m_\pi R^2T+2)^{3/2} , \tag{6}$$

where m_π is the mass of the pion.[10] Figure 6 shows the differences $\Delta\lambda$ between the fitted coherence factor λ for the two-pion analysis in the events of multiplicity 3, and the input Monte Carlo parameter λ, as a function of R^2T. The solid curve represents Eq. (6).

The distortion in the radius and the coherence factor increases as the temperature T and the radius R of the pion source decrease. The explanation is as follows: As the temperature decreases, the phase space contracts; as the radius R decreases, the coherence length becomes longer. In either of these situations, the relative correlation contribution from pure pion-triplet interference increases. Hence, there can be a strong distortion of the distribution of the correlation function in two-pion analysis. In the data samples collected thus far,[4,9] the distortion of the radius R and the coherence factor λ from pure pion triplet interference has not been detected, because the size of this effect is smaller than the statistical errors. The distortion from higher order multipion (≥ 4) correlations has also been calculated and is found to be even smaller.

We conclude that three-pion interferometry offers an independent alternative tool for testing and supplementing the findings of two-pion interferometry analysis. The fitted pion source parameters from two-pion and three-pion analyses are consistent with each other for our data samples. This fact shows that the Gaussian model is a consistent phenomenological model which describes the pion emitting source in relativistic heavy ion collisions at the present level of experimental accuracy. This fact also offers evidence that the enhancement in the low momentum region comes solely from Bose-Einstein statistics. With increased statistics in future experiments, the three-pion coherence factor ξ can be used to determine the ratio n_c/n_{in} for the pion source independently of λ. The distortion of the pion source parameters in two-pion interferometry arising from higher order pion interference becomes a serious problem only in the situation where the pion source has a small radius and low temperature, e.g., $R=3$ fm and $T=30$ MeV. The temperatures and radii for our existing streamer chamber data do not lie in the region of significant distortion of the source parameters.

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