Self-energies of nuclei and real and imaginary optical model potentials

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Previously the inverse mean field method has been shown to lead to an analytical expression for the volume integral $J_V(A)$ per nucleon over the self-energies of nuclei with mass numbers A and to specific energy dependences for real $f(E_p)$ and imaginary volume contributions to the optical model potential. These relations are now utilized to obtain $J_V(A,E_p)=J_V(A,0)f(E_p)$ and its imaginary counterpart, $J_{W_v}(A,E_p)=J_V(A,0)[1-f(E_p)]/3$. The results compare favorably with heuristic data and the findings of other approaches.

I. INTRODUCTION

In this paper we are concerned with the local optical model potential $V_{\rm OM}$ for elastic nucleon-nucleus scattering. For spherically symmetric nuclei, which we consider here, it is usually given in the form

$$V_{\rm OM}(r)=V_{\rm Cl}(r)+V(r)+iW_v(r)+iW_s(r)+V_{\rm SO}(r)+iW_{\rm SO}(r)\ , \tag{1}$$

with r standing for the spatial variable. The different terms (from left to right) denote Coulomb potential, real central nuclear interaction, imaginary volume and surface terms, and real and imaginary spin-orbit interactions, respectively. We are going to discuss $V(r;A,E_p)$ and $W_v(r;A,E_p)$, where A stands for the mass number of the target nucleus and E_p for the kinetic energy of the incident nucleon. The present consensus is that V and also W_v may be written in the factorized forms

$$V(r;A,E_p)\cong V(r;A,0)f(E_p)=U(r,A)f(E_p)\ ,$$
$$W_v(r;A,E_p)\cong W_v(r;A,0)f_W(E_p)\ , \tag{2}$$

with $f(E_p)$ and $f_W(E_p)$ containing the respective energy dependences of the two potentials and $U(r,A)$ representing the shell-model potential of the target nucleus.

There are various methods for evaluating V and W_v, e.g., via phenomenological macroscopic optical model search codes on the computer, via the microscopic folding model, by the aid of inversion procedures, dispersion relations, and the nuclear matter approach (see Refs. 1–5 and references therein). Recently,[6,7] a different route has been suggested. Guided by (2), it has been proposed to start the evaluation of the (real central part of the) scattering potential V from the nuclear bound-state problem to obtain first $U(r,A)$, which is then to be multiplied by the energy dependence $f(E_p)$ to yield the above $V(r;A,E_p)$.

Kohno *et al.*[6] computed the self-consistent Hartree-Fock field $U(r,A)$ of the target nucleus in order to proceed then to nucleon-nucleus scattering. As usual, they relied on the so-called *direct approach* to the solution of the nuclear bound-state problem (i.e., they started with a given nucleon-nucleon interaction to evaluate all further quantities of interest). However, the mathematical methods developed in the last few decades make it also feasible to apply *inverse methods* to the solution of the bound-state problem[8] (i.e., the respective stationary Schrödinger equations are solved with the experimentally accessible energy eigenvalues E_i as the main input). Application of this concept led not only to the qualitative and quantitative evaluation of some static nuclear properties[7,9] and a discussion of the optical model potential,[7,10,11] but also to the skeleton of a self-consistent dynamical approach[12–14] termed the inverse mean field method (Imefim). For review-type papers, see Ref. 14.

Within that approach the volume integral over the shell-model potential per nucleon $J_U(A)$ has been evaluated analytically[7] and a specific energy dependence $f(E_p)$ has been put forward.[10] It served as the starting point for deriving, as well, the function $f_W(E_p)$.[11] Separately, these results and predictions of Refs. 7, 10, and 11 have already been put in relation to some empirical data and the findings of other approaches. Here our aim is to synthesize these bits of information.

Explicit parameter free (except for a renormalization constant of the depth) A and E_p dependences of $J_V(A,E_p)=J_U(A)f(E_p)$ and its imaginary counterpart $J_{W_v}(A,E_p)$ will be given. J_V and J_{W_v} denote the volume integrals per nucleon over real and imaginary volume terms of the scattering potentials V and W_v, respectively. Both types of volume integrals are of particular interest in comparative studies involving different targets, projectile energies, and/or theoretical approaches. Their energy dependences will be compared to heuristic data (for the A dependence, see Ref. 7).

In Sec. II the main formulas are recalled and combined together to obtain the quantities J_V and J_{W_v} to be discussed in Sec. III. The final part contains some concluding remarks.

II. MAIN FORMULAS

For spherically symmetric target nuclei inverse methods yield an analytical expression for the volume integral per nucleon $J_U(A)$ over the single-particle shell-

model potential $U(r,A)$. To obtain that result, only the contributions of the occupied ground-state levels were taken into account. This implies the restriction to the self-energies or self-interactions[7,15] of the respective nuclei and has been asserted to be a good approximation to the exact results (see Ref. 7 and references therein). For nuclei with mass numbers A within the range $4 \leq A \leq 96$ the microscopic result derived in Ref. 7 is extremely well approximated by the macroscopic formula

$$J_U(A) = -(8\pi\hbar/\sqrt{2m})\sqrt{-B(A)/0.38} = -187\sqrt{-B(A)}\ , \tag{3}$$

where $B(A) \equiv B_t(A)/A < 0$ is the total binding energy (or mass) per nucleon of the target nucleus with its A nucleons. Experiment gives these values to a very high precision.[16] The A dependence predicted by (3) is in very nice agreement with the findings of the extensive study by Rapaport,[1] who considered about 100 empirical data points for $J_V(A) = J_U(A)$.[7]

In addition to (3), the inverse approach (Imefim) also yields the specific energy dependence

$$f(E_p) = (1+0.0027E_p)^{-3} \tag{4}$$

for the depth of the real central part of the optical potential V. This relation compares favorably with heuristic data and the results of other approaches.[10] Conservation laws, as established within the context of nonlinear evolution equations, led subsequently, with no adjustable parameters from (3) and (4), to

$$f_W(E_p) = 1 - f(E_p) = 1 - (1+0.0027E_p)^{-3}\ , \tag{5}$$

giving the energy dependence of the depth of the imaginary volume potential W_v, Eqs. (1) and (2). It is uniquely determined by $f(E_p)$ [except for a possible renormalization of its amplitude due to different choices for the geometry of $W_v(r,A,0)$]. At least qualitatively, relation (5) compares much better with empirical data than dispersion relations (though, quantitatively, there are some discrepancies for specific energy regions; see Ref. 11).

Combining (3) and (4), we obtain immediately

$$J_V(A,E_p) \cong J_U(A) f(E_p)$$

$$= -187\sqrt{-B(A)}(1+0.0027E_p)^{-3}\ . \tag{6}$$

Following, now, Ref. 11 in the use of a conservation law with respect to the volume integral (or mass/flux), yields, together with (5) and (6), the result

$$J_{W_v}(A,E_p) = -J_V(A,0)[1-f(E_p)]/3\ . \tag{7}$$

It should be stressed that (6) and (7) represent predictions and not adjusted formulas (except for the coefficient $\frac{1}{3}$, which is added empirically; see below). $J_{W_v}(A,E_p)$ is thus uniquely determined by its real counterpart $J_V(A,E_p)$—to our knowledge this is a qualitatively and quantitatively new result which can be easily tested against the phenomenological data.

III. DISCUSSION

The physical content and interpretation of Eqs. (6) and (7) are as follows: For $E_p = 0$ MeV, (6) gives just the self-energy of the target nucleus.[7,15] Varying projectile energies give rise to the energy dependence given by Eq. (4). As a consequence, an imaginary potential iW_v characterized by Eq. (7) is generated. Relations (6) and (7) contain neither specific structure nor shell effects, nor do they account for dynamical effects due to the presence of the projectile. Hence, their comparison to heuristic data (which *do* contain all these effects) has, necessarily, to give rise to some deviations that are a clean measure for the importance of contributions arising from sources other than the respective self-energies of the target nuclei. In the case of nucleon-nucleus scattering considered here, we expect only small deviations [except for low energies and small A (Ref. 7)]. The suppressed structure effects are traditionally associated with the surface absorption term W_s in (1), which is not treated within the present approach at all. Hence, we cannot expect a reasonable correspondence of Eq. (7) with the heuristic integrals over the total imaginary potential, $J_W = \int dx(W_v + W_s)$, but only with the ones over the volume term, $J_{W_v} = \int dx\, W_v$.

The projectile energy dependence of the real central part of $V_{\rm OM}$ represented by $V(r;A,E_p)$ is illustrated in Fig. 1, adopted from Ref. 1 and supplemented by the bold curve due to Eq. (6). The energy dependence of Eq. (6) is closer to the data than the heuristic logarithmic dependence of Ref. 17 (the dotted curve) and does certainly no worse than the linear one put forward in Ref. 1 (the dotted line).

As stated above, we cannot expect a good correspondence of (7) with the heuristic energy dependence of the

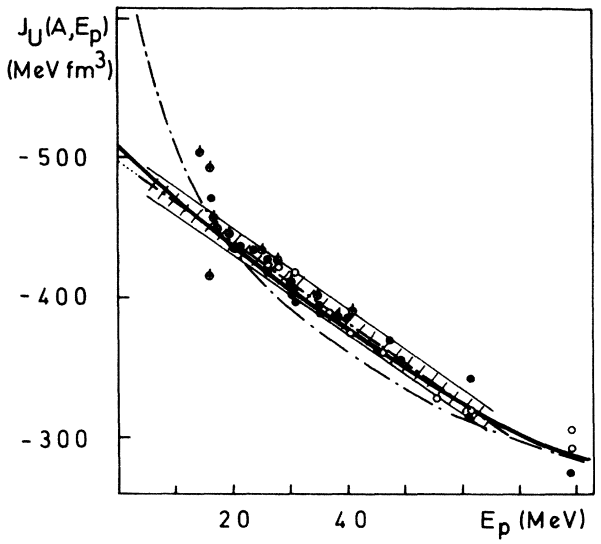

FIG. 1. The integral $J_V(E_p)$ due to Eq. (6) with $B(A) = -7.5$ MeV is plotted versus the kinetic energy E_p of the projectile (bold curve). Empirical data (points), the dotted curve [based on $J_V(E_p) = -765 + 110\ln E_p$; see Ref. 17], and the dotted line [due to $J_V(E_p) = -498 + 2.9E_p$] have also been discussed in Ref. 1, which should be consulted for further details.

integral per nucleon over the total imaginary potential, $J_W = \int d\mathbf{x}(W_v + W_s)$. Nevertheless, let us first consider that case. The prediction of Eq. (7) (times 3; see below) represented by the solid bold curve in Fig. 2 is obviously not at all satisfactory. Treating the amplitude and coefficient in front of the quantity E_p rather arbitrarily as adjustable parameters allows us to reproduce the data (see the dashed curve in Fig. 2). However, that would reduce (7) to a phenomenological parametrization of the data and contradict the physical content of (7) as discussed above. We do not want to suggest such a procedure.

As stated above, relation (7) contains only the global contributions to the volume term. Hence, it should be compared to the empirical $J_{W_v} = \int d\mathbf{x}\, W_v$ and not to the J_W discussed above. This is done in Fig. 3, adopted from Ref. 1, where the relation

$$J_{W_v}(E_p) = -100\{1 - \exp[-0.04(E_p - 18)]\}\ \text{MeV fm}^3\ , \qquad 18 < E_p(\text{MeV}) < 60 \quad (8)$$

was used to generate the dashed curve. In this heuristic formula the constant 18 appears in the argument of the exponential because W_v and hence J_{W_v} were set equal to zero for $E_p < 18$ MeV. This was motivated by the fact that the fits to experiment for small E_p are rather insensitive to variations in W_v, while they do respond strongly to changes in W_s. (For higher energies, say above 60 MeV, the situation is reversed, i.e., W_s is reduced to insignificance.[1])

Such empirical findings certainly do indicate that it is reasonable to use $W_v = 0$ MeV for $E_p < 18$ MeV. However, they by no means insinuate that one has necessarily to introduce such a procedure. On the contrary, they indicate that small values of the volume absorption term are compatible with low energy data. Hence, we feel that the smooth increase of J_{W_v} from $E_p = 0$ MeV onward as predicted by Eq. (7), and shown by the solid curve in Fig. 3 appears more sensible than a rather arbitrary brute-force cutoff at a fixed energy. (More detailed numerical work

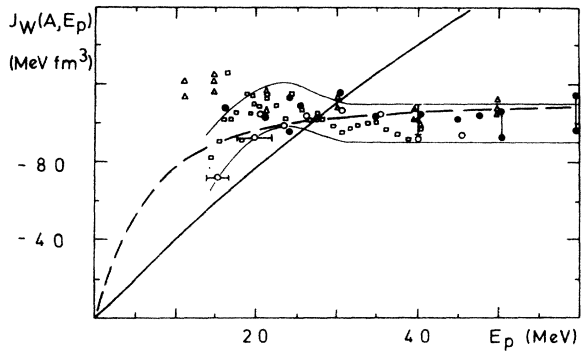

FIG. 2. The data points for the total proton absorption potential $J_W(E_p) = \int d\mathbf{x}(W_v + W_s)$ are adopted from Ref. 1, which contains further information on them. The thin solid curves are just meant to guide the eye. The parameterfree bold curve is due to Eq. (7) multiplied by 3. The parameters of the dashed curve, $J_W(E_p) = -110[1 - (1 + 0.005E_p)^{-3}]$, are superficially adjusted to reproduce the trend of the data.

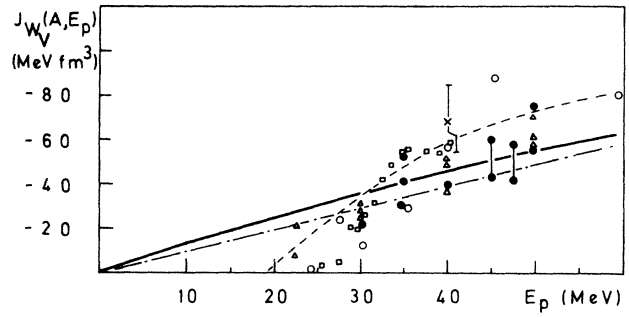

FIG. 3. The proton absorptive volume integral per nucleon $J_{W_v}(A,E_p)$ is plotted as a function of the proton energy E_p. The heuristic data are taken from Ref. 1, where they have been discussed and explained in detail. The dashed curve due to Eq. (8) is a fit to the data presented in Ref. 1. The bold curve is the prediction of Eq. (7). The dotted curve is evaluated using the relation $J_V(E_p) = -498 + 2.9E_p$ of Ref. 1 in conjunction with Eq. (7).

trying to clarify this point is still in progress.) In view of the well-known uncertainties and ambiguities in the determination of the heuristic J_{W_v}, the predictions of Eq. (7) agree rather well with the empirical data in Fig. 3. The same also holds for further data not discussed here.

It is worth mentioning that the interrelationship between real and imaginary volume integrals reflected by Eq. (7) is a rather general one. Hence, it should also be possible to use it together with any $f(E_p)$ taken from the literature to predict the related J_{W_v}. To illustrate that this is indeed the case, we take the linear energy dependence $J_V(E_p) = -498 + 2.9E_p$ proposed in Ref. 1, rewrite it in the form

$$J_V(E_p) = -498(1 - 0.0058E_p) \equiv -498f(E_p)\ ,$$

and use it in (7) to obtain

$$J_{W_v}(E_p) = -498[1 - (1 - 0.0058E_p)]/3 = -0.9628E_p\ , \quad (9)$$

corresponding to the dotted curve in Fig. 3. Applying (7) in this manner leads in many cases to results that are even more convincing than the dashed curve in Fig. 3.

It is inferred that Eqs. (6) and (7) provide quite useful predictions. However, now we have to return to the origin of the coefficient $\frac{1}{3}$ in (7). Comparison to the heuristic data showed its inclusion to be necessary to reproduce the magnitude of the data. This is certainly a perturbing feature, yet, it is some consolation that exactly the same value of the coefficient is demanded for a fairly large number of different cases (only part of which has been discussed in here) and that a similar renormalization is required when using dispersion relations. At present we can only speculate that the origin of this coefficient $\frac{1}{3}$ is related to the fact that binding or surface energy effects have been ignored in the derivation of J_{W_v} from J_V. Since about $\frac{2}{3}$ of the nuclear density and potential distributions are governed by surface effects, this may eventually turn out to be a plausible argument. Yet, at present, this is still

speculation. For the time being we accept this universal renormalization constant as an empirically enforced necessity and hope that we shall eventually be able to offer a consistent derivation and explanation of it.

IV. SUMMARY

The specific results of this paper are the explicit analytical formulas for mass number and projectile energy dependences of the volume integrals $J_V(A,E_p)$ and $J_{W_v}(A,E_p)$ as predicted by the dynamic inverse mean field method (Imefim). A more general prediction is seen in the determination of the imaginary volume term J_{W_v} in terms of its real counterpart. Only a small portion of the material studied could be displayed in here. It nevertheless provides a nice qualitative and (though less convincing) quantitative confirmation of the predicted features.

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