π-exchange NN interaction model with overlapping nucleon form factors

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(Received 24 February 1986)

The nucleon-nucleon (NN) interaction model includes a π-exchange and takes into account the first excited state $\Delta(1232)$ of the nucleon. It is supplemented by a short-range repulsion which has been derived from the nucleon form factor (rms radius b_f) combined with the three-quark wave function (rms radius b_q). The optimization of the model on empirical scattering phase shifts below 300 MeV gives, for a minimum χ^2, the root-mean-square radii $b_f = b_q = 0.51$ fm and a coupling constant $G_\pi^2/4\pi = 13$.

I. INTRODUCTION

It is generally accepted that the NN interaction at energies below 400 MeV arises from one pion exchange at large distances and from heavy meson exchange at intermediate and short distances. Furthermore, short-range interaction is treated either by high momentum regularization[1] (form factor) or by the introduction of phenomenological potentials.[2] These models give good quantitative results, but do contain a certain number of parameters (in the range of 8). On the other hand, the present understanding of the short-range part of the nucleon-nucleon potential as a residual interaction between quarks offers some interesting results,[3-7] but still remains qualitative.

Confronted with such a situation, we think it is still worthwhile to go on with a conventional approach to the NN interaction, taking particular account of the phenomenology of the extended structure of the nucleon (form factor). In order to do this, we first have to set the theoretical framework of our model. We limit ourselves to a π-exchange interaction model. However, besides the nucleon in the ground state, we consider the $\Delta(1232)$ excitation as a real state according to the SU(2) flavor symmetry. We admit that the one pion exchange constitutes, with the Δ isobars, an important part of the intermediate- and long-range nucleon-nucleon force. For the short range of the interaction we complete the phenomenology of the usual nucleon form factor with the wave function of the two clusters of three quarks. This yields a form factor depending not only on the position of each nucleon, but also on the distance between them, and leads to a description of the overlap of the two extended structures of the nucleons. The resulting potential shows a strong repulsion in the odd-parity states and a weak attraction in the even-parity states. We make up the deficiency of repulsion on the latter by cutting off the whole potential with a hard core of about 0.3 fm. Finally, we have to take into account the relativistic corrections that have a degree of importance even at energy below 300 MeV. The first of these will be introduced in the coupling constant by an energy dependence. The second will be given by the spin-orbit potential which results from the short-range repulsion.

The semiphenomenological potential one obtains shall be adapted to the experimental NN phase shifts. We will have to optimize five parameters. The first ones are the root-mean-square (rms) radii of the form factor and the three-quark cluster. They are defined in Sec. II. The third is the πN coupling constant. The hard core and the strength of the spin-orbit potential will complete the set of parameters. The development of the potential appears in Sec. III. Numerical results are given and discussed in Sec. IV.

II. THE NUCLEON FORM FACTOR

In a nonrelativistic static meson theory (relativistic corrections will be considered later), the interaction of the nucleon with the π field ϕ_α is fixed by the Hamiltonian

$$H = \frac{1}{2}\sum_{\alpha=1}^{3}\int d^3x[\pi_\alpha^2 + \phi_\alpha(\mu^2 - \Delta)\phi_\alpha] + \sqrt{4\pi}\frac{f}{\mu}\sum_{\alpha,j=1}^{3}\sigma_j\tau_\alpha\int d^3x\, U(|\mathbf{x}-\mathbf{R}_a|)\frac{\partial}{\partial x_j}\phi_\alpha , \tag{2.1}$$

where μ is the inverse of the pion wavelength and f the unrenormalized πN coupling constant. We denote the usual spin and isotopic spin operators by σ_j, τ_α. The form factor $U(|\mathbf{x}-\mathbf{R}_a|)$ represents the spatial extension of the nucleon localized at $\mathbf{R}_a$.

In the NN interaction, the representation of each nucleon with the function $U(|\mathbf{x}-\mathbf{R}_n|)$, $n = a,b$, is certainly sufficient when the two nucleons are separated. However, when they are brought together, the representation should take into account the overlap of the quark structure of the two nucleons. We will characterize this phenomenology by a new form factor, $W(R, |\mathbf{x}-\mathbf{R}_n|)$, which depends on the relative distance $R = |\mathbf{R}_b - \mathbf{R}_a|$ between the two nucleons and possesses the following property:

$$\lim_{R\to\infty} W(R, |\mathbf{x}-\mathbf{R}_n|) = U(|\mathbf{x}-\mathbf{R}_n|), \quad n = a,b . \tag{2.2}$$

In order to define the function $W(R, |\mathbf{x}-\mathbf{R}_n|)$, we consider the NN, N$\Delta$, and $\Delta\Delta$ pairs as a system of six quarks divided into two clusters of three quarks. Following the cluster model calculation, we assume that this system of six fermions can be described by the completely antisymmetric wave function

$$\Psi=\frac{1}{\sqrt{20}}\sum_{\alpha}(-1)^{\alpha}P_{\alpha}[\Phi_a(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)\Phi_b(\mathbf{r}_4,\mathbf{r}_5,\mathbf{r}_6)] \times[\chi_a(123)\chi_b(456)]\psi_r(\mathbf{R}) . \quad (2.3)$$

The P_α operators comprise all the possible permutations between the two clusters. $\Phi_a\Phi_b$ designates the symmetric space wave function, and $\chi_a\chi_b$ the symmetric spin, symmetric isotopic spin, and antisymmetric color wave functions. The relative motion of the two clusters is represented by $\psi_r(\mathbf{R})$. With the wave function Ψ [Eq. (2.3)] satisfying the Pauli exclusion principle, we complete the function $U(|\mathbf{x}-\mathbf{R}_n|)$. In order to do this, let us assume a separation between the quark wave functions and the wave function $\psi_r(|\mathbf{R}|)$ of the relative motion in the following way:

$$\Psi\sim\psi_{LST}\psi_r(|\mathbf{R}|) , \quad (2.4)$$

where

$$\psi_{LST}=\frac{1}{\sqrt{20}}\sum_{\alpha}(-1)^{\alpha}P_{\alpha}(\Phi_a\Phi_b)(\chi_a\chi_b)_{ST}Y_{LM}(\mathbf{R}/|\mathbf{R}|) . \quad (2.5)$$

Moreover, one here admits that P_α acts only on the parity of the spherical harmonics Y_{LM}. Under these assumptions, one sees that the ψ_{LST} part, which is not taken into account in the dynamics of the two nucleons, can be replaced by the normalization factor

$$N^2(R)=c^2(\psi_{LST},\psi_{LST}) , \quad (2.6)$$

where (,) denotes the scalar product in the configuration, spin, isotopic spin, and color spaces. Remembering the fact that the nucleon form factor can be interpreted as the probability density of the nucleon, we use the normalization (2.6) to define the new form factor,

$$W(R,|\mathbf{x}-\mathbf{R}_n|)=N(R)U(|\mathbf{x}-\mathbf{R}_n|), \quad n=a,b . \quad (2.7)$$

The constant c is determined by the boundary condition (2.2). For $U(|\mathbf{x}-\mathbf{R}_n|)$ normalized to unity, we take, in our calculations, the Yukawa function

$$U(r)=\frac{3}{2\pi}\frac{1}{b_f^2}\exp\left[-\sqrt{6}\frac{r}{b_f}\right]\Big/r, \quad r=|\mathbf{x}-\mathbf{R}_n| \quad (2.8)$$

with b_f its rms radius. Starting from a π-quark coupling, the π-N form factor can be determined.[8] We justify our choice by numerical calculations which show that the phase shifts depend essentially on the spatial extension b_f of $U(r)$ and not on its form.[9]

In the norm $N(R)$, the internal functions Φ_a and Φ_b are taken as Gaussian,

$$\Phi_a=\eta\exp\left[-\frac{1}{2b_q^2}(\mathbf{y}_1^2+\mathbf{y}_2^2+\mathbf{y}_3^2)\right], \quad \mathbf{y}_i=\mathbf{r}_i-\mathbf{R}_a, \quad i=1,2,3 \quad (2.9a)$$

$$\Phi_b=\eta\exp\left[-\frac{1}{2b_q^2}(\mathbf{y}_4^2+\mathbf{y}_5^2+\mathbf{y}_6^2)\right], \quad \mathbf{y}_i=\mathbf{r}_i-\mathbf{R}_b, \quad i=4,5,6 \quad (2.9b)$$

where b_q is the rms radius and $\eta=(\sqrt{3}\pi b_q^2)^{-3/2}$. The computation of $N^2(R)$ with the functions (2.9) and the boundary condition (2.2) gives (see Appendix)

$$N^2(R)=1-M(S,T)F\left[\frac{R^2}{b_q^2}\right], \quad (2.10)$$

where

$$F\left[\frac{R^2}{b_q^2}\right]=\frac{81}{61}\left\{\frac{1}{2}\left[\frac{1}{5}\right]^{3/2}\exp\left[-\frac{3}{5}\left[\frac{R}{b_q}\right]^2\right]+\left[\frac{3}{8}\right]^{3/2}\exp\left[-\frac{9}{8}\left[\frac{R}{b_q}\right]^2\right]+\left[\frac{3}{7}\right]^{3/2}\exp\left[-\frac{9}{7}\left[\frac{R}{b_q}\right]^2\right]\right\}, \quad (2.11)$$

and, for each pair of values (S,T), $M(S,T)$ is given by the expectation value of the spin and isotopic spin parity operators. Restricting ourselves to the NN transitions, one obtains the well known result

$$M(S,T)=\begin{cases}\frac{7}{9}, & (0,0) ,\\ \frac{31}{81}, & (1,1) ,\\ -\frac{1}{27}, & (0,1) \text{ and } (1,0) .\end{cases} \quad (2.12)$$

The new form factor (2.7) introduced into the dynamics of the two-nucleon system brings about a strong short-range repulsion in the odd-parity states and a weak attraction in the even-parity states.

III. THE NUCLEON-NUCLEON POTENTIAL

The static NN potential is derived classically, starting from the interaction Hamiltonian

$$H_I=-\sqrt{4\pi}\frac{f}{\mu}\sum_{n=a}^{b}\sum_{\alpha,j=1}^{3}e_{\alpha j}^{(n)} \times\int d^3x\left[\frac{\partial}{\partial x_j}W(R,|\mathbf{x}-\mathbf{R}_n|)\right]\phi_\alpha(\mathbf{x}) . \quad (3.1)$$

Here we represent the nucleon and the Δ isobar with the function $W(R, |\mathbf{x}-\mathbf{R}_n|)$ given by (2.7). In order to describe the NN, NΔ, and ΔΔ transitions we have replaced the $\sigma_j\tau_\alpha$ operators by the "strong-coupling" operators $e_{\alpha j}^{(n)}$ obeying the orthogonality relations[10]

$$\left.\begin{array}{l}\sum_{\alpha=1}^{3} e_{\alpha j}^{(n)}e_{\alpha k}^{(n)}=\delta_{jk}\\ \sum_{j=1}^{3} e_{\alpha j}^{(n)}e_{\beta j}^{(n)}=\delta_{\alpha\beta}\end{array}\right\} n=a,b\ . \tag{3.2}$$

These operators have the advantage of fixing all the possible transitions between the N and Δ states with only one coupling constant, $f_r^2=f^2/9$, satisfying the relations

$$f_{N\Delta\pi}^2=\tfrac{9}{2}f_r^2,\quad f_{\Delta\Delta\pi}^2=\tfrac{1}{25}f_r^2\ . \tag{3.3}$$

In the static approximation the equation of motion for the π field of the two-nucleon system reads

$$(\Delta-\mu^2)\phi_\alpha(\mathbf{x})=-\sqrt{4\pi}\frac{f}{\mu}\sum_{n=a}^{b}\sum_{j=1}^{3}e_{\alpha j}^{(n)}\frac{\partial}{\partial x_j}\times W(R,|\mathbf{x}-\mathbf{R}_n|)\ . \tag{3.4}$$

The solution

$$\phi_\alpha(\mathbf{x})=-\frac{1}{\sqrt{4\pi}}\frac{f}{\mu}\sum_{n=a}^{b}\sum_{j=1}^{3}e_{\alpha j}^{(n)}\int d^3x'\frac{e^{-\mu|\mathbf{x}-\mathbf{x}'|}}{|\mathbf{x}-\mathbf{x}'|}\frac{\partial}{\partial x_j'}\times W(R,|\mathbf{x}'-\mathbf{R}_n|) \tag{3.5}$$

introduced in the total Hamiltonian of the two-nucleon system yields the energy E_0. The self-energy of each nucleon is determined by the condition (2.2) and subtracted from E_0. One obtains the potential

$$V(R)=V_{\rm core}(R)+V_\pi(R)\ , \tag{3.6}$$

where

$$V_{\rm core}(R)=\frac{f^2}{\mu^2}[1-N^2(R)]\sum_{j=1}^{3}I_{jj}(0)\ , \tag{3.7}$$

$$V_\pi(R)=-\frac{f^2}{\mu^2}N^2(R)\sum_{\alpha,j,k=1}^{3}e_{\alpha j}^{(a)}e_{\alpha k}^{(b)}I_{jk}(\mathbf{R}_b-\mathbf{R}_a)\ , \tag{3.8}$$

and

$$I_{jk}(\mathbf{R}_b-\mathbf{R}_a)=\int d^3x\,d^3x'\left[\frac{\partial}{\partial x_j}U(|\mathbf{x}-\mathbf{R}_a|)\right]\times\frac{e^{-\mu|\mathbf{x}-\mathbf{x}'|}}{|\mathbf{x}-\mathbf{x}'|}\left[\frac{\partial}{\partial x_k'}U(|\mathbf{x}'-\mathbf{R}_b|)\right]\ . \tag{3.9}$$

The formulae (3.7) and (2.12) show that the potential $V_{\rm core}(R)$ is strongly repulsive in the odd-parity states. For example, given the Yukawa function (2.8), we have

$$V_{\rm core}(R)=m_\pi f_r^2\tfrac{9}{2}\alpha^4\frac{\alpha+2}{(\alpha+1)^2}M(S,T)F\left[\frac{R^2}{b_q^2}\right]\ ,\quad \alpha=\sqrt{6}/(\mu b_f)\ . \tag{3.10}$$

For the first relativistic correction we introduce a spin-orbit potential. This can be freely chosen since we will determine its strength by fitting the phase shifts. However, we think that an appropriate form can be given by

$$V_{\rm so}(R)=a_{\rm so}\frac{1}{\mu R}\frac{d}{d(\mu R)}V_{\rm core}(R)(\mathbf{L}\cdot\mathbf{S})\ . \tag{3.11}$$

Such a choice is convenient because it takes into account the strong repulsion which we encounter in the odd-parity states. In the even-parity states, we know that the spin-orbit potential is not very important. We therefore admit that it results from a weak repulsion with a strength corresponding to that of $V_{\rm core}(R)$. The value of $a_{\rm so}$ will be essentially fixed by the $^3P_2$-$^3F_2$ partial wave. For the second relativistic correction we attribute to the coupling constant an energy dependence. We take it from the first term (the term which remains in the static approximation) of the nonstatic one pion-exchange potential.[11] In the center-of-mass system of energy $E_{\rm c.m.}$ we have

$$f_r^2(E_{\rm c.m.})=f_r^2(0)\left[\frac{2M}{E_{\rm c.m.}}\right]^2\ . \tag{3.12}$$

The thus-completed potential (3.6) will be put into the Schrödinger equation, where the energy of the Δ isobars is represented by

$$\omega^{(a)}+\omega^{(b)}=[j_a(j_a+1)+j_b(j_b+1)-\tfrac{3}{2}]\frac{E_r}{3}\ , \tag{3.13}$$

with $E_r=300$ MeV, and j_a,j_b are the spins of the nucleons and Δ. The Schrödinger equation will be solved according to the Rarita-Schwinger prescriptions. This means that for each partial wave we have to solve a system of from four to nine differential equations, and this poses a great technical difficulty.[12] The results are given in the next section.

IV. NUMERICAL RESULTS AND DISCUSSION

After inserting our potential into the Schrödinger equation, we calculate the scattering phase shifts $\delta(^{2S+1}L_J)$ and compare them to the empirical phases (error bars) taken from the analysis of Arndt *et al.*[13] We consider 100 experimental phase shifts in an energy range lying between 25 and 300 MeV. The parameters of the potential are optimized with the help of a χ^2 analysis. In order to do this we have to consider five parameters: the hard core R_c, the coefficient $a_{\rm so}$ of the spin orbit, the rms radii b_f and b_q of the nucleon form factor and the three-quark cluster, and the pion-nucleon coupling constant f_r^2. A global optimization gives for $\chi^2=20.7$ a coupling constant $f_r^2=0.07$ ($G_\pi^2/4\pi=13$), a hard core $R_c=0.34$ fm, and a spin-orbit coefficient $a_{\rm so}=0.073$. For the two root-

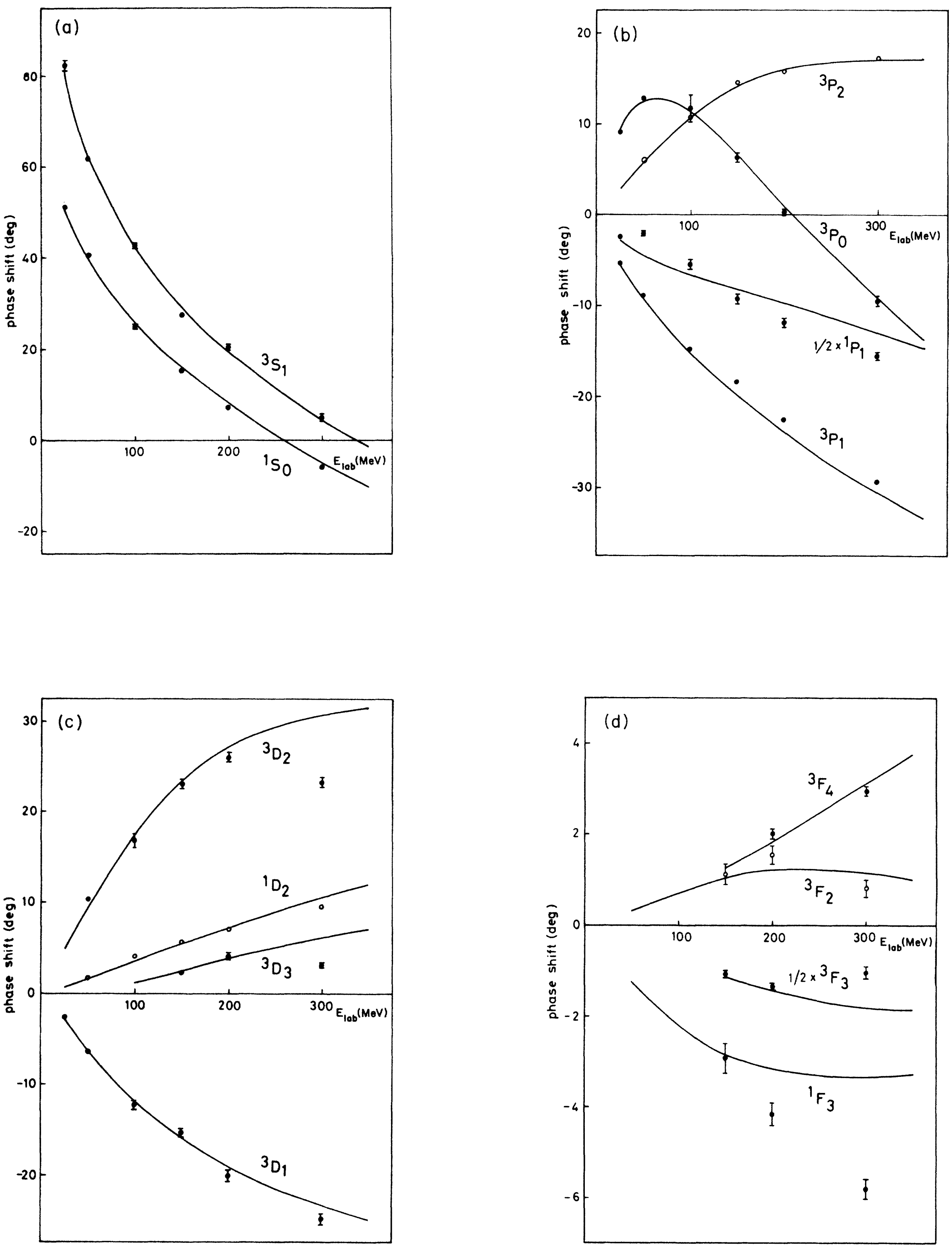

FIG. 1. Curves of the nucleon-nucleon nuclear bar phase shifts (as functions of the laboratory energy $E_{\rm lab}$) predicted by the model. The error bars are taken from the energy-independent analysis of Arndt *et al.* (Ref. 13).

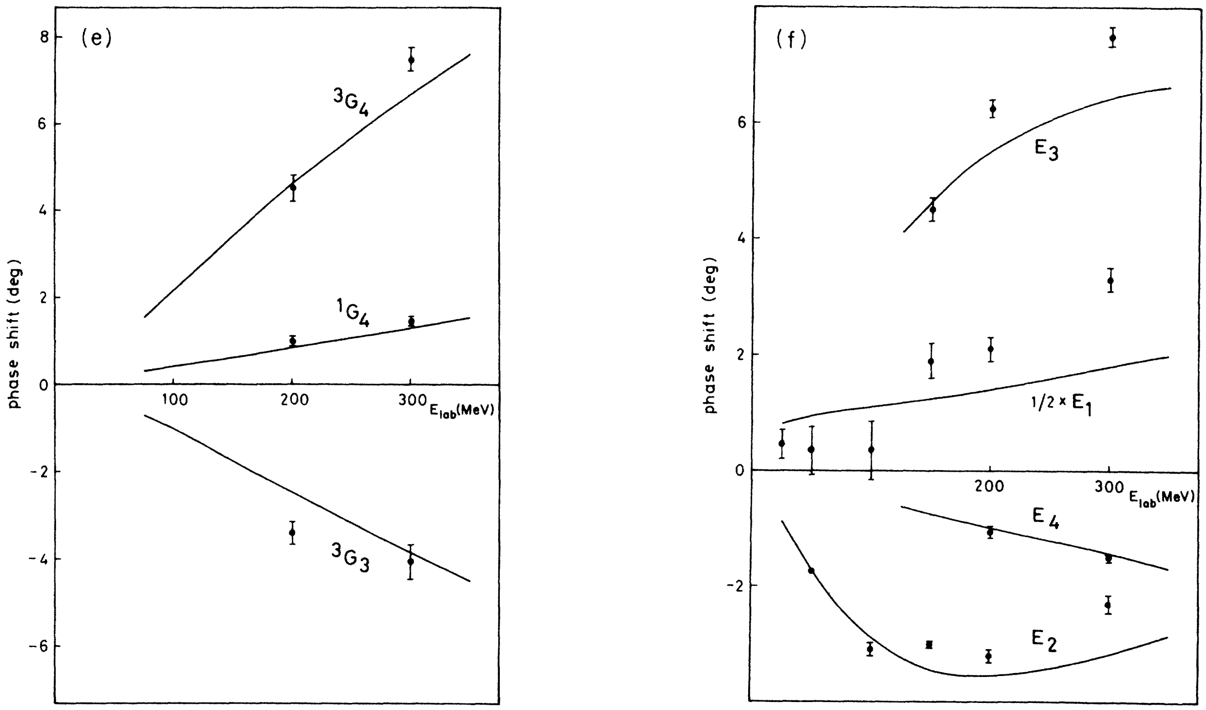

FIG. 1. (*Continued*).

mean-square radii one obtains

$$b_f = b_q = 0.51 \text{ fm} . \qquad (4.1)$$

The phase shift curves are shown in Figs. 1(a)—1(f) and the deuteron data are presented in Table I.

The relatively high value of χ^2 is essentially due to the precision of the experimental phase shifts which we have chosen. The choice of the older phase shift analysis of the Livermore group would have furnished a value of χ^2 from 5 to 6 times weaker. Nevertheless, it is remarkable to observe that the size of the rms radii b_f and b_q of the nucleon form factor and of the three-quark cluster turn out to be the same. The value given in (4.1) is in agreement with that already published by other authors. It closely corresponds to the value 0.6 fm taken, in numerous papers,[6] as the standard value for describing the nucleon without its meson cloud. We also have to emphasize that this value of $b_f = b_q$ is very close to those found in the determination of the N and Δ spectrum with a quark model[4] or in the computation of the reduced mass of the two-nucleon system and the mass difference $M_\Delta - M_N$.[7] As for the hard core, we use it essentially for reasons of convenience. In fact, its effect is negligible in the odd-parity states where the repulsion of $V_{core}(R)$ is strong enough. Any other phenomenological potential acting specifically on the even-parity states could replace it.

Throughout this paper we have especially presented the strong short-range repulsion of $V_{core}(R)$ derived from the application of the Pauli principle to the quark clusters. Besides this we must stress the importance of the isobars in the description of the intermediate-range interaction. Hence, our π-exchange NN, NΔ, and ΔΔ potentials completed with the above-described phenomenology fit well the nucleon-nucleon phase shifts and lead to a NN interaction model of only a few parameters.

The authors would like to thank A. Houriet and K. Holinde for helpful discussions.

TABLE I. Deuteron data: binding energy E_d, quadrupole moment Q, D-state probability p_D, percentage of ΔΔ component, $p_{\Delta\Delta}$, asymptotic (D/S)-wave ratio η, effective range $\rho_t(E_d,0)$, scattering length a_t, and root-mean-square radius r_d.

$E_d = -2.224$ MeV	$\eta = 0.0251$
$Q = 0.269$ fm	$\rho_t(E_d,0) = 1.71$ fm
$p_D = 5.43\%$	$a_t = 5.42$ fm
$p_{\Delta\Delta} = 0.62\%$	$r_d = 1.95$ fm

APPENDIX

Using the simplified notation $\Phi = \Phi_a \Phi_b$, $\chi = [\chi_a \chi_b]_{ST}$, and $Y = Y_{LM}$, the norm (2.6) can be expressed by

$$N^2(R) = \frac{c^2}{5} \left(\left[1 - \sum_{i,j} P_{ij} \right] \Phi \chi Y , \left[1 - \sum_{i,j} P_{ij} \right] \Phi \chi Y \right), \qquad \text{(A1)}$$

where P_{ij}, $i \in [123], j \in [456]$, denotes the even permutations between the two clusters and admits the decomposition

$$P_{ij}=P_{ij}^{x}\otimes P_{ij}^{L}\otimes P_{ij}^{\sigma}\otimes P_{ij}^{\tau}\otimes P_{ij}^{c} . \tag{A2}$$

The operator $1-\sum_{i,j}P_{ij}$ is no longer symmetric in our function space. We have then to compute

$$N^2(R)=\frac{c^2}{5}\left[1-2\sum_{i,j}(\Phi\chi Y,P_{ij}\Phi\chi Y)+\sum_{i,j,k,l}(P_{ij}\Phi\chi Y,P_{kl}\Phi\chi Y)\right] . \tag{A3}$$

In the space of the cluster functions, we define the scalar product

$$(\Phi,\Phi)=\int\prod_{j=1}^{6}d^3r_j\delta\left[\frac{\mathbf{r}_1+\mathbf{r}_2+\mathbf{r}_3}{3}+\frac{\mathbf{R}}{2}\right]\times\delta\left[\frac{\mathbf{r}_4+\mathbf{r}_5+\mathbf{r}_6}{3}-\frac{\mathbf{R}}{2}\right]\Phi^2 , \tag{A4}$$

whereas in the spaces of the spin, isospin, color, and orbital functions we take the usual scalar product. With the cluster functions (2.9) one obtains

$$(P_{ij}^{x}\Phi,P_{ij}^{x}\Phi)=3^3 ,$$

$$(\Phi,P_{ij}^{x}\Phi)=3^3(\tfrac{1}{5})^{3/2}\exp\left[-\frac{3}{5}\left[\frac{R}{b_q}\right]^2\right] ,$$

$$(P_{ij}^{x}\Phi,P_{kl}^{x}\Phi)=\begin{cases}3^3(\tfrac{3}{8})^{3/2}\exp\left[-\frac{9}{8}\left[\frac{R}{b_q}\right]^2\right], & i\neq k \text{ or } j\neq l\\ 3^3(\tfrac{3}{7})^{3/2}\exp\left[-\frac{9}{7}\left[\frac{R}{b_q}\right]^2\right], & i\neq k \text{ and } j\neq l .\end{cases} \tag{A5}$$

One admits that $P_{ij}^{L}Y\sim Y$, but in the case where $i\neq k$ and $j\neq l$ we have to take into account the exchange of the two clusters by introducing the factor $(-1)^L$. Then

$$\begin{aligned}N^2(R)=\frac{c^2}{5}\Bigg\{&1+9\times3^3-2\times9\times3^3(\tfrac{1}{5})^{3/2}\exp\left[-\frac{3}{5}\left[\frac{R}{b_q}\right]^2\right]\tfrac{1}{3}M(S,T)\\&-4\times9\times3^3(\tfrac{3}{8})^{3/2}\exp\left[-\frac{9}{8}\left[\frac{R}{b_q}\right]^2\right]\tfrac{1}{3}M(S,T)\\&+(-1)^{L+S+T}4\times9\times3^3(\tfrac{3}{7})^{3/2}\exp\left[-\frac{9}{7}\left[\frac{R}{b_q}\right]^2\right]\tfrac{1}{3}M(S,T)\Bigg\} .\end{aligned} \tag{A6}$$

The factor $\frac{1}{3}M(S,T)$ comes from the mean value of P_{36} on the symmetric spin and isotopic as well as antisymmetric color functions. By determining c with the boundary condition (2.2), we obtain the result (2.11).

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