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Few-channel models of nuclear reactions: Three-body model for deuteron elastic scattering and breakup

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This paper is concerned with the reduction of the general deuteron-nucleus collision problem to a three-body model describing deuteron elastic scattering and elastic breakup. A formally exact reduction is carried out using an antisymmetrized, multiparticle scattering theory, viz., the Bencze-Redish-Sloan theory in precursor form. All effects of the Pauli principle due to the target nucleons are thus included in the Hamiltonian H_3 describing the three-body model. Since deuteron elastic scattering and breakup have been treated for quite some time via an empirical, three-body model Hamiltonian H_M, the main purpose of this work has been to establish the relation between H_3 and H_M. It is shown that, even with inclusion of the Pauli principle, H_3 has exactly the form conjectured some years ago by Austern and Richards using a distinguishable particle ansatz. That is, H_3 is a sum of the following terms: the two kinetic energy operators, the neutron-proton interaction V_{np} binding the deuteron, the sum of the exact (antisymmetrized) neutron-nucleus and proton-nucleus optical potentials, each evaluated at an "energy" shifted by the kinetic energy operator of the other (spectator) nucleon, and a three-body interaction. Contrary to other conjectures, the Pauli principle does not give rise to a term $V_{np}Q$ (or QV_{np}), where Q is a Pauli blocking factor, projecting off states occupied in the (exact) target ground state. The deuteron in a deuteron-nucleus collision is thus not like a nucleon pair in the structure problem described by the Bethe-Goldstone theory. The three-body interaction W_{np} is sufficiently complicated to necessitate approximate evaluation. Some relatively simple approximations to W_{np} are described within a multiple scattering type of framework.

I. INTRODUCTION

The standard treatment of deuteron elastic scattering and elastic breakup—the processes A(d,d)A and A(d,np)A—is via a model three-body Hamiltonian H_M. If n and p are used to label the neutron and proton of the deuteron, then the form of H_M used in most analyses of these processes is[1]

$$H_M = K_R + K_{np} + V_{np} + \mathscr{V}_n + \mathscr{V}_p , \tag{1.1}$$

where K_R is the kinetic energy operator for the relative motion between the c.m. of the np pair and the c.m. of the target nucleus A, K_{np} is the kinetic energy operator for the relative motion of the neutron and proton, V_{np} is the neutron-proton interaction (which binds the deuteron), and $\mathscr{V}_n$ ($\mathscr{V}_p$) is an absorptive potential acting between n (p) and the c.m. of A. It is easy to see that due to the absorption H_M allows only for deuteron elastic scattering and breakup.

When deuterons collide with a nucleus, the (d,d) and (d,np) processes are only two among a very large number that usually can occur. The general deuteron-nucleus system is described not by a three-body Hamiltonian but by an $(A+2)$-body Hamiltonian H_{A+2}, which depends on the labels of the $A+2$ nucleons forming the collision system. H_M thus represents one possible reduction of H_{A+2} to a form describing the elastic events alone. The questions we pose and answer in this paper concern the general form taken by such a reduction of H_{A+2}, and how H_M is related to it. In particular, since we regard the nucleons in d and A as identical, we wish to determine the effects of the Pauli principle in such a reduction. We also wish to determine if operators occur that correspond to $\mathscr{V}_n$ and $\mathscr{V}_p$, and if they do occur, whether (and how) they are related to the optical potentials describing elastic scattering of the neutron and proton by A.

An alternate phrasing of these points arises on noting that the reduction of H_{A+2} takes the general form (see

Sec. III)

$$H_{A+2}\rightarrow H_3=K_R+K_{\rm np}+V_{\rm np}+\tilde{U}({\rm n,p})\ . \tag{1.2}$$

The main purpose of this paper is to investigate the points posed above by studying the properties of $\tilde{U}$(n,p).

A number of other authors have investigated this reduction problem, but all of their analyses are essentially *ad hoc,* due to the introduction, *ab initio,* of one or more approximations. Among such investigations we single out three of particular interest.

Austern and Richards[1] studied a distinguishable particle version of this problem and used a coupled channel ansatz to effect the reduction. They inferred that the correct reduction would be

$$H_{A+R}=H_M+W_3\ , \tag{1.3}$$

where W_3 is an unspecified three-body interaction while the $\mathscr{V}_{\rm n}$ ($\mathscr{V}_{\rm p}$) term in H_M is the neutron- (proton-) target-nucleus optical potential evaluated at an "energy" $E_{\rm d}-K_{\rm p}$ ($E_{\rm d}-K_{\rm n}$), where $E_{\rm d}$ is the incident deuteron kinetic energy. A suitable averaging procedure presumably replaces $K_{\rm p}$ ($K_{\rm n}$) by $E_{\rm d}/2$.

The use of optical potentials which fit elastic scattering data already includes some effects of antisymmetry due to the identity of the nucleons in the deuteron and those in the target. It was argued by Johnson and Soper[2] and by Austern and Pong[3] that a further effect of including nucleon identity would be to modify the potential $V_{\rm np}$ appearing in H_M. They drew an analogy between the incident deuteron moving in the interior of the target nucleus and the treatment of a pair of nucleons in a finite or infinite system via the Bethe-Goldstone equation.[4] Assuming that the target nucleus ground state is a Slater determinant of occupied orbitals, these authors replaced H_M by a Bethe-Goldstone–type Hamiltonian H'_M,

$$H'_M=K_R+K_{\rm np}+V_{\rm np}Q_{\rm occ}+\mathscr{V}_{\rm n}+\mathscr{V}_{\rm p}\ . \tag{1.4}$$

where $Q_{\rm occ}$ projects off the orbitals occupied in the target. Thus H'_M contains the effects of Pauli blocking directly in the isolated $V_{\rm np}$ interaction term.

While the modified interaction $V_{\rm np}Q_{\rm occ}$ is physically reasonable and, by comparison with the Bethe-Goldstone equation might actually be anticipated, it or its analogue for a non-Hartree-Fock ground state *does not occur* in the general $H_{A+2}\rightarrow H_3$ reduction. The incident deuteron in a scattering problem is thus only superficially like the behavior of a pair of nucleons in the structure problem: we find no isolated term such as $V_{\rm np}Q_{\rm occ}$ (or $Q_{\rm occ}V_{\rm np}$). Terms of this latter type will occur as part of more complex terms containing other interactions due to the effects of the Pauli principle, but no Pauli blocking effects arise in the isolated $V_{\rm np}$ term. Hence, apart from the fully connected, absorptive, three-body interaction $W_{\rm np}$, we find that $\tilde{U}$(n,p) of Eq. (1.2) contains only the sum of the antisymmetrized neutron-nucleus and proton-nucleus optical potentials evaluated at $E_{\rm d}-K_{\rm p}$ and $E_{\rm d}-K_{\rm n}$, respectively. Thus, the "leading" terms in H_3 are precisely H_M, as originally inferred by Austern and Richards.[1]

In addition to the structure of H_3 [i.e., of $\tilde{U}$(n,p)], we are also concerned with other questions, viz., the meaning of the model wave function and the accuracy of the numerical procedures used to solve the Schrödinger equation for the model wave function, viz.,

$$(E-H_M)\,|\,\psi_M\rangle=0\ . \tag{1.5}$$

We answer the former in this paper and comment on the latter in a succeeding article.[5]

In order to carry out the reduction of H_{A+2} to H_3, we describe the $d+A$ system using the general, antisymmetrized, many-particle collision formalism discussed by Adhikari, Kozack, and Levin,[6] and then apply the cluster decomposition/connectivity expansion methods[7] used in an earlier paper in this series.[8] We therefore take account of the Pauli principle *ab initio.* The $A+2$ particle collision system is assumed to consist of $A+2$ identical nucleons labeled n,p,1,2, . . . , A; as in Refs. 6 and 8, isospin is thus understood to be one of the state labels. Since the formalism has been described several times before, only a brief review of notation is given in Sec. II, along with a statement of the relevant equations. Unlike our earlier work on multiple scattering type expansions,[8] where the N-particle collision formalism was the extended Faddeev theory,[9] in the present case of the coupled (d,d) and (d,np) processes, we use the precursor form of the Bencze-Redish-Sloan (BRS) theory[10(a)] in wave function form.[9] As we shall see, this is a requirement imposed by the inclusion of the breakup channel. We note here that the BRS theory has also been used by Bencze, Polyzou, and Redish to derive general three-body models of nuclear reactions, but for the case in which the particles are distinguishable.[10(b)]

II. NOTATION AND THEORY

The labels n,p,1,2, . . . , A can be partitioned in a variety of ways corresponding to possible bound configurations of the particles n,p,1,2, . . . , A into clusters. We use the notation $b_m(j)$ for an m-cluster partition with particle labeling indicated by j. In general, the subscript m will be suppressed. When $m=2$, b is replaced by lower case Greek letters. The canonical labels[11] are denoted by $j=0$; in the present case $\alpha(0)=({\rm np})(1,\ldots,A)$ is the incident partition. Only the set of canonical labels $\{b(0)\}$ are needed in describing collisions involving identical particles.

Corresponding to $b(0)$ is a partition of the $(A+2)$-particle Hamiltonian H $(=H_{A+2})$: $H=H_{b(0)}+V^{b(0)}$, where $H_{b(0)}$ describes the internal states of the clusters and their relative plane wave motion. $V^{b(0)}$ is the set of intercluster interactions in partition $b(0)$, and for simplicity, we assume all interactions to be pairwise. The outgoing-wave, partition-$b(0)$ Green's function $G^{(+)}_{b(0)}$ is defined by

$$G^{(+)}_{b(0)}=\lim_{\epsilon\downarrow 0}(E+i\epsilon-H_{b(0)})^{-1}\ ,$$

where E is the total energy. The initial plane wave state $|\,\Phi_{\alpha(0)}\rangle$ is a product of the target and deuteron ground states times a relative motion plane wave state:

$$|\,\Phi_{\alpha(0)}\rangle=|\,\phi_{\rm d}({\rm n,p})\phi_T(1,\ldots,A)\rangle\,|\,\mathbf{k}_\alpha\rangle\ ,$$

and obeys $[G^{(+)}_{\alpha(0)}]^{-1}\,|\,\Phi_{\alpha(0)}\rangle=0$.

In the wave function component approach to multiparticle collisions,[6] the antisymmetrized Schrödinger solution $|\Psi^A(\alpha)\rangle$ is expressed as an antisymmetrized linear combination of partition-labeled components $|\psi_{b(0)}(\alpha)\rangle$,

$$|\Psi^A(\alpha)\rangle=\sum_b A_{b(0)}|\psi_{b(0)}(\alpha)\rangle\ , \tag{2.1}$$

where $A_{b(0)}$ is an antisymmetrizer. In the present case, these $|\psi_{b(0)}(\alpha)\rangle$ are the solutions of the following set of coupled integral equations:[6]

$$|\psi_{b(0)}(\alpha)\rangle=\delta_{b\alpha}|\Phi_{\alpha(0)}\rangle + G^{(+)}_{b(0)}\sum_d \tilde{V}^{\rm BRS}_{b(0)d(0)}|\psi_{d(0)}(\alpha)\rangle\ . \tag{2.2}$$

Since in this article we are choosing the BRS precursor formalism as our basic theory, the superscript BRS has been appended to the exchange effect potential $\tilde{V}$. It is given by[6]

$$\tilde{V}^{\rm BRS}_{b(0)d(0)}=(\hat{N}_b/\hat{N}_d)^{1/2}\sum_k R_{b(0)}V^{\rm BRS}_{b(0)d(k)}\hat{P}_{d(k)d(0)}\ , \tag{2.3}$$

where, e.g., $\hat{N}_b$ is the number of partitions of type $b_M(j)$, i.e., $0\le j\le\hat{N}_b=N_b+1$, $R_{b(0)}$ is the antisymmetrizer for the internal states in partition $b(0)$, $\hat{P}_{d(k)d(0)}$ is the operator that changes partition $d(0)$ to $d(k)$ and multiplies by the proper fermion phase factor, and $V^{\rm BRS}_{b_m(0)d_n(k)}$ is given by[9,10]

$$V^{\rm BRS}_{b_m(0)d_n(k)}=C_{d_n}V^{b_m(0)}_{d_n(k)}\ , \tag{2.4}$$

with $C_{d_n}=(-)^n(n-1)!$, while V^b_d is the set of interactions external to partition b and internal to partition d.[10] By assuming, as we do, that the internal states in all partitions are antisymmetrized, the $R_{b(0)}$ factors may be set equal to unity.

Corresponding to the $|\psi_{b(0)}(\alpha)\rangle$ are a set of antisymmetrized transition operators $T^A_{b(0)\alpha(0)}$, which obey[6]

$$T^A_{b(0)\alpha(0)}=\tilde{V}^{\rm BRS}_{b(0)\alpha(0)}+\sum_d \tilde{V}^{\rm BRS}_{b(0)d(0)}G^{(+)}_{d(0)}T^A_{d(0)\alpha(0)}\ , \tag{2.5}$$

and are related to the $|\psi_{b(0)}(\alpha)\rangle$ via[6]

$$T^A_{b(0)\alpha(0)}|\Phi_{\alpha(0)}\rangle=\sum_d \tilde{V}^{\rm BRS}_{b(0)d(0)}|\psi_{d(0)}(\alpha)\rangle\ . \tag{2.6}$$

In a coordinate representation, on-shell matrix elements of the $T^A_{b(0)\alpha(0)}$ are obtained from the asymptotic form of $|\psi_{b(0)}(\alpha)\rangle$; these on-shell matrix elements have been shown[6] to be equal to the exact, antisymmetrized transition amplitudes.[11] Thus, even though $|\psi_{b(0)}(\alpha)\rangle$ is not fully antisymmetric, it yields the correctly symmetrized amplitudes. We note here that since the BRS formalism is being used, the components $|\psi_{b(0)}(\alpha)\rangle$ are each equal to $|\Psi^A(\alpha)\rangle$, independent of $b(0)$:[9,6]

$$|\psi_{b(0)}(\alpha)\rangle=|\Psi^A(\alpha)\rangle\ \ \text{(BRS case)}\ . \tag{2.7}$$

Since we are interested only in the elastic processes (d,d) and (d,np), we can restrict our attention to $|\psi_{\alpha(0)}(\alpha)\rangle$. It is useful to reexpress (2.3) and (2.5) in vector/matrix notation and then employ a projection operator to single out the partition of interest, viz., $\alpha(0)$. The details and notation are given in Ref. 6, and we simply quote the results here. Equations (2.3) and (2.5) become

$$|\psi_0\rangle=|\Phi_0\rangle+\underline{G}_0\underline{\tilde{V}}^{\rm BRS}_0|\psi_0\rangle \tag{2.8}$$

and

$$\underline{T}^A_0=\underline{\tilde{V}}^{\rm BRS}_0+\underline{\tilde{V}}^{\rm BRS}_0\underline{G}_0\underline{T}^A_0\ . \tag{2.9}$$

Introducing the matrix projector $\underline{P}_0$, these equations go over to

$$\underline{P}_0|\psi_0\rangle=\underline{P}_0|\Phi_0\rangle+\underline{G}_0\underline{P}_0\underline{\tilde{U}}^{\rm BRS}_0\underline{P}_0|\psi_0\rangle \tag{2.10}$$

and

$$\underline{P}_0\underline{T}^A_0\underline{P}_0=\underline{P}_0\underline{\tilde{U}}^{\rm BRS}_0\underline{P}_0+\underline{P}_0\underline{\tilde{U}}^{\rm BRS}_0\underline{P}_0\underline{G}_0\underline{P}_0\underline{T}^A_0\underline{P}_0\ , \tag{2.11}$$

where $\underline{\tilde{U}}^{\rm BRS}_0$ is the solution to

$$\underline{\tilde{U}}^{\rm BRS}_0=\underline{\tilde{V}}^{\rm BRS}_0+\underline{\tilde{V}}^{\rm BRS}_0\underline{Q}_0\underline{G}_0\underline{\tilde{U}}^{\rm BRS}_0\ , \tag{2.12}$$

with $\underline{Q}_0=\underline{I}_0-\underline{P}_0$.

III. FORMAL REDUCTION TO A THREE-BODY MODEL

The restrictions to the elastic channel and to the target nucleus remaining unexcited mean that $\underline{P}_0$ is given by

$$(\underline{P}_0)_{bd}=\delta_{b(0)d(0)}\delta_{b(0)\alpha(0)}\pi\ , \tag{3.1}$$

where

$$\pi=|\phi_T(1,\ldots,A)\rangle\langle\phi_T(1,\ldots,A)| \tag{3.2}$$

projects onto the target ground state $|\phi_T\rangle$ in partition $\alpha(0)=(\text{np})(1,\ldots,A)$. The projected state $\underline{P}_0|\psi_0\rangle$ thus becomes

$$\underline{P}_0|\psi_0\rangle=|\phi_T(1,\ldots,A)\rangle|\psi(\text{n,p})\rangle\ , \tag{3.3}$$

where

$$|\psi(\text{n,p})\rangle=\langle\phi_T(1,\ldots,A)|\psi_{\alpha(0)}(\alpha)\rangle \equiv\langle\phi_T(1,\ldots,A)|\Psi^A(\alpha)\rangle \tag{3.4}$$

is a three-body state involving particles n and p and the c.m. of A. Since $|\Psi^A(\alpha)\rangle$ is the antisymmetrized Schrödinger solution, then $|\psi(\text{n,p})\rangle$, which is antisymmetric in n and p, will yield the complete, antisymmetrized amplitudes for all processes leaving the target nucleus unexcited, i.e., the antisymmetrized amplitudes for the processes $A(\text{d,d})A$ and $A(\text{d,np})A$. This, then, is the basic meaning of $|\psi(\text{n,p})\rangle$. It differs in an important way from the analogous state that one would obtain if the extended Faddeev theory had been used; viz., in the latter case, not all of the breakup amplitude would be contained in the projection of the $\alpha(0)$-partition component onto π: there would also be contributions to breakup from the continuum portions of the (d,p) and (d,n) channels (see Sec. IV). Thus the use of the BRS formalism has the advantage that the model yields the full breakup amplitude.

Equation (2.10) describes $|\psi(\text{n,p})\rangle$. The relevant portion of $\underline{P}_0\underline{\tilde{U}}^{\rm BRS}_0\underline{P}_0$ is

$$\tilde{U}(\text{n,p})=\langle\phi_T(1,\ldots,A)|\tilde{U}^{\rm BRS}_{\alpha(0)\alpha(0)}|\phi_T(1,\ldots,A)\rangle\ , \tag{3.5}$$

which is the sum of various two- and three-body, complex, nonlocal, energy-dependent effective potentials. Acting on both sides of (2.10) with $\underline{G}_0^{-1}$ and then projecting onto $\langle \phi_T(1,\ldots,A) |$ leads to

$$(E-H_3)\,|\,\psi(\mathrm{n,p})\rangle=0\ , \tag{3.6}$$

where

$$H_3=K_R+K_{\mathrm{np}}+V_{\mathrm{np}}+\tilde{U}(\mathrm{n,p})\ , \tag{3.7}$$

and we have set the target ground state energy ϵ_T equal to zero. It is evident that (3.7) and (1.3) are the same. Note that the form of (3.7) follows from the structure of $H_{\alpha(0)}$, viz.,

$$H_{\alpha(0)}=H_T+K_R+K_{\mathrm{np}}+V_{\mathrm{np}}\ ,$$

with H_T being the target nucleus Hamiltonian acting on particles $1,\ldots,A$. Equations (3.6) and (3.7) are the formal solution to the reduction problem, where in the present notation the Hamiltonian H_{A+2} is just the full Hamiltonian H for the nucleons n,p,1,...,A. Our task is thus to determine $\tilde{U}$(n,p). We undertake this using a connectivity expansion, as described in the next section.

IV. CONNECTIVITY EXPANSION FOR $\tilde{U}$(n,p)

In using a connectivity expansion[7] to evaluate $\tilde{U}$(n,p), we follow the same procedure as in Ref. 8. We start with Eq. (2.11) in the form

$$\pi T^A_{\alpha(0)\alpha(0)}\pi=\pi[\tilde{U}^{\mathrm{BRS}}_{\alpha(0)\alpha(0)}+\tilde{U}^{\mathrm{BRS}}_{\alpha(0)\alpha(0)}G^{(+)}_{\alpha(0)}\pi T^A_{\alpha(0)\alpha(0)}]\pi\ . \tag{4.1}$$

The aim is to obtain a cluster decomposition for $\tilde{U}^{\mathrm{BRS}}_{\alpha(0)\alpha(0)}$ from that of $T^A_{\alpha(0)\alpha(0)}$. Unlike the analogous, pure elastic scattering situation of Ref. 8, here π projects onto $|\phi_T(1,\ldots,A)\rangle$ and not onto $|\phi_T\phi_{\mathrm{d}}\rangle$. Therefore

$$\pi\tilde{U}^{\mathrm{BRS}}_{\alpha(0)\alpha(0)}G_{\alpha(0)}\pi T^A_{\alpha(0)\alpha(0}\pi$$

is not connected. Hence this latter term will eventually contribute portions to $\tilde{U}$(n,p) having two-particle as well as three-particle connectivity, the latter corresponding to full connectivity for the three-body model. Note that lack of complete connectivity of the kernel term in (4.1) is no problem, since we do not solve for $\pi T^A_{\alpha(0)\alpha(0)}\pi$ from it.

To proceed further, we use

$$T^A_{\alpha(0)\alpha(0)}=\sum_i T^{\mathrm{BRS}}_{\alpha(0)\alpha(i)}\hat{P}_{\alpha(i)\alpha(0)}\ , \tag{4.2}$$

where $T^{\mathrm{BRS}}_{\alpha(0)\alpha(i)}$ is the unsymmetrized BRS transition operator. The key element in the analysis is the on-shell relation[9,12]

$$T^{\mathrm{BRS}}_{\alpha(0)\alpha(i)}\,|\,\Phi_{\alpha(i)}\rangle=U^{(+)}_{\alpha(0)\alpha(i)}\,|\,\Phi_{\alpha(i)}\rangle \tag{4.3a}$$

$$=V^{\alpha(0)}[1+G^{(+)}V^{\alpha(i)}]\,|\,\Phi_{\alpha(i)}\rangle\ , \tag{4.3b}$$

where $U^{(+)}_{\alpha(0)\alpha(i)}$ is the "post" form of the transition operator[13] and $G^{(+)}=\lim_{\epsilon\downarrow0}(E+i\epsilon-H)^{-1}$ is the full outgoing wave Green's function. Since $T^{\mathrm{BRS}}_{\alpha(0)\alpha(i)}$ and $U^{(+)}_{\alpha(0)\alpha(i)}$ are right-half-shell equivalent when acting on an on-shell, two-cluster state, it does not matter that the theory is to be used for both two-cluster and three-cluster (i.e., breakup) final states. On the other hand, had we started with the extended Faddeev (EF) theory,[9] then the analogue of (4.3) that would be required is

$$\langle\Phi_{b(i)}\,|\,T^{\mathrm{EF}}_{b(i)\alpha(0)}=\langle\Phi_{b(i)}\,|\,U^{(-)}_{b(i)\alpha(0)}\ ,$$

with $U^{(-)}$ being the "prior" form of transition operator,[13] and this latter relation holds *only* when $\langle\Phi_{b(i)}|$ is a *two-cluster* state.[14] Thus, in the EF case we could not use a cluster decomposition applied to a known expression for the breakup final state.

As in Ref. 8, to which we refer for details, we write the cluster decomposition or connectivity expansion of an arbitrary operator B as[7]

$$B=\sum_a[B]_a\ , \tag{4.4}$$

where a runs over all partitions and $[B]_a$ has the connectivity of a. Substituting (4.3b) into (4.2) into (4.1), and then using (4.4), we find

$$[\pi\tilde{U}^{\mathrm{BRS}}_{\alpha(0)\alpha(0)}\pi]_a=\sum_i[\pi V^{\alpha(0)}(1+G^{(+)}V^{\alpha(i)})\pi-\pi\tilde{U}^{\mathrm{BRS}}_{\alpha(0)\alpha(0)}G^{(+)}_{\alpha(0)}\pi V^{\alpha(0)}(1+G^{(+)}V^{\alpha(i)})\pi]_a\hat{P}_{\alpha(i)\alpha(0)} \tag{4.5a}$$

$$\equiv[\pi T^A_{\alpha(0)\alpha(0)}\pi-\pi\tilde{U}^{\mathrm{BRS}}_{\alpha(0)\alpha(0)}\pi G^{(+)}_{\alpha(0)}\pi T^A_{\alpha(0)\alpha(0)}\pi]_a\ , \tag{4.5b}$$

where a will, in effect, take on only the values "two-body contribution" and "three-body contribution." We consider these types of contributions separately.

V. DETERMINATION OF $[\tilde{U}(\mathrm{n,p})]_{\text{2-body contribution}}$

The first step is to identify the two-body or two-particle terms in

$$T^A_{\alpha(0)\alpha(0)}=\sum_i V^{\alpha(0)}[1+G^{(+)}V^{\alpha(i)}]\hat{P}_{\alpha(i)\alpha(0)}\ .$$

To do so means finding the portions with connectivity $\beta(0)=$(n)(p,1,...,A) and $\gamma(0)=$(p)(n,1,...,A). Any fully connected contributions must be eliminated, since they will yield terms of three-particle connectivity.

It is helpful to rewrite $V^{\alpha(0)}$ in (4.5a) as

$$V^{\alpha(0)}=V^{\alpha(0)}_{\beta(0)}+V^{\alpha(0)}_{\gamma(0)}\ , \tag{5.1}$$

which holds due to our assumption of pairwise interactions. Then the two-body contributions to $T^A_{\alpha(0)\alpha(0)}$ are obtained from

$$[T^A_{\alpha(0)\alpha(0)}]_{\text{2-body}}=\sum_i[V^{\alpha(0)}_{\beta(0)}(1+G^{(+)}V^{\alpha(i)})]_{\text{2-body}}\hat{P}_{\alpha(i)\alpha(0)}+\sum_i[V^{\alpha(0)}_{\gamma(0)}(1+G^{(+)}V^{\alpha(i)})]_{\text{2-body}}\hat{P}_{\alpha(i)\alpha(0)}\ . \tag{5.2}$$

The equivalence class (or channel) α contains four types of partitions $\alpha(i)$, denoted

$$\alpha(0)=(\mathrm{np})(1,\ldots,A)\ ,$$

$$\alpha(\mathrm{n}i)=(i\mathrm{p})(1,\ldots,i-1,n,i+1,\ldots,A)\ ,$$

$$\alpha(\mathrm{p}i)=(\mathrm{n}i)(1,\ldots,i-1,p,i+1,\ldots,A)\ ,$$

and

$$\alpha(ij)=(ij)(1,\ldots,i-1,\mathrm{n},i+1,\ldots,j-1,\mathrm{p},j+1,\ldots,A)\ ,$$

where we have introduced a new notation for convenience. Since all $V^{\alpha(i)}$ involve external interactions, it is straightforward to determine which of the above partitions contributes to (5.2). None of the partitions $\alpha(ij)$ can contribute, since, due to the $\pi,\ldots,\hat{P}_{\alpha(ij)\alpha(0)}\pi$ factors, they give rise only to three-body portions; on the other hand, $\alpha(0)$ will contribute to both terms in (5.2). Furthermore, the $\alpha(\mathrm{n}i)$ partitions do not contribute to the first sum and the $\alpha(\mathrm{p}i)$ partitions do not contribute to the second sum in (5.2) because in each case the results are fully connected, i.e., are of three-body type. It thus follows that

$$[T^{A}_{\alpha(0)\alpha(0)}]_{\text{2-body}}=\left[V^{\alpha(0)}_{\beta(0)}\left\{(1+G^{(+)}V^{\alpha(0)})+\sum_i(1+G^{(+)}V^{\alpha(\mathrm{p}i)})\hat{P}_{\mathrm{p}i}\right\}\right]_{\text{2-body}}$$
$$+\left[V^{\alpha(0)}_{\gamma(0)}\left\{(1+G^{(+)}V^{\alpha(0)})+\sum_i(1+G)^{(+)}V^{\alpha(\mathrm{n}i)})\hat{P}_{\mathrm{n}i}\right\}\right]_{\text{2-body}}\ . \tag{5.3}$$

Next, we must isolate the relevant contributions from those terms involving $G^{(+)}$. To illustrate the procedure, we consider the first [ ] term in (5.3). Noting that

$$G^{(+)}V^{\alpha'}=\sum_d[G^{(+)}V^{\alpha'}]_d\ , \tag{5.4}$$

where α' denotes $\alpha(0)$ or $\alpha(\mathrm{p}i)$, the two-particle connectivity restriction means that the relevant portion of (5.4) is the term $G^{(+)}_{\beta(0)}V^{\alpha'}_{\beta(0)}$. A similar analysis applied to the second [ ] term in (5.3) leads, in this case, to $G^{(+)}_{\gamma(0)}V^{\alpha'}_{\gamma(0)}$. Hence, (5.3) becomes

$$[T^{A}_{\alpha(0)\alpha(0)}]_{\text{2-body}}=V^{\alpha(0)}_{\beta(0)}\left\{(1+G^{(+)}_{\beta(0)}V^{\alpha(0)}_{\beta(0)})+\sum_i(1+G^{(+)}_{\beta(0)}V^{\alpha(\mathrm{p}i)}_{\beta(0)})\hat{P}_{\mathrm{p}i}\right\}+V^{\alpha(0)}_{\gamma(0)}\left\{(1+G^{(+)}_{\gamma(0)}V^{\alpha(0)}_{\gamma(0)})+\sum_i(1+G^{(+)}_{\gamma(0)}V^{\alpha(\mathrm{n}i)}_{\gamma(0)})\hat{P}_{\mathrm{n}i}\right\} \tag{5.5a}$$

$$=\left\{T^{\alpha(0)\alpha(0)}_{\beta(0)}+\sum_i T^{\alpha(0)\alpha(\mathrm{p}i)}_{\beta(0)}\hat{P}_{\mathrm{p}i}\right\}+\left\{T^{\alpha(0)\alpha(0)}_{\gamma(0)}+\sum_i T^{\alpha(0)\alpha(\mathrm{n}i)}_{\gamma(0)}\hat{P}_{\mathrm{n}i}\right\}\ , \tag{5.5b}$$

where (5.5b) is a definition of the various transition operators.

These transition operators are straightforwardly interpreted. $\pi T^{\alpha(0)\alpha(0)}_{\beta(0)}\pi$ corresponds to elastic scattering of the proton by the target (which contains the nucleons $1,\ldots,A$), during which the neutron behaves like a free particle. This is due to the propagator

$$G^{(+)}_{\beta(0)}=\left[E+i0-K_R-K_{\mathrm{np}}-H_T-\sum_{i=1}^{A}V_{\mathrm{p}i}\right]^{-1}\ .$$

Hence $\pi T^{\alpha(0)\alpha(0)}_{\beta(0)}\pi$ is the unsymmetrized, proton-target-nucleus elastic scattering transition operator embedded in the $(A+2)$-particle Hilbert space. Similarly, $\pi T^{\alpha(0)\alpha(\mathrm{p}i)}_{\beta(0)}\hat{P}_{\mathrm{p}i}\pi$ describes an "elastic" rearrangement of the form

$$i+(1,\ldots,i-1,\mathrm{p},i+1,\ldots,A)\rightarrow\mathrm{p}+(1,\ldots,A)\ ,$$

also embedded in the $(A+2)$-particle Hilbert space. The presence of the factor $\hat{P}_{\mathrm{p}i}$ plus the sum over all $i\in A$ means that the first term in curly braces in (5.5b) is just the properly antisymmetrized, $\mathrm{p}+A$ elastic scattering transition operator embedded in the $(A+2)$-particle space. The second term in curly braces is the analogous operator for $\mathrm{n}+A$ elastic scattering.

In view of the above comments, it is useful to introduce two $(A+1)$-particle partition labels: $\nu(0)=(\mathrm{n})(1,\ldots,A)$ and $\rho(0)=(\mathrm{p})(1,\ldots,A)$. Then Eq. (5.5b) can be written as

$$[\pi T^{A}_{\alpha(0)\alpha(0)}\pi]_{\text{2-body}}=\{\pi T^{A}_{\rho(0)\rho(0)}(E-K_{\mathrm{n}})\pi+\pi T^{A}_{\nu(0)\nu(0)}(E-K_{\mathrm{p}})\pi\}\ , \tag{5.6}$$

where the shifted energy dependence of and the antisymmetry label A on the new $(A+1)$-particle transition operators is made manifest.

Returning to Eq. (4.5), we now have, for the two-body part,

$$[\pi\tilde{U}^{\rm BRS}_{\alpha(0)\alpha(0)}\pi]_{2\text{-body}}=\{\pi T^A_{\rho(0)\rho(0)}(E-K_{\rm n})\pi+\pi T^A_{\nu(0)\nu(0)}(E-K_{\rm p})\pi\}$$
$$+[\pi\tilde{U}^{\rm BRS}_{\alpha(0)\alpha(0)}\pi G^{(+)}_{\alpha(0)}\{\pi T^A_{\rho(0)\rho(0)}(E-K_{\rm n})\pi+\pi T^A_{\nu(0)\nu(0)}(E-K_{\rm p})\pi\}]_{2\text{-body}}\ . \tag{5.7}$$

From the structure of $H_{\alpha(0)}$, which contains $V_{\rm np}$, it is evident that the product of $G^{(+)}_{\alpha(0)}$ and the terms in the curly braces in Eq. (5.7) will contain both two- and three-body contributions. To eliminate the latter, we must replace $G^{(+)}_{\alpha(0)}$ by

$$G^{(+)}_{a(0)}=(E^+-K_R-K_{\rm np}-H_T)^{-1}\ ,$$

where $a(0)=({\rm n})({\rm p})(1,\ldots,A)$. To obtain the desired two-body parts, we may iterate (5.7), using $G^{(+)}_{a(0)}$ in place of $G^{(+)}_{\alpha(0)}$. It should be obvious that any term in the iteration which will contain as a factor the cross terms

$$\pi T^A_{\rho(0)\rho(0)}\pi T^A_{\nu(0)\nu(0)}\pi$$

or

$$\pi T^A_{\nu(0)\nu(0)}\pi T^A_{\rho(0)\rho(0)}\pi$$

must be eliminated, since these factors are fully connected, i.e., any term containing them will be a three-body contribution. The right hand side of (5.7) will therefore consist of two infinite series, one involving $\pi T^A_{\rho(0)\rho(0)}\pi$, the other $\pi T^A_{\nu(0)\nu(0)}\pi$, both identical in structure. The result is

$$[\pi\tilde{U}^{\rm BRS}_{\alpha(0)\alpha(0)}\pi]_{2\text{-body}}=\{T^A_{\rho(0)\rho(0)}\pi-\pi T^A_{\rho(0)\rho(0)}\pi G^{(+)}_{a(0)}\pi T^A_{\rho(0)\rho(0)}\pi+\cdots\}_{\rm p}$$
$$+\{\pi T^A_{\nu(0)\nu(0)}\pi-\pi T^A_{\nu(0)\nu(0)}\pi G^{+}_{a(0)}\pi T^A_{\nu(0)\nu(0)}\pi+\cdots\}_{\rm n}\ . \tag{5.8}$$

Equation (5.8) is the desired result. We can put it in a more recognizable form by noting that each of the two series is an iterated form of an integral equation. In particular, $\{\ \}_{\rm p}$ results from iterating

$$\pi U_{pA}\pi=\pi T^A_{\rho(0)\rho(0)}\pi-\pi U_{pA}\pi G^{(+)}_{a(0)}\pi T^A_{\rho(0)\rho(0)}\pi\ , \tag{5.9}$$

while $\{\ \}_{\rm n}$ is the iterated form of

$$\pi U_{nA}\pi=\pi T^A_{\nu(0)\nu(0)}\pi-\pi U_{nA}\pi G^{(+)}_{a(0)}\pi T^A_{\nu(0)\nu(0)}\pi\ , \tag{5.10}$$

where $\pi U_{pA}\pi$ and $\pi U_{nA}\pi$ are defined by (5.9) and (5.10). Rearranging these latter two equations, we get (restoring the suppressed energy dependence)

$$\pi T^A_{\rho(0)\rho(0)}(E-K_{\rm n})\pi$$
$$=\pi U_{pA}(E-K_{\rm n})\pi[1+G^{(+)}_{a(0)}\pi T^A_{\rho(0)\rho(0)}(E-K_{\rm n})\pi] \tag{5.11}$$

and

$$\pi T^A_{\nu(0)\nu(0)}(E-K_{\rm p})\pi$$
$$=\pi U_{nA}(E-K_{\rm p})\pi[1+G^{(+)}_{a(0)}\pi T^A_{\nu(0)\nu(0)}(E-K_{\rm p})\pi]\ . \tag{5.12}$$

Since $T^A_{\rho(0)\rho(0)}$ and $T^A_{\nu(0)\nu(0)}$ are the antisymmetrized ${\rm p}+A$ and ${\rm n}+A$ elastic scattering transition operators, then we can immediately identify

$$\langle\phi_T(1,\ldots,A)\,|\,U_{pA}(E-K_{\rm n})\,|\,\phi_T(1,\ldots,A)\rangle$$

and

$$\langle\phi_T(1,\ldots,A)\,|\,U_{nA}(E-K_{\rm p})\,|\,\phi_T(1,\ldots,A)\rangle$$

as the antisymmetrized, optical potential operators describing ${\rm p}+A$ and ${\rm n}+A$ elastic scattering, respectively, at the appropriate *shifted* energies. We denote these optical potential operators by $\mathscr{V}_{pA}(E-K_{\rm n})$ and $\mathscr{V}_{nA}(E-K_{\rm p})$. We now see that Eq. (5.8) is equivalent to

$$[\pi\tilde{U}^{\rm BRS}_{\alpha(0)\alpha(0)}\pi]_{2\text{-body}}$$
$$=|\phi(1,\ldots,A)\rangle\{\mathscr{V}_{pA}(E-K_{\rm n})+\mathscr{V}_{nA}(E-K_{\rm p})\}$$
$$\times\langle\phi_T(1,\ldots,A)|\ , \tag{5.13}$$

which leads to the result

$$[\tilde{U}({\rm n},{\rm p})]_{2\text{-body}}=\mathscr{V}_{pA}(E-K_{\rm n})+\mathscr{V}_{nA}(E-K_{\rm p})\ . \tag{5.14}$$

Since the remaining contributions to $\tilde{U}({\rm n},{\rm p})$ are of a fully connected, three-body nature, we write

$$\tilde{U}({\rm n},{\rm p})=\mathscr{V}_{pA}(E-K_{\rm n})+\mathscr{V}_{nA}(E-K_{\rm p})+W_{\rm np}\ , \tag{5.15}$$

where $W_{\rm np}$ is the three-body part. Equation (3.7) thus takes on the form

$$H_3=K_R+K_{\rm np}+V_{\rm np}+\mathscr{V}_{pA}(E-K_{\rm n})$$
$$+\mathscr{V}_{nA}(E-K_{\rm p})+W_{\rm np}\ . \tag{5.16}$$

With the interpretation of $\mathscr{V}_{pA}$ and $\mathscr{V}_{nA}$ given above, we see that H_3 is in precisely the form of the Austern-Richards Hamiltonian H_{A+R}, Eq. (1.3). When $W_{\rm np}$ can be neglected, $H_3\rightarrow H_M$ and we recover the starting point of typical ${\rm d}+A$ analyses.[1]

The final task is to determine $W_{\rm np}$. An exact determination is not possible, but low order contributions to it can be estimated, the analysis of which we undertake in the next two sections. In particular, we shall show that $W_{\rm np}$ does not contain an isolated (three-body) term like $-V_{\rm np}\rho({\rm n},{\rm p})$, where $\rho({\rm n},{\rm p})$ is the two-particle density in the target ground state, thus establishing that no combination such as $V_{\rm np}[1-\rho({\rm n},{\rm p})]$ occurs. That is, the analogue of the $V_{\rm np}Q_{\rm occ}$ term conjectured for a Hartree-Fock ground state [see Eq. (1.4)] is absent.

VI. PAULI BLOCKING EFFECTS

Pauli blocking arises when a system contains identical fermions. The present case is no exception, of course, al-

though, as we show in this section, no such effects are associated with the isolated V_{np} interaction: they will occur only when two or more interactions and, correspondingly, one or more Green's functions are present in one term.

Suppose Pauli blocking could occur in conjunction with the isolated V_{np} potential. Then the matrix element (3.5) would necessarily contain terms like

$$M_{np}^{(i)}=V_{np}\langle\phi_T(1,\ldots,A)\,|\,\hat{P}_{\alpha(0)\alpha(i)}\,|\,\phi_T(1,\ldots,A)\rangle\ , \tag{6.1}$$

where $i\neq 0$ and $\alpha(0)=(\mathrm{np})(1,\ldots,A)$. The only type of term in (4.5) that could give rise to (6.1) is the leading one, viz., $V^{\alpha(0)}\hat{P}_{\alpha(0)\alpha(i)}$, since all others will contain at least two interactions and one Green's function. But, V_{np} does *not* occur in $V^{\alpha(0)}$. Hence, no portion of $\tilde{U}_{np}$ contains a factor such as (6.1) and therefore no Pauli blocking effects are associated with the isolated V_{np} interaction. It is therefore incorrect to replace V_{np} in H_M by a "Pauli-principle-corrected," three-body interaction of the form $V_{np}Q$.

There are, of course, other Pauli blocking terms associated with $\tilde{U}(\mathrm{n,p})$ and thus with W_{np}. The lowest order Pauli blocking terms that will contain V_{np} are of the form

$$I_{np}^{(i)}=\langle\phi_T(1,\ldots,A)\,|\,V^{\alpha(0)}Q_{\alpha(0)}G_{\alpha(0)}^{(+)}V_{np}\hat{P}_{\alpha(i)\alpha(0)}\,|\,\phi_T(1,\ldots,A)\rangle\ , \tag{6.2}$$

where

$$\alpha(i)=(i\mathrm{p})(1,\ldots,i-1,\mathrm{n},i+1,\ldots,A)$$

or

$$\alpha(i)=(\mathrm{n}i)(1,\ldots,i-1,\mathrm{p},i+1,\ldots,A)\ .$$

Higher order terms, analogous to (6.2), are easily written down; all are three-body contributions. Such terms are identified as being of Pauli blocking type due to the appearance in the matrix element of both $Q_{\alpha(0)}$ and $\hat{P}_{\alpha(i)\alpha(0)}$. These types of terms, and others not containing V_{np}, will arise from the exchange-effect contributions to W_{np}. It serves no useful purpose for this article to single them out, and we have not done so in the following discussion of W_{np}.

VII. MULTIPLE SCATTERING TYPE APPROXIMATIONS TO W_{np}

It follows from Eq. (5.6) that the three-body contributions to the transition operator $\pi T^A_{\alpha(0)\alpha(0)}\pi$ [Eqs. (4.5a) and (4.5b)] are given by

$$[\pi T^A_{\alpha(0)\alpha(0)}\pi]_{3\text{-body}}=\pi T^A_{\alpha(0)\alpha(0)}\pi-\pi\{T^A_{\rho(0)\rho(0)}(E-K_{\mathrm{n}})+T^A_{\nu(0)\nu(0)}(E-K_{\mathrm{p}})\}\pi \tag{7.1a}$$

$$\equiv\pi T^A_{\alpha(0)\alpha(0)}\pi-T_{\mathrm{p}}-T_{\mathrm{n}}\ ; \tag{7.1b}$$

Eq. (7.1b) defines T_{p} and T_{n}.

We have found no ways to determine $[\pi T^A_{\alpha(0)\alpha(0)}\pi]_{3\text{-body}}$ exactly. On the other hand, there are many possibilities for approximating this quantity and thus W_{np}. We have studied two types, and shall describe the simpler of them in detail in this section, ending it with a qualitative description of the other.

The simpler of these two classes of approximations is characterized as a multiple scattering type of approximation.[15] In its least complex form, $\pi\hat{U}^{\mathrm{BRS}}_{\alpha(0)\alpha(0)}\pi$ in Eq. (4.1) is replaced by $T_{\mathrm{p}}+T_{\mathrm{n}}$—the leading (curly-bracketed) term in (5.7)—and then (4.1) is iterated. The leading three-body terms in this iteration are from (7.1)

$$[\pi T^A_{\alpha(0)\alpha(0)}\pi]_{3\text{-body}}\cong T_{\mathrm{p}}G_{\mathrm{d}}^{(+)}T_{\mathrm{n}}+T_{\mathrm{n}}G_{\mathrm{d}}^{(+)}T_{\mathrm{p}}\ , \tag{7.2}$$

where

$$G_{\mathrm{d}}^{(+)}=(E+i0-K_R-K_{\mathrm{np}}-V_{\mathrm{np}})^{-1}\ , \tag{7.3}$$

and, as before, the target ground state energy ϵ_T has been set equal to zero.

A further approximation is to replace $G_{\mathrm{d}}^{(+)}$ by $G_{\mathrm{np}}^{(+)}$, defined by

$$G_{\mathrm{np}}^{(+)}=(E+i0-K_R-K_{\mathrm{np}})^{-1}\ , \tag{7.4}$$

so that an alternate low-order approximation is

$$[\pi T^A_{\alpha(0)\alpha(0)}\pi]_{3\text{-body}}\cong T_{\mathrm{p}}G_{\mathrm{np}}^{(+)}T_{\mathrm{n}}+T_{\mathrm{n}}G_{\mathrm{np}}^{(+)}T_{\mathrm{p}}\ . \tag{7.5}$$

If all iterations can be ignored, so that (7.4) or (7.5) and

$$\pi\tilde{U}^{\mathrm{BRS}}_{\alpha(0)\alpha(0)}\pi\cong\pi T^A_{\alpha(0)\alpha(0)}\pi$$

are reasonable approximations, then the simplest approximation to W_{np} has the form

$$W_{\mathrm{np}}\cong W_{\mathrm{np}}^{(1)}=T_{\mathrm{p}}G_{\lambda}T_{\mathrm{n}}+T_{\mathrm{n}}G_{\lambda}T_{\mathrm{p}}\ , \tag{7.6}$$

where $\lambda=\mathrm{d}$ or $\lambda=\mathrm{np}$. If the choice $\lambda=\mathrm{np}$ is made, the T_{p} and T_{n} terms can be replaced by their expressions in terms of the antisymmetrized, one-body optical potentials $\mathscr{V}_{\mathrm{p}A}$ and $\mathscr{V}_{\mathrm{n}A}$ (evaluated at the properly shifted energies). Use of a few operator manipulations and Green's function identities then yield ($\lambda=\mathrm{np}$)

$$\begin{aligned}W_{\mathrm{np}}^{(1)}=\ &\mathscr{V}_{\mathrm{p}A}(E+i0-K_R-K_{\mathrm{np}}-\mathscr{V}_{\mathrm{n}A}-\mathscr{V}_{\mathrm{p}A}+\mathscr{V}_{\mathrm{n}A}G_{\mathrm{np}}^{(+)}\mathscr{V}_{\mathrm{p}A})^{-1}\mathscr{V}_{\mathrm{n}A}\\&+\mathscr{V}_{\mathrm{n}A}(E+i0-K_R-K_{\mathrm{np}}-\mathscr{V}_{\mathrm{n}A}-\mathscr{V}_{\mathrm{p}A}+\mathscr{V}_{\mathrm{p}A}G_{\mathrm{np}}^{(+)}\mathscr{V}_{\mathrm{n}A})^{-1}\mathscr{V}_{\mathrm{p}A}\ .\end{aligned} \tag{7.7}$$

The physical interpretation of this result is straightforward, the propagators containing a modified neutron-proton interaction involving the optical potentials $\mathscr{V}_{nA}$ and $\mathscr{V}_{pA}$.

Equation (7.6) is especially simple because no iterations are included. Their inclusion involves three-body terms of various kinds. Probably the most familiar looking of these arises from iterations in which T_p and T_n alternate. Such an iteration arises from the decomposition

$$T_{np}^F = T^{(n)} + T^{(p)} , \tag{7.8a}$$

with

$$T^{(j)} = T_j + T_j G_\lambda^{(+)} T^{(k)}, \quad k \neq j, \; k \text{ and } j = \text{n or p} . \tag{7.8b}$$

The pair (7.8) is an analogue of the well-known set of Faddeev equations.[16] The iteration of them yields a multiple scattering expansion,[15] to which the title of this subsection refers.

If we subtract T_n and T_p from T_{np}^F of (7.8a), the result is a pure three-body term, which we denote T_3^F:

$$T_3^F = T_{np}^F - T_n - T_p . \tag{7.9}$$

This is the three-body portion of the amplitude for simultaneous scattering of the neutron and proton due, respectively, to their interactions with the target *via* $\mathscr{V}_{nA}$ and $\mathscr{V}_{pA}$ and, for $\lambda = d$, with each other *via* V_{np}.

Substituting (7.9) into (4.1) and then dropping the terms with $G_{\alpha(0)}^{(+)}$ yields a second, Faddeev-like approximation for W_{np}, viz.,

$$W_{np} \cong W_{np}^{(2)} = T_3^F . \tag{7.10}$$

Notice that we cannot get a pure three-body contribution by first iterating

$$T_{np} = (T_n + T_p)(1 + G_\lambda^{(+)} T_{np})$$

and then subtracting $T_n + T_p$, since such an iteration will contain pure two-body terms like $T_n G_\lambda^{(+)} T_n$, etc. Subtracting all such two-body terms from T_{np} would, of course, yield a third approximation to W_{np} of the same type as $W_{np}^{(1)}$ and $W_{np}^{(2)}$.

The "simplicity" of the $W_{np}^{(1)}$ and $W_{np}^{(2)}$ approximations results from not solving the relevant forms of (4.1). Such solutions can be obtained, in much the same way as the two-body contributions were determined in Sec. V. They do not lead to results that are as simple in form as in the case of the two-body contributions, however, and we do not consider them further.

The other procedure we have studied for approximating the left hand side of (7.1) involves restricting the incident deuteron to interact with only one target nucleon at a time. Such a restriction may be referred to as a quasiimpulse approximation (QIA). The resulting expression for W_{np}, assuming no iteration of Eq. (4.1), is the ground state matrix element of a symmetrized, three-body scattering operator which has subtracted from it the sum of two two-body scattering operators. This difference of operators is analogous to any of the three terms in the expression for the symmetrized three-body contribution to the deuteron-nucleus elastic scattering optical potential we have derived in Ref. 8, an expression which may be referred to for structural details. We do not display the analogous expression in the present case because to do so involves an inconsistency, viz., W_{np} is treated in the QIA, but the potentials $\mathscr{V}_{pA}$ and $\mathscr{V}_{nA}$ are not, though they must be also. Treating the QIA approach consistently, however, leads to approximations on both the two-body and three-body levels. Since only formal expressions for $\mathscr{V}_{pA}^{QIA}$ and $\mathscr{V}_{nA}^{QIA}$ can be written down, while their detailed properties remain unknown, then both the two-body and three-body parts of H_3 become indeterminate. It therefore suffices simply to note that this alternate approach exists.

VIII. SUMMARY

In this paper we have used a fully antisymmetrized, multiparticle scattering theory to reduce the deuteron-nucleus collision system to an effective three-body model consisting of the neutron and proton forming the deuteron plus an absorptive core representing the unexcited (and unexcitable) target nucleus. The Hamiltonian H_3 for this model is a sum of a "two-body" portion and a "three-body" portion W_{np}. The two-body part is in precisely the form employed as the theoretical basis of the standard deuteron-nucleus three-body collision model as used to analyze (d,d) and (d,np) collision data. In addition to the kinetic energy operators, this two-body part of H_3 consists of three other terms: the exact neutron-nucleus and the exact proton-nucleus elastic scattering optical potentials, each evaluated at an energy shifted by the kinetic energy of the other (spectator) nucleon, plus the bare neutron-proton interaction V_{np} that binds the deuteron. Pauli-principle effects occur at this level only in the optical potentials.

The three-body part of H_3 is too complicated to be amenable to exact calculation. A multiple-scattering type of approximation is introduced, which allows for relatively "simple" evaluations of W_{np}. However, we are able to show exactly that W_{np} does not contain any terms which, when used in conjunction with the V_{np} part of H_3, would lead to a contribution of the form $V_{np}Q$, where Q represents a Pauli blocking effect. Pauli blocking involving V_{np} will arise only when at least one other interaction and a Green's function are present in the relevant (target ground state) matrix element. Thus, one should not attempt to augment[2,3] the Hamiltonian H_M of the standard deuteron-nucleus collision three-body collision model by a term such as $V_{np}(1-Q)$ in an attempt to include Pauli principle effects: the model *as it is used* already properly takes them into account.

Finally, we note that this paper is exclusively concerned with the formal reductions $H_{A+2} \to H_3 \to H_M$ and not with the validity of the means used to approximate H_M so as to extract numbers from it. In the next paper in this series,[5] we shall examine the accuracy of the diagonalization and expansion methods used in numerically solving the H_M problem [after suitable (and standard) approximations to H_M have been made].

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