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Rapid Communications

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$^2H(\vec{d},\gamma)^4He$ reaction and the ^4He *D* state

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The $^2H(\vec{d},\gamma)^4He$ reaction observables resulting from $M1$, $E1$, $M2$, and $E2$ transitions are studied quantitatively. The calculations are in agreement with recent data for the reaction vector (A_y) and tensor (A_{yy}) analyzing powers at 5 MeV center of mass energy and also with the best available theoretical predictions for the ^4He D state wave function.

Since the recent first measurement of T_{20} for $^2H(\vec{d},\gamma)^4He$ by Weller *et al.*,[1] it has been clear that the ^4He D state plays a significant role in the capture reaction. Subsequent to these data and associated theoretical work[2,3] there has been intense experimental interest in this reaction for incident deuteron energies from 50 keV to 100 MeV.[4-6] A common aim of the experiments is to understand quantitatively these D-state contributions to the reaction and thus to extract an empirical measure of the magnitude of the D state from the data. Analyses to date[1-3] assume the reaction is pure $E2$ in nature.

In this Rapid Communication we confine discussion to the very recent data of Mellema *et al.*[7] for the reaction vector (A_y) and tensor (A_{yy}) analyzing powers at $E_{c.m.}=5$ MeV. These data are significant for two reasons. First, they show unambiguously, through a large measured A_y, the presence of multipole transitions (e.g., $E1$ and $M2$) other than simply $E2$. Second, A_{yy} is the analyzing power least sensitive to ambiguities in the present theoretical treatment of the initial state interaction[2] and thus a good observable to study ^4He D-state effects. This Rapid Communication reports calculations which address these two points and which incorporate the most reliable available theoretical estimates of the ^4He D-state amplitude.[8,9]

The probability amplitude for transition from a continuum two-deuteron initial state $|d_1^{\sigma_1}d_2^{\sigma_2};\mathbf{k}\rangle$ to the $J^\pi=0^+$ ^4He ground state $|\alpha;0^+\rangle$ with the emission of a photon of circular polarization $\boldsymbol{\varepsilon}_q$ ($q=\pm1$) relative to the photon momentum $\mathbf{k}_\gamma$ is

$$T(\sigma_1\sigma_2,\mathbf{k}\rightarrow\alpha\mathbf{k}_\gamma q)=\langle\alpha;0^+|H_e(\mathbf{k}_\gamma,\boldsymbol{\varepsilon}_q)|d_1^{\sigma_1}d_2^{\sigma_2};\mathbf{k}\rangle\ . \quad (1)$$

Here σ_1 and σ_2 are the projections of the intrinsic spins ($S_1=S_2=1$) of the incident (2) and target (1) deuterons and $\mathbf{k}$ their asymptotic wave numer in the c.m. frame. The interaction Hamiltonian for emission is

$$H_e(\mathbf{k}_\gamma,\boldsymbol{\varepsilon}_q)=-\sum_{LM\pi}q^\pi T_{LM}^\dagger(\pi)\mathcal{D}_{Mq}^L(R)^*\ , \quad (2)$$

where the $T_{LM}(\pi)$ are multipole operators for electric ($e,\pi=0$) and magnetic ($m,\pi=1$) transitions.[10,11] The rotation R takes the fixed coordinate system z axis into $\mathbf{k}_\gamma$ and, in the Madison system (z axis along $\mathbf{k}$, y axis along $\mathbf{k}\times\mathbf{k}_\gamma$), $R=(0,\theta,0)$ where $\theta=\cos^{-1}(\hat{\mathbf{k}}\cdot\hat{\mathbf{k}}_\gamma)$.

Viewed as a one-step process, the capture amplitude can be expressed, using the Wigner-Eckart theorem,[11] as a sum of terms involving matrix elements of the form

$$\langle\alpha;0^+|T_{LM}^\dagger(\pi)|^{2s+1}l_J;JM'\rangle=-(2L+1)^{-1/2}\delta_{LJ}\delta_{MM'}\langle\alpha\|T_L(\pi)\|^{2s+1}l_J\rangle\ , \quad (3)$$

where $|^{2s+1}l_J;JM'\rangle$ is a two-deuteron initial state with channel spin s, orbital angular momentum l, and total angular momentum J. Symmetry of the $d+d$-wave function also requires that $l+s$ is even. Thus, for $L\leq2$ the reaction can proceed only through the following transitions $\langle\alpha|E1|^3P_1\rangle$, $\langle\alpha|M1|^5D_1\rangle$, $\langle\alpha|E2|^5S_2\rangle$, $\langle\alpha|E2|^1D_2\rangle$, $\langle\alpha|E2|^5D_2\rangle$, $\langle\alpha|E2|^5G_2\rangle$, $\langle\alpha|M2|^3P_2\rangle$, and $\langle\alpha|M2|^3F_2\rangle$. These are listed in Table I.

We need to consider the explicit forms of the T_{LM}. These are[11]

$$T_{LM}(e)=\alpha_L^e\sum_i[Q_{LM}(\mathbf{r}_i)+Q_{LM}'(\mathbf{r}_i)]\ , \quad (4)$$

$$T_{LM}(m)=\alpha_L^m\sum_i[M_{LM}(\mathbf{r}_i)+M_{LM}'(\mathbf{r}_i)]\ , \quad (5)$$

for electric and magnetic transitions, respectively. They are sums over all nucleons of one-body operators, functions of the position $\mathbf{r}_i$ of nucleon i relative to the c.m. of the sys-

TABLE I. Tabulation, by increasing multipole order, of the coefficients $C_L^\pi(ls;l's')$ in the radial overlaps $\Delta_L^\pi(ls;l's')$ of Eq. (12).

	Multipole L	π	α_L^π	Transition $(^{2s+1}l_L \rightarrow {}^{2s'+1}l'_0)$	Coefficient $C_L^\pi(ls;l's')$	
$E1$	1	0	ik_γ	$^3P_1 \rightarrow {}^1S_0$	$k_\gamma\mu/(2\sqrt{4\pi})$	E
				$^3P_1 \rightarrow {}^5D_0$	$-k_\gamma\mu/(4\sqrt{8\pi})$	
$M1$	1	1	k_γ	$^5D_1 \rightarrow {}^5D_0$	$\sqrt{15}(\mu-\beta/2)/\sqrt{8\pi}$	H
$E2$	2	0	$-k_\gamma^2/(2\sqrt{3})$	$^1D_2 \rightarrow {}^1S_0$	$-\sqrt{3}e/(4\sqrt{4\pi})$	A
				$^5S_2 \rightarrow {}^5D_0$	$-\sqrt{3}e/(20\sqrt{4\pi})$	B
				$^5D_2 \rightarrow {}^5D_0$	$+\sqrt{3}e/(4\sqrt{14\pi})$	C
				$^5G_2 \rightarrow {}^5D_0$	$-9\sqrt{3}/(20\sqrt{14\pi})$	D
$M2$	2	1	$ik_\gamma^2/(2\sqrt{3})$	$^3P_2 \rightarrow {}^1S_0$	$-\sqrt{3}\mu/\sqrt{4\pi}$	F
				$^3P_2 \rightarrow {}^5D_0$	$-\sqrt{3}\mu/(10\sqrt{8\pi})$	
				$^3F_2 \rightarrow {}^5D_0$	$3\sqrt{21}\mu/(10\sqrt{4\pi})$	G

tem. The constants α_L^π are collected in Table I for the EL and ML transitions of interest here. Under the assumption that the nucleon-nucleon interaction is charge independent, and hence, that isospin is a good quantum number, both the initial and final states have total isospin $T=0$. The capture therefore proceeds only through the isoscalar components of these operators. In the long wavelength approximation, these are

$$Q_{LM}(\Delta T=0)=(e/2)r_i^L C_{LM}(\hat{\mathbf{r}}_i) \ , \tag{6}$$

$$Q'_{LM}(\Delta T=0)=-k_\gamma\mu/(L+1)\mathbf{l}_i[r_i^L C_{LM}(\hat{\mathbf{r}}_i)]\cdot\mathbf{s}_i \ , \tag{7}$$

$$M_{LM}(\Delta T=0)=\beta/(L+1)\nabla_i[r_i^L C_{LM}(\hat{\mathbf{r}}_i)]\cdot\mathbf{l}_i \ , \tag{8}$$

$$M'_{LM}(\Delta T=0)=\mu\nabla_i[r_i^L C_{LM}(\hat{\mathbf{r}}_i)]\cdot\mathbf{s}_i \ , \tag{9}$$

where ∇_i, $\mathbf{l}_i$, and $\mathbf{s}_i$ are gradient, orbital, and spin angular momentum operators for nucleon i, β in the nuclear magneton, and $\mu=(\mu_n+\mu_p)\beta$, μ_n, μ_p in nuclear magnetons, the sum of the neutron and proton magnetic moments. The C_{LM} are the normalized spherical harmonics.[11] In making the long wavelength approximation we assume $(k_\gamma r_i)^2$ is small compared with unity. This approximation should be reasonable in the present application, where the r_i are restricted by the finite extent of the ^{4}He wave function, and at the energy of interest $k_\gamma \approx 0.15$ fm^{-1}. Referring to the allowed one-step transitions, it is clear that in this long wavelength, $\Delta T=0$ limit, $\sum_i Q_{1M}=0$, and hence, that the $\langle\alpha|E1|{}^3P_1\rangle$ transition must proceed via the spin-term Q' of $E1$. No other terms vanish due to isospin considerations.

When calculating the EL, ML matrix elements we assume, as in previous work,[1-3] that the nucleon coordinates $\mathbf{r}_i$ in the T_{LM} are proportional to the vector $\rho\{=[(\mathbf{r}_3+\mathbf{r}_4)-(\mathbf{r}_1+\mathbf{r}_2)]/2\}$ joining the center of mass of deuteron 1 to deuteron 2. That is, the deuterons are "pointlike" for the purposes of estimating the transition operators. So, subsequent to operating with $\mathbf{l}_i$ in Eq. (7) and ∇_i in Eqs. (8) and (9) we set $\mathbf{r}_1=\mathbf{r}_2=-\rho/2$, $\mathbf{r}_3=\mathbf{r}_4=+\rho/2$. We also must set $\mathbf{l}_i=\mathbf{l}/4$ in Eq. (8), where $\mathbf{l}$ is the $d+d$ relative orbital angular momentum operator. With this "point-deuteron" approximation, and assuming that the internal wave functions of the deuterons are their dominant $^3S_1$ configurations, the structure of ^{4}He appears only through the two-deuteron-^{4}He overlap

$$\langle\rho;d_1^{\sigma_1}d_2^{\sigma_2}|\alpha;0^+\rangle=\tfrac{1}{2}\sum_{l'=0,2}(-1)^{\sigma_1}(l'M'S_2\sigma_2|S_1-\sigma_1)u_{l'}(\rho)Y_{l'M'}(\hat{\rho}) \ , \tag{10}$$

which is an admixture of $^1S_0$ and $^5D_0$ two-deuteron configurations. The $M2$ transitions, Table I, thus require a deuteron spin-flip (change in channel spin) and therefore only the M' term of $T_{LM}(m)$ can contribute.

To describe the initial state distortions we utilize the results of the one-channel resonating group model (RGM) calculations of Chwieroth, Tang, and Thompson,[12] which give a good account of low energy $d+d$ phenomena.[12,13] The channel wave functions of that analysis, which conserves both l and channel spin s, are not however explicitly J dependent and are thus denoted χ_{ls} with phase shifts δ_{ls}.

In this model (and the Madison coordinate system) all EL, ML matrix elements reduce to the following form:

$$\langle\alpha;0^+|T_{LM}^\dagger(\pi)|d_1^{\sigma_1}d_2^{\sigma_2};\mathbf{k}\rangle=\sum_{lsl's'}\alpha_L^\pi i^l(S_1\sigma_1S_2\sigma_2|sM)(l0sM|LM)\Delta_L^\pi(ls;l's') \ , \tag{11}$$

where the radial overlaps $\Delta_L^\pi(ls;l's')$ for transition from the $^{2s+1}l_L$ component of the initial state to the $^{2s'+1}l'_0$ component of ^{4}He are

$$\Delta_L^\pi(ls;l's')=C_L^\pi(ls;l's')\int d\rho\,\rho^{2+L-\pi}\chi_{ls}(\rho)u_{l'}(\rho) \ . \tag{12}$$

The coefficients C_L^π are given in Table I. For reference, the last column of the table assigns each amplitude a single letter identifier ($A-H$). Amplitudes $A-D$, for the $E2$ transition, are precisely those of Ref. 2 [but for an i^l phase factor, Eq. (11)]. As regards the description of the initial state, our treatment of amplitudes $A-D$ follows exactly the technique of Ref. 2. The required χ_{ls} (ls =02,20,22, 42) are calculated from separable potentials fitted to the RGM phase shifts of Chwieroth *et al.*[12] Only two additional ^{2s+1}l channels are introduced by the $E1$, $M1$, and $M2$ multipoles, χ_{11} in amplitudes E ($E1$) and F ($M2$), and χ_{31} in G ($M2$). Amplitude H ($M1$) uses χ_{22} which enters C ($E2$). The ^3F channel is very weakly distorted, with $\delta_{31}\approx 1°$; we thus set $\chi_{31}=4\pi j_3(k\rho)$. The ^3P channel, however, is strongly distorted with $\delta_{11}\approx 110.5°$. In this channel χ_{11} is calculated in an attractive spherical square well chosen to reproduce δ_{11}. To simulate the interaction of two extended deuterons ($\langle r^2\rangle^{1/2}\approx 2$ fm) we take a well of radius 4 fm, although calculations show little sensitivity to this choice.

The two-deuteron-^{4}He overlap functions $u_{l'}$, Eq. (10), have recently been the subject of two theoretical studies.[8,9] We will utilize the analysis of Schiavilla, Pandharipande, and Wiringa[9] whose ^{4}He wave function, unlike that used in Ref. 8, includes the effects of realistic three-nucleon forces and is in better agreement with the experimental ^{4}He g.s. energy. They tabulate the $u_{l'}$, in momentum space, for two realistic two-body interaction models, the Argonne and Illinois interactions, which yield ^{4}He D-state parameters $D_2(d,\alpha)$ (Ref. 14) of -0.16 fm^2 and -0.24 fm^2, respectively. Here the $u_{l'}$ are calculated in Woods-Saxon wells,

$$V_{dd}(\rho)=V_0/\{1+\exp[(\rho-\rho_0)/a_0]\} \ ,$$

with geometries (ρ_0 fm, a_0 fm) chosen to model, as closely as possible, the momentum space forms of Ref. 9 and depths adjusted to reproduce the $d-d$ separation energy. For the Argonne interaction the geometries are (2.11, 0.75) for $l'=0$ and (2.65, 0.9) for $l'=2$. These normalized $u_{l'}$ are now scaled to the tabulated values of Ref. 9 at low momenta.

We now consider the reaction observables. In the case of A_y, the largest contributions will arise due to $E2/E1$ [Im(E/A)] and $E2/M2$ [Im(F/A)] interference terms, in particular the indicated cross terms of E and F with the dominant $E2$ amplitude A. Both the $E1$ (E) and $M2$ (F) amplitudes are in fact dominated by the transition to the $^1S_0$ state of ^{4}He. In the present model, which contains no ^3P_J phase shift splitting, these large S-state contributions to the interference terms are in fact equal and have a $-\cos\theta/\sin(2\theta)$ angular distribution.

For A_{yy}, $E2/E2$ interference terms dominate, and explicitly[2]

$$A_{yy}\cong 4/\sqrt{7}\,\mathrm{Re}[C/A-5D/(12A)] \ . \qquad (13)$$

The important point is that, whereas all T_{2q} contain amplitude B, to first order $A_{yy}[=-(T_{20}+\sqrt{6}T_{22})/\sqrt{2}]$ is independent of B. Amplitudes A, C, and D are only weakly distorted, show very little sensitivity to the detailed short range behavior of the χ_{ls} (provided they are regular at the origin and have the same phase shift), and are thus expected to be well described by the model used. Amplitude B, on the other hand, is strongly distorted and should include (potentially large) contributions from the deuteron D state to the $\langle\alpha(^1S_0)|E2|^5S_2\rangle$ transition, leading to the dominant S state of ^{4}He. This contribution vanished upon making the "point deuteron" approximation for $E2$. These uncertainties in amplitude B are not present in the observable A_{yy}.

In Fig. 1 the solid curve shows the calculated A_y which result from $E1+M1+E2+M2$ transitions using the model detailed above. The dashed and dot-dashed curves show the results of the $E2+E1$ and $E2+M2$ calculations only. As stated above, these are essentially equal in the present model, except near 90° where other small interference terms contribute. In the presence of $E2$ alone the calculated A_y has modulus $\lesssim 0.01$ and is asymmetric about 90°. The $M1$ contributions are very small. The slight asymmetry seen in the angular distribution is the result of small cross terms of E and F with B and C. The agreement with the data is quite satisfactory, though there appears to be a small overestimation (25–30%) in the E and/or F amplitudes of the present calculation.

Figure 2 shows the calculated A_{yy}. The solid curve corresponds to the same $E1+M1+E2+M2$ calculation as shown for A_y, for the ^{4}He wave function of the Argonne interaction [$D_2(d,\alpha)=-0.16$ fm^2]. The dashed curve shows just the $E2$ contribution in this calculation. Clearly, and as expected, there being no Re(E/A) and Re(F/A) type cross terms in the tensor observables, the $E1$ and $M2$ contributions to A_{yy} are small. They are, however, responsible for the small asymmetry about 90° observed in the calculation and evident in the data. The dot-dashed curve is the full $E1+M1+E2+M2$ calculation with $D_2(d,\alpha)=-0.24$ fm^2 of the Illinois interaction. It appears that with only moderately improved A_{yy} data very useful limits could be placed upon $D_2(d,\alpha)$, which

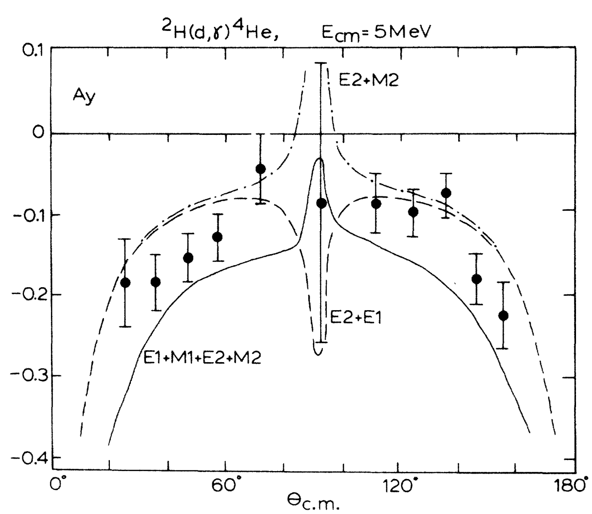

FIG. 1. Calculated vector analyzing power A_y for the ^{2}H(d,γ)^{4}He reaction at $E_{\mathrm{c.m.}}=5$ MeV obtained when including $E1+M1+E2+M2$ transitions (solid curve), $E1+E2$ transitions (dashed curve), and $M2+E2$ transitions (dot-dashed curve). The ^{4}He wave function is that of the Argonne interaction (Ref. 9). The data are from Ref. 7.

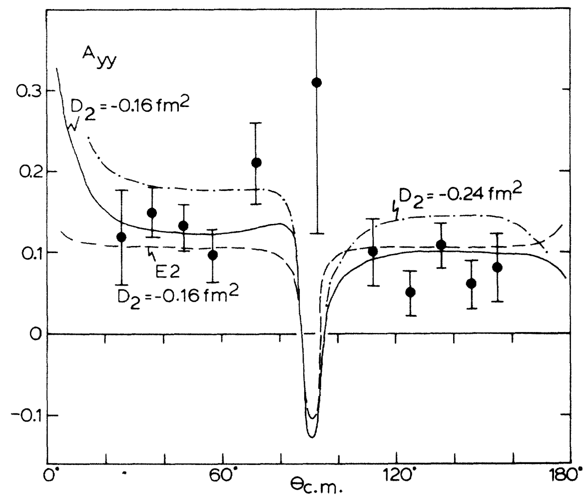

FIG. 2. Calculated tensor analyzing power A_{yy} for the ^{2}H(d,γ)^{4}He reaction at $E_{c.m.}=5$ MeV when including $E1+M1+E2+M2$ transitions (solid curve) and pure $E2$ transitions (dashed curve) for the ^{4}He wave function of the Argonne interaction (Ref. 9) with $D_2(d,\alpha)=-0.16$ fm^2. The dot-dashed curve is the full $E1+M1+E2+M2$ calculation with $D_2(d,\alpha)=-0.24$ fm^2. The data are from Ref. 7.

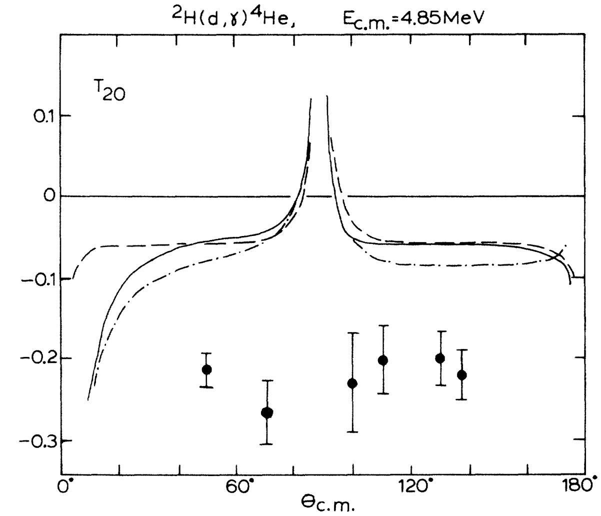

FIG. 3. Calculated tensor analyzing power T_{20} for the ^{2}H(d,γ)^{4}He reaction at $E_{c.m.}=4.85$ MeV. The curves have the same meaning as in Fig. 2 and the data are from Ref. 1.

could, in turn, allow one to reject certain otherwise realistic two-body nucleon-nucleon interaction models.

We remark in concluding that the present calculations, which reproduce A_y and A_{yy}, seriously underpredict the T_{20} data of Weller *et al.*,[1] as shown in Fig. 3. This indicates that the $E2$ amplitude B, which plays an important role in T_{20} (Ref. 2) but which is absent (in first order) from A_{yy}, is very poorly described by the present model, as was discussed earlier. Clearly a microscopic calculation of the $^5S_2$ $d-d$ channel would clarify this point. Alternatively, given that the $E2$ amplitudes A, C, and D are essentially model-independent with regard to the initial state distortions; together, the A_{yy} and T_{20} data are sufficient to determine B empirically [actually $\mathrm{Re}(B/A)$]. The B so determined not only reproduces T_{20} but produces a positive going peak in A_{yy} near 90° as required by the data (Fig. 2) but absent from the present calculation. We will report fully on the results of this investigation in a subsequent article.

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