$M1$ transitions between low-lying levels in ^{128}Xe and the proton-neutron interacting boson model

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The branching ratios of the collective levels in ^{128}Xe were discussed in the framework of the proton-neutron interacting boson model . It is shown that the experiment is only consistent with rather small $M1$ admixtures among the low-lying collective levels. These small $M1$ matrix elements imply strong constraints on the proton-neutron interacting boson model Hamiltonian.

The various collective models have focused hitherto mostly on the excitation energies and $E2$ transitions. With the discovery of the 3.08 MeV isovector (1^+) excitation in ^{156}Gd and of similar excitations in neighboring nuclei by the Darmstadt group,[1] there has been a surge of interest in a theoretical description of the magnetic properties of collective nuclei, viz., g factors and $M1$ transitions. Clearly, besides the experimental evidence on the isovector excitations there exists in the literature a large body of information on $M1$ transitions in the form of intensity branching ratios and less accurately in the form of $E2/M1$ mixing ratios of transitions. These data clearly involve both $E2$ and $M1$ matrix elements, and in order to obtain reliable information on the $M1$ matrix elements one should consider only those transitions which have strong $E2$ matrix elements in the collective model considered. In the following we will consider as an example the nucleus ^{128}Xe, because there exist particularly detailed data for this nucleus from recent experiments at Köln and Jyväskylä,[2] and also because this nucleus belongs to a large class of nuclei in the Xe-Ba region which have been shown to belong to the O(6) limit in the interacting boson model (IBM-1).[2] Thus we have a rather good understanding of the energies and of the $E2$ transitions of the collective levels of ^{128}Xe. In order to describe $M1$ transitions we will use the IBM-2 version of the interacting boson model which comprises both proton and neutron bosons, which has been introduced by Arima *et al.*[3] In the IBM-2 model one can describe $M1$ transitions with the operator $T(M1)$,

$$T(M1)=g_pL_p+g_nL_n =(N_p+N_n)^{-1}[(N_pg_p+N_ng_n)(L_p+L_n) +(g_p-g_n)(N_nL_p-N_pL_n)] , \quad (1)$$

in nuclear Bohr magneton units, where g_p and g_n are the g factors for the proton and neutron bosons. The values of these g factors of the bosons are not completely known at the moment. The experimental evidence comes from the $M1$ transitions to the mixed symmetry 1^+ state usually called the isovector state[1] which leads to $0.6 \leq g_p-g_n \leq 1$.[1] Similar values are also found from systematic fits to the g factors of the 2_g^+ states which have been performed by Wolf *et al.*[4] which yield $g_p=(0.63\pm0.04)$ and $g_n=(0.05\pm0.05)$.

In the following we will use the value $g_p-g_n=1$. This value is obtained if the magnetic properties of the proton boson and the neutron boson arise from orbital contributions only.

With this transition operator we can calculate the experimental energies, branching ratios, and mixing ratios when we have the IBM-2 wave functions, i.e., if we have a proper IBM-2 Hamiltonian.

In order to obtain a proper IBM-2 Hamiltonian we note that the absolute $M1$ transition intensities needed to reproduce the data are rather small. The $B(M1)$ values are of order of $\leq 10^{-2}\,\mu_N^2$ for the low-lying levels of the ground and gamma bands in ^{128}Xe. Thus, it seems reasonable to start with an IBM-2 Hamiltonian, which has no $M1$ transitions at all between the low-lying members of the ground and gamma bands. This Hamiltonian will subsequently be perturbed by a small additional term which induces the $M1$ transitions. We have chosen the following form of the IBM-2 Hamiltonian introduced originally by Dieperink *et al.*:[5]

$$H=\epsilon(n_{dp}+n_{dn})+k(Q_pQ_n)+k'(Q_p+Q_n)^2-\lambda M ,$$

where $n_{dr}=(d_r^\dagger\cdot\tilde{d}_r)$,

$$Q_r=(d_r^\dagger s_r+\tilde{d}_r s_r^\dagger)^{(2)}+\chi_r[d_r^\dagger\times\tilde{d}_r]^{(2)}, \quad r=\text{n or p} ,$$

and where

$$M=(s_p^\dagger d_n^\dagger-s_n^\dagger d_p^\dagger)(s_p\tilde{d}_n-s_n\tilde{d}_p) -2\sum_{k=1,3}[d_p^\dagger\times d_n^\dagger]^{(k)}[\tilde{d}_p\times\tilde{d}_n]^{(k)} . \quad (2)$$

In the following we will denote the Hamiltonian $H=H(k=0)$ by H_1 and the Hamiltonian $H=H(k'=0)$ by H_2. There are at least two limiting cases of H, which lead to vanishing $M1$ transitions between the low lying members of the ground and gamma bands, namely the Hamiltonians $H_1(\chi_p=\chi_n=0)$ and $H_2(\chi_p=\chi_n=0)$. By fitting energies and $E2$ transitions in ^{128}Xe from Table I within the O(6) limit the following parameters were found for H_1:

$$\epsilon=0,\quad k'=-0.07\text{ MeV} ,$$
$$\chi_p=\chi_n=0,\quad \lambda=0.183\text{ MeV} . \quad (3)$$

TABLE I. Relative gamma ray intensity values for ^{128}Xe. Calculation of the theoretical values for the Hamiltonian $H_1(\chi')$ [effective charge obtained from $B(E2;2_g\rightarrow 0_{1g})=1500\ e^2\text{fm}^4$ (Ref. 15); experimental intensities from Refs. 15–17; parameters as in Fig. 1 but for the variation of χ'].

I_i	I_f	Expt.	$\chi'=0.0$	Intensities $\chi'=-0.155$	$\chi'=-0.300$	$\chi'=-1.0$
3_1	4_1	17.8(3)	19.0	22.9	27.7	29.7
3_1	2_2	100	100	100	100	100
4_2	4_1	78.6(15)	53.4	53.4	56.2	65.9
4_2	2_2	100	100	100	100	100
5_1	4_2	13.8(11)	7.4	15.3	35.2	76.7
5_1	6_1	$\leq 4.1(4)$	0.9	4.1	11.7	23.8
5_1	3_1	100	100	100	100	100

Similarly, we found the following parameters for H_2:

$$\epsilon=0.28\text{ MeV},\quad k=-0.263\text{ MeV}\,,$$
$$\chi_p=\chi_n=0,\quad \lambda=0.1\text{ MeV}\,. \tag{4}$$

The value of λ has been chosen to set the 1^+ isovector level at $E_x(1^+)=2.3$ MeV. Presumably the true isovector level lies higher and thus a larger value of λ is needed, as discussed below. The value of the parameter λ of the Majorana force can be determined from the energy of the lowest (1^+) isovector state. The energy of these isovector excitations is not known in the barium-xenon region. We find a value $\lambda=0.22$ MeV for $E_x=(1^+)=3$ MeV and a value $\lambda=0.55$ MeV for $E_x(1^+)=5$ MeV. The fits to the energies with the two Hamiltonians are shown in Fig. 1; the fits to the branching ratios are shown in Tables I and II. These fits seem quite reasonable and they correspond in quality to fits which were obtained in the IBM-1 with the O(6) limit.[2] Actually the above IBM-2 Hamiltonians were obtained by using as a start the O(6) IBM-1 Hamiltonian[2] and by the help of a projection method which connects an IBM-2 Hamiltonian with an IBM-1 Hamiltonian for states of maximum F spin $F=F_{\max}$.[6] When we consider the fit to the energies in Fig. 1 it is rather good, although there are some deviations for the quasibeta band and in the staggering of the gamma band.

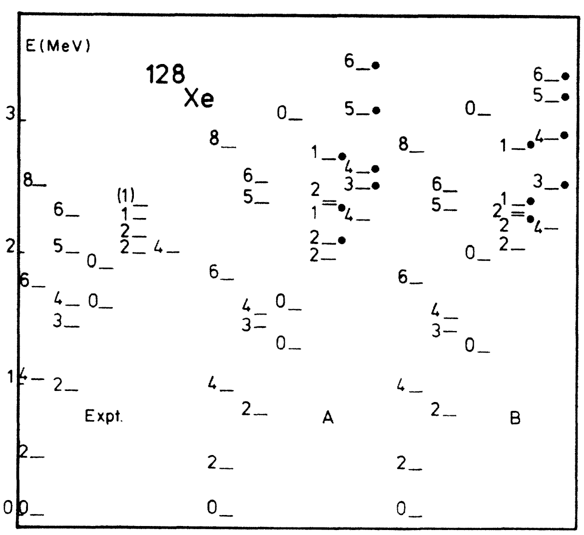

FIG. 1. Excitation energies: A: Hamiltonian $H_2(\chi')$; parameters $\epsilon=0.28$ MeV, $k=-0.263$ MeV, $\bar{\chi}=0$, $\chi'=0$, $\lambda=0.1$ MeV, and $k'=0.0$ MeV. B: Hamiltonian $H_1(\chi')$; parameters $\epsilon=0.0$ MeV, $k'=-0.07$ MeV, $\bar{\chi}=0$, $\chi'=0$, $\lambda=0.183$ MeV, and $k=0.0$ MeV. (The dots label these levels which are nearly pure F_m-1 states.)

Those deviations can be removed in the IBM-1 by the introduction of a three-body term which induces triaxiality. In a nucleus described by IBM-2 triaxiality appears if χ_p and χ_n are large and have opposite signs.

In order to induce $M1$ transitions we have chosen to vary the parameter $\chi'=\frac{1}{2}(\chi_p-\chi_n)$ while keeping the parameter $\bar{\chi}=\frac{1}{2}(\chi_p+\chi_n)=0$. We will now denote the Hamiltonians $H_1(\chi')$ and $H_2(\chi')$, respectively. The dependence of our results on the parameter χ' is shown in Tables I and II. If we look at the tables, we note that some of the branching ratios depend very strongly on the parameter χ' like, e.g., the branching ratio from the 5^+ state, whereas other branching ratios such as the branching ratios from the 3^+ state depend less strongly on χ'.

If we consider a deviation between experiment and theory of less than 25% satisfactory, then it is clear that a value $\chi'=-0.15$ for the IBM-2 Hamiltonian $H_1(\chi')$ with the $(Q_p+Q_n)^2$ term and $\chi'=-0.22$ for the usual IBM-2 Hamiltonian $H_2(\chi')$ with the Q_pQ_n term gives a fair overall agreement. One notes also that larger values of $\chi'=-1.0$ lead to strong deviations between the experiment and the theory. In Fig. 2 we show the dependence of the $M1$ matrix element of the $2_\gamma^+\rightarrow 2_g^+$ and the $5_\gamma^+\rightarrow 6_g^+$ transitions vs χ'. One notes that these matrix elements are linear for small values of χ', which shows that one actually can calculate those matrix elements by perturbation theory.

In ^{124,126}Xe we got the same results with one exception. The branching ratio $(4_\gamma^+\rightarrow 4_g^+)/(4_\gamma^+\rightarrow 2_\gamma^+)$ of ^{126}Xe is two times weaker than the predicted O(6) ratio without an $M1$ admixture. Thus this ratio cannot be used to extract the $M1$ strength. Comparing the experimental values of ^{124}Xe and ^{128}Xe we consider it possible that the $4_\gamma^+\rightarrow 2_g^+$ line is a doublet.

Important observables are also the g factors and $E2/M1$ mixing ratios. In the case of ^{128}Xe most of the $E2/M1$ mixing ratios are not well measured. Only the value for $2_\gamma^+\rightarrow 2_g^+$ is well known. Table III shows that we were able to reproduce them within experimental error

TABLE II. Relative gamma ray intensity values for ^{128}Xe. Calculation of the theoretical values for the Hamiltonian $H_2(\chi')$ [effective charge obtained from $B(E2;2_g \to 0_{1g}) = 1500\ e^2\text{fm}^4$ (Ref. 15); experimental intensities from Refs. 15–17; parameters as in Fig. 1 but for the variation of χ'].

			Intensities			
I_i	I_f	Expt.	$\chi'=0.0$	$\chi'=-0.22$	$\chi'=-0.300$	$\chi'=-1.0$
3_1	4_1	17.8(3)	19.0	21.9	23.9	28.6
3_1	2_2	100	100	100	100	100
4_2	4_1	78.6(15)	53.5	64.1	73.0	104.2
4_2	2_2	100	100	100	100	100
5_1	4_2	13.8(11)	7.4	15.4	21	42.8
5_1	6_1	$\leq 4.1(4)$	0.9	4.2	6	14.2
5_1	3_1	100	100	100	100	100

TABLE III. $E2/M1$ mixing ratios for the Hamiltonian $H_2(\chi')$. (Parameters as in Fig. 1 but with $\chi'=-0.22$.)

			$E2/M1$ mixing ratios		
I_i	I_f	Expt.	$\chi'=-0.220$	$\chi'=-0.300$	$\chi'=-1.000$
2_2	2_1	6.1 ± 0.5[b] $3.8^{+1.6}_{-0.8}$[a]	4.66	3.44	1.01
3_1	2_2	$6.3^{+3.2}_{-1.8}$ $-0.45^{+0.08}_{-0.05}$[c]	2.98	2.20	0.71
3_1	4_1	≥ 0.11[c]	1.88	1.38	0.49
4_2	4_1	$1.48^{+0.40}_{-0.29}$ $0.25^{+0.09}_{-0.09}$[c]	2.24	1.66	0.58
5_1	4_2		0.96	0.72	0.29
5_1	6_1		0.53	0.40	0.21

[a]J. Srebrny *et al.* (Ref. 7).
[b]I. Lange *et al.* (Ref. 10).
[c]L. Goettig *et al.* (Ref. 15).

TABLE IV. Calculated F-spin amplitudes for the Hamiltonian $H_1(\chi')$ (parameters as in Fig. 1 but with $\chi'=-0.15$.)

			Amplitudes	
I	E_{exc} (MeV)	F_m	F_m-1	F_m-2
2_1	0.443	−0.9995	0.0321	−0.0014
4_1	1.033	−0.9994	0.0343	−0.0014
2_2	0.969	−0.9982	0.0593	−0.0015
3_1	1.430	−0.9944	0.1059	−0.0021
4_2	1.603	−0.9986	0.0524	−0.0021
5_1	1.997	−0.9915	0.1301	−0.0034

TABLE V. Calculated F-spin amplitudes for the Hamiltonian $H_2(\chi')$ (parameters as in Fig. 1 but with $\chi'=-0.22$.)

			Amplitudes	
I	E_{exc} (MeV)	F_m	F_m-1	F_m-2
2_1	0.443	−0.9827	0.1313	−0.1306
4_1	1.033	−0.9819	0.1461	−0.1209
2_2	0.969	−0.9806	0.1547	−0.1207
3_1	1.430	−0.9786	0.1776	−0.1049
4_2	1.603	−0.9794	0.1727	−0.1043
5_1	1.997	−0.9746	0.2090	−0.0808

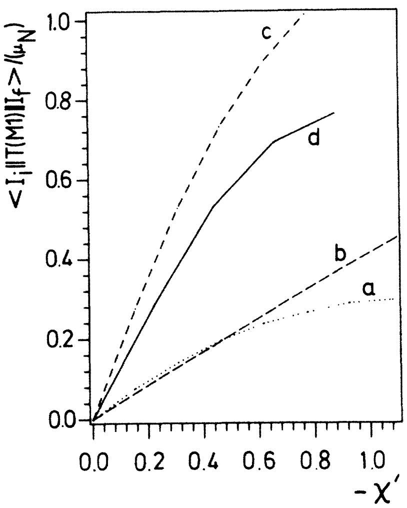

FIG. 2. $M1$ matrix element $\langle I_i||T(M1)||I_f\rangle$ plotted vs $\chi'=\frac{1}{2}(\chi_p-\chi_n)$. a—$I_i=2_2,I_f=2_1$, Hamiltonian $H_1(\chi')$; b—$I_i=2_2,I_f=2_1$, Hamiltonian $H_2(\chi')$; c—$I_i=5_1,I_f=6_1$, Hamiltonian $H_1(\chi')$; d—$I_i=5_1,I_f=6_1$, Hamiltonian $H_2(\chi')$.

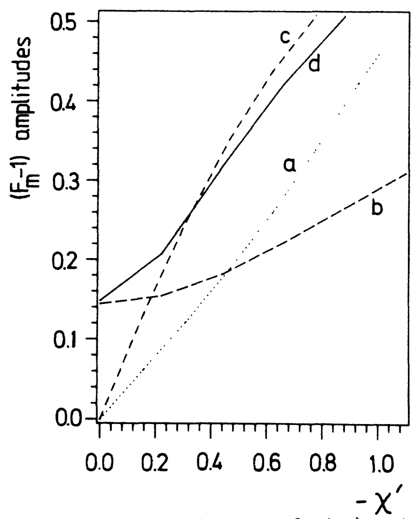

FIG. 3. (F_m-1) amplitudes of $|I\rangle$ plotted vs $\chi'=\frac{1}{2}(\chi_p-\chi_n)$. a—$I=2_2$, Hamiltonian $H_1(\chi')$; b—$I=2_2$, Hamiltonian $H_2(\chi')$; c—$I=5_1$, Hamiltonian $H_1(\chi')$; d—$I=5_1$, Hamiltonian $H_2(\chi')$.

bars. As a result of recent Coulomb excitation experiments[7] $B(E2;2_\gamma\rightarrow 2_g)=1800(20)$ $e^2\text{fm}^4$ and $B(M1;2_\gamma\rightarrow 2_g)$ have been determined. The resulting δ, which is also in agreement with our calculation, is given in Table III. Moreover, there is good agreement between the calculated $B(E2;2_\gamma\rightarrow 2_g)=1959$ $e^2\text{fm}^2$ (Hamiltonian H_1) and the experimental one. The g factor of the first 2^+ state is $g=0.31(3)$. The Hamiltonian H_1 gives $g=0.33$ and the Hamiltonian H_2 gives $g=0.24$; the optimum value of χ' mentioned above has been used.

A particularly simple interpretation of our results can be given for the Hamiltonian H_1 $(\chi'=0)$ with the $(Q_p+Q_n)^2$ term by means of the F-spin concept. This concept was introduced by Arima *et al.*[8] F spin is the isospin for a system of proton and neutron bosons. Since $T(M1)$ is an F-spin vector, only transitions with $\Delta F=0,\pm 1$ are allowed. For operators of the form $N_nT_p-N_pT_n$ the matrix elements between states with maximal F spin $F=N/2$ are zero.[9] Only the second part of the $M1$ transition operator which has this form will contribute to transition matrix elements, because the first part [formula (1)] is proportional to the angular momentum. Thus $M1$ transitions are rigorously forbidden between states with maximum F spin. If we use the parameters $\overline{\chi}=0$ and $\chi'=0$, we find that the Hamiltonian $H_1(\chi')$ with the $(Q_p+Q_n)^2$ term is a pure F-scalar operator and leads to wave functions with a pure F spin. Thus, indeed, we find no $M1$ transitions between $F=F_{\max}$ states for $\chi'=0$. If we now introduce $\chi'\neq 0$, then it will induce $F=(F_m-1)$ components into the IBM-2 wave functions. The existence of the $F=F_m-1$ components is a necessary condition for the appearance of $M1$ transitions. (See Fig. 3.)

It is clearly of interest to investigate the purity of the wave functions of the two Hamiltonians in terms of F spin. In order to do so we have modified the IBM-2 program NPBOS by adding a subroutine which decomposes the wave functions according to their F spin [program NPBOS-F (Ref. 11)]. The calculations were repeated with a new version of the IBM-2 code by Otsuka which runs on

TABLE VI. Calculated F-spin amplitudes for the Hamiltonian $H_2(\chi'=0),N_p=3,N_n=4$.

		Amplitudes			
I	E (MeV)	$F=\frac{7}{2}$	$F=\frac{5}{2}$	$F=\frac{3}{2}$	$F=\frac{1}{2}$
0_1	0.000	−0.985	0.048	−0.162	0.019
0_2	1.328	−0.990	0.068	−0.162	0.012
0_3	2.112	−0.909	0.036	−0.205	0.066
2_1	0.371	−0.986	0.056	−0.154	0.018
2_2	0.814	−0.988	0.063	−0.141	0.015
2_3	2.147	−0.993	0.069	−0.099	0.007
3_1	1.488	−0.990	0.068	−0.123	0.012
3_2	2.654	−0.000	0.980	−0.092	0.178
4_1	0.989	−0.988	0.063	−0.141	0.016
4_2	1.578	−0.990	0.068	−0.123	0.012

a VAX, and complete numerical agreement was obtained.

In Tables IV and V we decompose the amplitudes of the two Hamiltonians for which we obtained the best fit. One notes that the F-spin breaking is quite small for the $(Q_p+Q_n)^2$ type of Hamiltonian, $H_1(\mathcal{X}')$, whereas the situation is more complicated for the usual Q_pQ_n IBM-2 Hamiltonian $H_2(\mathcal{X}')$. In this case the Hamiltonian is a combination of an F-spin scalar and an F-spin tensor.[6] Thus the states considered have an F-spin mixing even at $\mathcal{X}'=0$, although the $M1$ transition strengths vanish. We want to note that the components with $F<F_m-2$ are generally much smaller than the admixtures with $F=F_m-1$ or F_m-2 due to the tensor part of the Hamiltonian. Table VI shows the F-spin amplitudes in a case with $N_p=3$, $N_n=4$. The possible values of F spin are now $F=\frac{7}{2},\frac{5}{2},\frac{3}{2},\frac{1}{2}$. Note that with one exception all amplitudes with $(F_m-3)=\frac{1}{2}$ are rather small compared to F_m-1 and F_m-2. The exception is the second 3^+ state. The largest component of this state is F_m-1 with an amplitude of 98.0%. Therefore it is not surprising that the F_m-3 component is 17.8%.

Due to the F-spin selection rule, in the case of the Hamiltonian $H_1(\mathcal{X}')$ the $M1$ transitions between the gamma and the ground state bands can occur only in the presence of F-spin mixing. To this extent the $M1$ transitions indicate F-spin mixing. However, the situation is complex, as shown by the properties of the Hamiltonian H_2, as we noted above.

This unexpected behavior is due to the symmetry properties of Q_pQ_n. This operator can be decomposed into the Casimir operators of $O_p(6)$,$O_n(6)$,$O_p(5)$,$O_n(5)$,$O_{p+n}(6)$, $O_{p+n}(5)$. In this sense it belongs to an intermediate case between the two dynamic symmetries, which have the $O_{p+n}(5)$ subgroup in common

$$O_p(6)\times O_n(6)\supset \left\{\begin{matrix} O_p(5)\times O_n(5) \\ O_{p+n}(6) \end{matrix}\right\} \supset O_{p+n}(5)\supset O_{p+n}(3)$$

The eigenstates of H_1 are representations of $O_{p+n}(5)$ characterized by two labels (τ_1,τ_2); the lowest states considered by us are of the $(\tau,0)$ type. It has been shown[14] that an $M1$ transition between states $(\tau,0)$ and $(\tau',0)$ is forbidden if $\tau\neq\tau'$. Besides, transitions with $\Delta L=\pm1$ inside a $(\tau,0)$ multiplet are also forbidden.

An IBA-2 calculation of the Xe, Ba, and Pt nuclei was recently carried out by Novoselsky and Talmi.[12] An excellent fit of energies and a quite good agreement for the $B(E2)$'s have been achieved. On the other hand, $M1$ strengths calculated with their wave functions and with $(g_p-g_n)=1$ would be too large as compared to the data, due to strong F-spin mixing in their wave functions. This disagreement could be removed either by reducing the value of (g_p-g_n) or by introducing higher-order terms in the $M1$ operator. This question is open to further investigation.

Finally, we have to add a word of caution. Our results depend on two parameters which are only partly known, namely on the value of (g_p-g_n) and on the value of the parameter λ of the Majorana force. It seems that both of these parameters are, however, known to within a factor 2 and that our results thus are valid. In this case the $\mathcal{X}'$ has to be changed and can be larger if λ is larger.

Summing up, we have shown that one can obtain a satisfactory IBM-2 fit of the energies and of the branching ratios for the low-lying collective levels in ^{128}Xe. We have shown in particular that the fit to the branching ratios which includes $M1$ transitions is a very sensitive way of determining the proton neutron degree of freedom of the IBM-2 Hamiltonian. Caution in this conclusion is needed, however, in view of special selection rules for $M1$ transitions, which are independent of the F spin.

It would be interesting to generalize our results to nuclei with other symmetries as, e.g., SU(3) nuclei. An example of such work has recently been performed by Lipas and Helimäki.

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