# Applications of the $\pi$NN bound-state problem: The deuteron and the 4,4 resonance

Humberto Garcilazo*
*Kernforschungszentrum Karlsruhe, Institut für Kernphysik and Institut für Experimentelle Kernphysik der Universität Karlsruhe, D-7500 Karlsruhe, Federal Republic of Germany*

Leopold Mathelitsch
*Institut für Theoretische Physik, Universität Graz, A-8010 Graz, Austria*

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The relativistic three-body equations for the bound-state problem proposed by the authors are solved for the $\pi$NN system, in particular for the states with total isospin zero and two. In the case of isospin zero, only one bound state is found which corresponds precisely to the deuteron and whose wave function is quite similar to those of phenomenological models. In the case of isospin two, the most favorable configuration to possess a bound state is with total angular momentum two and positive space parity. There is experimental evidence, however, that the state is unbound; that means that it will appear as a resonance of spin two and isospin two which we call the 4,4 resonance since it is the analog to the well-known 3,3 resonance in the $\pi$N system which has spin $\frac{3}{2}$ and isospin $\frac{3}{2}$.

## I. INTRODUCTION

In this paper we are going to study some applications of the $\pi$NN bound-state problem with the generalization to resonances; that means that we investigate not only the poles of the $S$ matrix in the negative real axis which give evidence of true bound states, but study also the poles in the complex plane which correspond to resonances.

It has been, in our opinion, one of the great successes of the $\pi$NN Faddeev theory to explain the observed $^1D_2$ and $^3F_3$ resonances of proton-proton scattering by means of conventional $\pi$N and NN dynamics.[1-4] These so-called dibaryon resonances were originally thought to be of quark origin and have aroused a great deal of interest among nuclear physicists. Since these isospin-one nucleon-nucleon resonances have been studied extensively,[1-4] we will not investigate them again in this paper, but we will concentrate on the less well-known channels with isospin zero and two.

The isospin zero channel is interesting in its own right, since there the only known bound state with baryon number two, the deuteron, is located. Thus, this channel provides us with a unique opportunity to test the reliability of the $\pi$NN dynamics contained in the Faddeev theory. The isospin-two channel recently received a considerable amount of attention[5-7] after it was pointed out by the authors of Refs. 8 and 9 that possible bound state solutions exist in this channel. The bound system would consist of a negative pion and two neutrons (or, its isobaric analog, of a positive pion and two protons). Such a possibility is a very attractive one, since, as the bound state can only decay by weak interactions, it means that it will be stable with a lifetime comparable to that of the charged pion. Even if it turns out that the system is unbound, as the result is in some of our solutions, this would mean that the state lies in the continuum; therefore, it should still be possible to observe it as a three-body resonance. Since this isospin-two resonance cannot couple directly to the nucleon-nucleon system, which can have only isospin zero or one, the observation is only possible via reactions like $\pi^- + d \to \pi^+ + \pi^- + n + n$ or $\pi^- + t \to p + \pi^- + n + n$, where in the first case the $\pi^+$ should be detected and in the second one the proton. Recent experiments on the first of these reactions[6,7] have found no evidence of a $\pi^-$nn bound state; the data, however, show a resonantlike behavior at 256 MeV incident pion energy, which can be interpreted as a resonance of mass equal to 2056 MeV, that is 38 MeV above the $\pi$NN threshold. Thus, if this result is confirmed by other experiments, it is very likely that one has observed the 4,4 resonance predicted by our three-body theory.

In Sec. II, we review the relativistic three-body equations for the bound-state problem including the treatment of angular momentum, spin, and isospin. Section III gives a brief description of the two-body interactions for all the necessary two-body channels. In Sec. IV we apply this formalism to study the three-body channels with isospin zero, placing special emphasis on the deuteron, and in Sec. V we study the three-body channels with isospin two, especially with respect to the 4,4 resonance. Finally, we summarize our results in Sec. VI.

## II. RELATIVISTIC THREE-BODY EQUATIONS

In Ref. 9 we pointed out that the relativistic version of the Faddeev equations proposed by Aaron, Amado, and Young[10] (AAY) possesses spurious bound-state solutions; therefore, it is not suitable for applications in the bound-state problem. In the same work, however, we modified the AAY theory in such a way that no spurious bound-state solutions appear and that the equations are well behaved as a function of the energy throughout the bound-state region $-\infty \leq \sqrt{S} \leq m_i+m_j+m_k$. The modified equations for the bound-state problem are in the case of spinless particles, of $S$-wave two-body interactions, and for a state with total angular momentum $J$ as follows:

$$F_i^J(k_i;S)=\sum_{j\neq i}\int_0^\infty \frac{k_j^2dk_j}{2\omega_j(k_j)}B_{ij}^J(k_i,k_j;S)\frac{1}{D_j(k_j;\sqrt{S})}F_j^J(k_j;S) , \tag{1}$$

where $k_i$ and $k_j$ are the magnitudes of the three-momenta of particles $i$ and $j$ in the three-body c.m. frame. The driving term $B_{ij}^J$ reads as

$$B_{ij}^J(k_i,k_j;S)=\int_{-1}^{1} d\cos\chi P_J(\cos\chi)\frac{g_i(p_i)g_j(p_j)}{2\omega_k(|\mathbf{k}_i+\mathbf{k}_j|)}\frac{\omega_i(k_i)+\omega_j(k_j)+\omega_k(|\mathbf{k}_i+\mathbf{k}_j|)}{S-[\omega_i(k_i)+\omega_j(k_j)+\omega_k(|\mathbf{k}_i+\mathbf{k}_j|)]^2} , \tag{2}$$

with

$$\cos\chi=\hat{\mathbf{k}}_i\cdot\hat{\mathbf{k}}_j , \tag{3}$$

$$\omega_i(k)=(m_i^2+k^2)^{1/2} , \tag{4}$$

$$p_i^2=\frac{1}{4s_i}[s_i-(m_j+m_k)^2][s_i-(m_j-m_k)^2] , \tag{5}$$

$$p_j^2=\frac{1}{4s_j}[s_j-(m_i+m_k)^2][s_j-(m_i-m_k)^2] , \tag{6}$$

$$s_i=[\omega_j(k_j)+\omega_k(|\mathbf{k}_i+\mathbf{k}_j|)]^2-k_i^2 , \tag{7}$$

$$s_j=[\omega_i(k_i)+\omega_k(|\mathbf{k}_i+\mathbf{k}_j|)]^2-k_j^2 . \tag{8}$$

The functions $g_i(p_i)$ and $g_j(p_j)$ are the form factors of the $S$-wave separable potentials

$$V_i(p_i,p_i')=\gamma_i g_i(p_i)g_i(p_i') , \tag{9}$$

and the propagator $1/D_j(k_j;\sqrt{S})$ appearing in Eq. (1) is given by

$$D_j(k_j;\sqrt{S})=\frac{1}{\gamma_j}-\int_0^\infty \frac{p_j^2dp_j}{4\omega_i(p_j)\omega_k(p_j)}\frac{\omega_i(p_j)+\omega_k(p_j)}{\{k_j^2+[\omega_i(p_j)+\omega_k(p_j)]^2\}^{1/2}}\frac{g_j^2(p_j)}{W_j(k_j;\sqrt{S})-\{k_j^2+[\omega_i(p_j)+\omega_k(p_j)]^2\}^{1/2}} , \tag{10}$$

with

$$W_j(k_j;\sqrt{S})=\sqrt{S}-\omega_j(k_j) . \tag{11}$$

In the rest frame of the pair $ik$ we have $k_j=0$, and Eq. (10) becomes

$$D_j(0;\sqrt{S})=\frac{1}{\gamma_j}-\int_0^\infty \frac{p_j^2dp_j}{4\omega_i(p_j)\omega_k(p_j)}\frac{g_j^2(p_j)}{W_j(0;\sqrt{S})-\omega_i(p_j)-\omega_k(p_j)} , \tag{12}$$

which is the solution of the two-body Kadyshevski equation[11] for the separable potential of Eq. (9). In the original theory of Aaron, Amado, and Young, on the other hand, the two-body amplitudes were obtained from the solutions of the two-body Blankenbecler-Sugar equation.[12]

We can straightforwardly generalize this three-body theory to the case of particles with spin and interactions with arbitrary angular momentum using Wick's three-body helicity formalism.[13–15] Thus, the partial-wave decomposed three-body equations obtained by the application of the Wick formalism are

$$F_i^{\alpha_i}(k_i;S)=\sum_{j\neq i}\sum_{\alpha_j}\int_0^\infty \frac{k_j^2dk_j}{2\omega_j(k_j)}B_{ij}^{\alpha_i,\alpha_j}(k_i,k_j;S)\frac{1}{D_j^{\alpha_j}(k_j;\sqrt{S})}F_j^{\alpha_j}(k_j;S) . \tag{13}$$

The discrete quantum numbers $\alpha_i$ in Eq. (13) are

$$\alpha_i=\{JT,l_is_ij_it_i,m_i\nu_i\} , \tag{14}$$

where $J$ and $T$ are the total angular momentum and total isospin of the three-body system; $l_i$, $s_i$, $j_i$, and $t_i$ are the orbital angular momentum, spin, total angular momentum, and isospin of the pair $jk$; while $m_i$ is the helicity of the pair $jk$ and $\nu_i$ the helicity of particle $i$.

The driving terms $B_{ij}^{\alpha_i,\alpha_j}$ in Eq. (13) are now given by

$$B_{ij}^{\alpha_i,\alpha_j}(k_i,k_j;S)=\int_{-1}^{1} d\cos\chi\, A_{ij}^{\alpha_i,\alpha_j}(k_i,k_j,\cos\chi)\frac{g_i^{\alpha_i}(p_i)g_j^{\alpha_j}(p_j)}{2\omega_k(|\mathbf{k}_i+\mathbf{k}_j|)}\frac{\omega_i(k_i)+\omega_j(k_j)+\omega_k(|\mathbf{k}_i+\mathbf{k}_j|)}{S-[\omega_i(k_i)+\omega_j(k_j)+\omega_k(|\mathbf{k}_i+\mathbf{k}_j|)]^2}\ , \tag{15}$$

where the function $A_{ij}^{\alpha_i,\alpha_j}$, that replaced the Legendre polynomial $P_J$ in Eq. (2), is given by

$$A_{ij}^{\alpha_i,\alpha_j}(k_i,k_j,\cos\chi)=(-)^{t_j+\tau_j-T}(2t_i+1)^{1/2}(2t_j+1)^{1/2}W(\tau_j\tau_kT\tau_i;t_it_j)$$
$$\times\sum_{\lambda_j\lambda_k}\sum_{\mu_k\mu_i} b_{\lambda_j\lambda_k}^{l_is_ij_i} b_{\mu_k\mu_i}^{l_js_jj_j}(-)^{\sigma_j-\nu_j+\sigma_k+\mu_k} d^{J}_{m_j-\nu_j,m_i-\nu_i}(\chi)d^{j_i}_{m_i,\lambda_j-\lambda_k}(\theta_i)$$
$$\times d^{j_j}_{m_j,\mu_k-\mu_i}(\theta_j)d^{\sigma_i}_{\nu_i,\mu_i}(\beta_i)d^{\sigma_j}_{\nu_j,\lambda_j}(\beta_j)d^{\sigma_k}_{\mu_k,\lambda_k}(\rho_k)\ , \tag{16}$$

where $W$ is a Racah coefficient, and[16]

$$b_{\lambda_j\lambda_k}^{l_is_ij_i}=\left[\frac{2l_i+1}{2j_i+1}\right]^{1/2} C_{0,\lambda_j-\lambda_k}^{l_is_ij_i} C_{\lambda_j,-\lambda_k}^{\sigma_j\sigma_ks_i}\ . \tag{17}$$

$\tau_i,\tau_j,\tau_k$ and $\sigma_i,\sigma_j,\sigma_k$ are the isospins and spins of the three particles, and the arguments of the rotation matrices $\chi$, $\theta_i$, $\theta_j$, $\beta_i$, $\beta_j$, and $\rho_k$ are the angles of the Wick triangle.[13–15] The angular momentum coefficient [Eq. (16)] satisfies the parity relation

$$A_{ij}^{-\alpha_i,-\alpha_j}(k_i,k_j,\cos\chi)=(-)^{\eta_i+\eta_j}A_{ij}^{\alpha_i,\alpha_j}(k_i,k_j,\cos\chi)\ , \tag{18}$$

with

$$\eta_i=l_i-j_i+\sigma_j+\sigma_k\ , \tag{19}$$

and with the definition

$$-\alpha_i\equiv\{JT,l_is_ij_it_i,-m_i-\nu_i\}\ . \tag{20}$$

Using the parity relation of Eq. (18), the integral equations [Eq. (13)] can be decoupled into two sets which correspond to the states of positive and negative parity.[14,15] Since two of the particles are identical in the case of the $\pi$NN system, the integral equations can be further reduced following the same steps as for the nonrelativistic equation.[17] Thus, if we take particle 1 to be the pion and particles 2 and 3 the two nucleons, we obtain the final set of equations

$$F_P^{\gamma_2}(k_2;S)=\sum_{\gamma_2'}\int_0^\infty \frac{k_2'^2dk_2'}{2\omega_2(k_2')}B_P^{\gamma_2,\gamma_2'}(k_2,k_2';S)\frac{1}{D_2^{\gamma_2'}(k_2';\sqrt{S})}F_P^{\gamma_2'}(k_2';S)\ , \tag{21}$$

where $P$ is the parity quantum number that can take the values $P=\pm1$. The new discrete quantum numbers $\gamma_2$ are

$$\gamma_2=\{JT,l_2s_2j_2t_2;m_2\tfrac{1}{2}\}\ , \tag{22}$$

where we have fixed the helicity of the spectator 2 as $\nu_2=\frac{1}{2}$, so that

$$-\gamma_2\equiv\{JT,l_2s_2j_2t_2;-m_2-\tfrac{1}{2}\}\ . \tag{23}$$

The new driving terms are

$$B_P^{\gamma_2,\gamma_2'}(k_2,k_2';S)=-(-)^{(1/2)+t_2+l_2}[B_{23}^{\gamma_2,\gamma_2'}(k_2,k_2';S)+P(-)^{\eta_2'}B_{23}^{\gamma_2,-\gamma_2'}(k_2,k_2';S)]$$
$$+2\sum_{\alpha_1}\int\frac{k_1^2dk_1}{2\omega_1(k_1)}B_{21}^{\gamma_2,\alpha_1}(k_2,k_1;S)\frac{1}{D_1^{\alpha_1}(k_1;\sqrt{S})}$$
$$\times[B_{12}^{\alpha_1,\gamma_2'}(k_1,k_2';S)+P(-)^{\eta_2'}B_{12}^{\alpha_1,-\gamma_2'}(k_1,k_2';S)]\ . \tag{24}$$

The integral equations (21) depend only on the variables with index 2 in which the pion and one of the nucleons are in a state with quantum numbers $l_2,s_2,j_2,t_2$, while $k_2$ is the momentum of the spectator nucleon. The dependence on the variables with index 1 has been eliminated, since in Eq. (24) we have summed over the discrete quantum numbers $\alpha_1$ and integrated over the momentum $k_1$. The three-body integral equations (21) are allowed to go into states corresponding to a nucleon-nucleon configuration when the discrete quantum numbers of the pion-nucleon subsystem are

$$\{l_2 s_2 j_2 t_2\} = \{1\tfrac{1}{2}\tfrac{1}{2}\tfrac{1}{2}\} ,$$

which are the quantum numbers of the pion-nucleon $P_{11}$ channel. Thus, in this case the helicity $m_2$ must be restricted to have only those values which are allowed by the Pauli principle in the case of two nucleons.

Finally, the $P=\pm 1$ solutions of Eq. (21) correspond to the usual states of positive and negative space parity according to the prescription[15]

$$\text{space parity} = P(-)^{J+1} . \tag{25}$$

If we take into account additionally the intrinsic parity of the pion, then the total parity of the solutions of Eq. (21), characterized by the quantum numbers $J$ and $P$, is

$$\text{total parity} = P(-)^{J} . \tag{26}$$

## III. THE TWO-BODY INPUT

Since the pion has isospin 1 and the two nucleons isospin $\frac{1}{2}$, a $\pi$NN state with total isospin $T=0$ requires the nucleon-nucleon subsystem to be in a state with isospin $t_i=1$ and the pion-nucleon subsystem to be in a state with isospin $t_i=\frac{1}{2}$. Thus, only the $^1S_0$, $^3P_0$, $^3P_1$, $^3P_2$, and $^1D_2$ partial waves will contribute out of all nucleon-nucleon channels with angular momentum $j_i \le 2$. Similarly, of the six $S$- and $P$-wave pion-nucleon channels, only the $S_{11}$, $P_{11}$, and $P_{13}$ will enter. In the case of states with total isospin $T=2$ the nucleon-nucleon subsystem again can only be in states with isospin $t_i=1$, while the pion-nucleon subsystem can now only be in a state with isospin $t_i=\frac{3}{2}$; in this case the contributing pion-nucleon channels are $S_{31}$, $P_{31}$, and $P_{33}$.

We have found that the solutions with total isospin $T=0$ are dominated by the pion-nucleon $P_{11}$ channel, and that the solutions with total isospin $T=2$ are governed by the pion-nucleon $P_{33}$ channel. Thus, we have constructed a family of two-body interactions with different ranges for these two channels. In the $P_{11}$ case we have used rank-one separable potentials [Eq. (9)], which give rise to a pole in the two-body propagator $1/D_j^{\alpha_j}(0;\sqrt{S})$, given by Eq. (12), at an energy $W_j(0,\sqrt{S})=M$, the mass of the nucleon; the corresponding residue gives the pion-nucleon coupling constant. We used the form factors

$$g_i(p_i)=\frac{p_i}{p_i^2+\beta^2}\left[1+\frac{\alpha p_i^2}{p_i^2+\beta^2}\right] , \tag{27}$$

and considered for the range $\beta$ the values $\beta=2$, 3, 4, and 5 $\text{fm}^{-1}$ as shown in Table I. Although these potentials reproduce the position of the nucleon pole and have the right pion-nucleon coupling constant, they do not fit well the $P_{11}$ phase shift which changes the sign around 170 MeV laboratory energy. To reproduce this fact, at least a rank-two potential or a potential with an explicit energy dependence would be required. However, in our bound-state calculations the two-body subenergy is always restricted to $W_i(q_i,\sqrt{S})<M$, which is far below the pion-nucleon threshold $M+\mu$; that means the behavior of the interaction in the region above threshold is not so important.

TABLE I. Parameters of the four models with $\beta=2$, 3, 4, and 5 $\text{fm}^{-1}$ in Eq. (27) for the pion-nucleon $P_{11}$ channel.

| $\alpha$ ($\text{fm}^0$) | $\gamma$ ($\text{fm}^{-2}$) | $\beta$ ($\text{fm}^{-1}$) |
|---|---|---|
| 2.28 | −16.752 43 | 2.0 |
| 1.80 | −42.273 36 | 3.0 |
| 1.25 | −99.295 54 | 4.0 |
| 0.78 | −211.270 1 | 5.0 |

For the pion-nucleon $P_{33}$ channel we have used also rank-one separable potentials with form factors given by

$$g(p_i)=p_i\left[\frac{A}{\alpha^2+p_i^2}+\frac{1-A}{\beta^2+p_i^2}\right] , \tag{28}$$

where we took the values 2000, 4000, 7000, and 10 000 MeV/$c$ for the second range $\beta$. These potentials fit the $P_{33}$ scattering volume and phase shift from 0 to 350 MeV, and the parameters are given in Ref. 9. We should mention that the requirement of an energy-independent potential has the consequence that one needs two ranges in Eq. (28), one very small ($\alpha \sim 200$ MeV/$c$) and the other one very large. In order to fit the two-body data the actual magnitude of the second range is not essentially provided it is large enough ($\beta > 1000$ MeV/$c$).[18] Thus, one of the most interesting features of our three body results will be that they are very sensitive to the value of this second range; so that by going into the three-body system one may be able to learn something about the off-shell behavior of the two-body subsystem.

The separable potentials for the remaining $\pi$N and NN channels have been constructed already in Ref. 9.

## IV. RESULTS FOR THE ISOSPIN ZERO CHANNEL

We will study the $T=0$ states with total angular momentum $J \le 2$ by calculating the Fredholm determinant of the relativistic Faddeev equations (21) below the nucleon-nucleon threshold, that is for invariant energies $\sqrt{S}<2M$, where $M$ is the mass of the nucleon. Of the six possible states $J^P$ corresponding to $J=0,1,2$ and $P=\pm 1$ only three are allowed to also be nucleon-nucleon states in accordance with the Pauli principle. These are the states $J^P=1^-$, $1^+$, and $2^+$, corresponding to the $^3S_1$-$^3D_1$, $^1P_1$, and $^3D_2$ nucleon-nucleon channels. Thus, it is important to calculate the Fredholm determinant of these three channels in order to compare it with features known from low energy nucleon-nucleon phase shift analyses. For example, in the case of the $^3S_1$-$^3D_1$ channel with the deuteron bound state we expect that the Fredholm determinant will pass through zero. In the case of the $^1D_1$ state the low-energy phase shift is negative; that means that the channel is repulsive and we expect the Fredholm determinant to be positive and larger than one. Finally, in the case of the $^3D_2$ state the low-energy phase shift is positive, i.e., the channel is attractive, although not attractive enough to produce a bound state; therefore, we expect that the Fredholm determinant will be positive but smaller than one.

In Fig. 1 we show the Fredholm determinant of the $J^P=1^-$, $1^+$, and $2^+$ channels as a function of the energy. The curves labeled 2 to 5 correspond to the models of the $P_{11}$ channel with ranges $\beta=2$–$5\ \text{fm}^{-1}$ given in Table I. In case (a) only the pion-nucleon $P_{11}$ channel, in case (b) all the pion-nucleon channels, and in case (c) all pion-nucleon and all nucleon-nucleon channels were taken into account. As we see, there is quite good agreement with the qualitative behavior expected from the low-energy phase shifts, that is, the ${}^3S_1$-${}^3D_1$ channel can have a bound state, the ${}^1P_1$ channel is repulsive, and the ${}^3D_2$ channel is weakly attractive. Although the ${}^3S_1$-${}^3D_1$ Fredholm determinant can go through zero, it can also have values larger than one, which means that this channel has both attractive and repulsive components. The trend of the results for the different models of the $P_{11}$ channel consists of a decrease of the attraction in the three-body states while increasing the range $\beta$.

We see that using as input only the $P_{11}$ channel (model 2) produces a bound state in the ${}^3S_1$-${}^3D_1$ channel of about 2.5 MeV. Including the other pion-nucleon channels decreases the binding energy to about 0.7 MeV, while the additional inclusion of the nucleon-nucleon channels raises the binding energy again to about 1.5 MeV. In order to get a bound state with approximately the correct deuteron binding energy, we constructed a fifth model of the $P_{11}$ channel ($\alpha=1.35$, $\beta=1.855\ \text{fm}^{-1}$, $\gamma=-14.395\,172\,9\ \text{fm}^{-2}$), for which the full calculation gives a binding energy of 2.22 MeV.

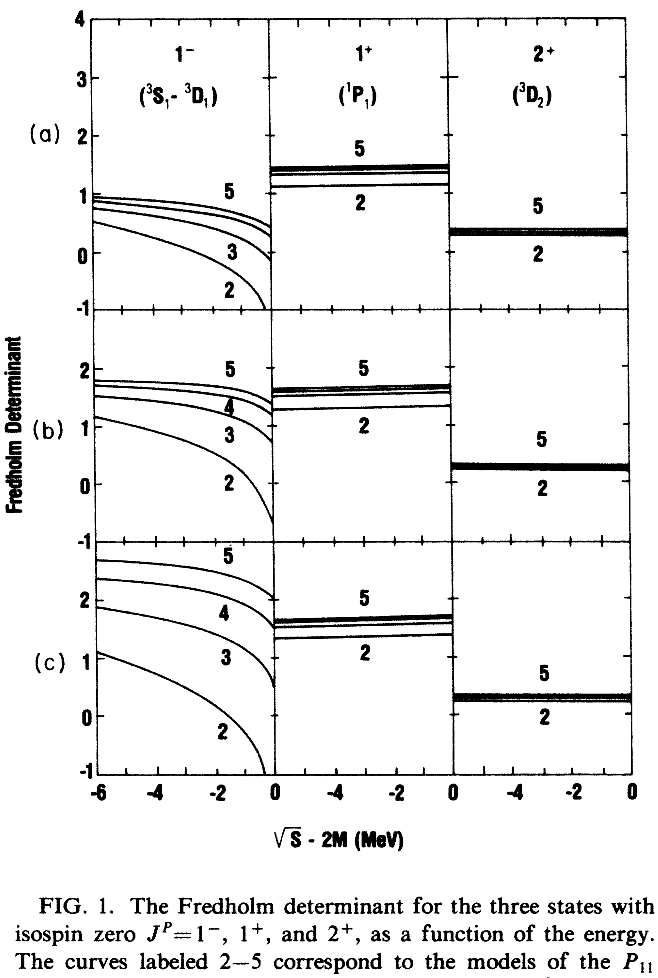


FIG. 1. The Fredholm determinant for the three states with isospin zero $J^P=1^-$, $1^+$, and $2^+$, as a function of the energy. The curves labeled 2–5 correspond to the models of the $P_{11}$ channel given by Eq. (27) with ranges $\beta=2$–$5\ \text{fm}^{-1}$. (a) Only the pion-nucleon $P_{11}$ channel, (b) all the pion-nucleon channels, and (c) all the pion-nucleon and nucleon-nucleon channels are taken into account in the calculation.

We have also calculated the wave function of this bound state. If we consider the solutions $F^{\gamma_2}_{-1}$ of Eq. (21) with the quantum numbers

$$\gamma_2=\{10,1\tfrac{1}{2}\tfrac{1}{2}\tfrac{1}{2},m_2\tfrac{1}{2}\}\equiv m_2 \tag{29}$$

(that means that the pion-nucleon subsystem is in the $P_{11}$ channel), we can construct the two linear combinations of the amplitudes $F^{m_2}_{-1}$:

$$\psi_0(k_2)=\frac{1}{D_2^{\gamma_2}(k_2;\sqrt{S}\,)}\left[\sqrt{2/3}F^{1/2}_{-1}(k_2)+\frac{1}{\sqrt{3}}F^{-1/2}_{-1}(k_2)\right], \tag{30}$$

$$\psi_2(k_2)=\frac{1}{D_2^{\gamma_2}(k_2;\sqrt{S}\,)}\left[\frac{1}{\sqrt{3}}F^{1/2}_{-1}(k_2)-\sqrt{2/3}F^{-1/2}_{-1}(k_2)\right], \tag{31}$$

which correspond to the usual $S$- and $D$-wave components of the deuteron wave function.

We show this "three-body" deuteron wave function in Fig. 2, where we compare it with the deuteron wave function of the Paris potential,[19] and with the Moravcsik-Gartenhaus wave function.[20] As we see, there is quite good agreement between the three-body wave function and the two phenomenological ones, particularly with regard to the change of sign of the $S$-state wave function at $k\sim 2\ \text{fm}^{-1}$; this feature is well known for all models having a strong repulsive core at short distances. In the tradi-

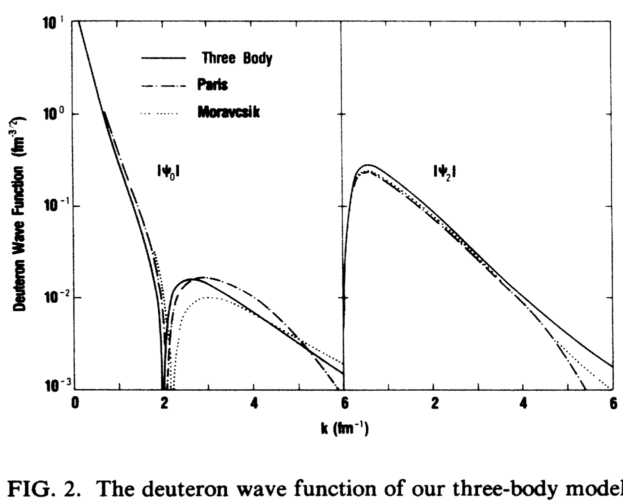


FIG. 2. The deuteron wave function of our three-body model as compared with the wave functions of the Paris potential (Ref. 19) and the Moravcsik-Gartenhaus (Ref. 20) wave function.

tional picture of the nucleon-nucleon interaction, this strong short range repulsion is associated with the exchange of heavy mesons, such as $\rho$ and $\omega$. In our picture, on the other hand, the deuteron is considered as a $\pi$NN bound state and the short range repulsion is generated by the three-body dynamics alone. The $D$-state probability of the three-body wave function is 8.9%, which is somewhat higher than the values obtained from the Paris (5.8%) and Moravcsik-Gartenhaus wave functions (6.7%).

The components of the bound-state wave function, shown in Fig. 2, have the quantum numbers corresponding to the pion-nucleon subsystem being in the $P_{11}$ channel. However, as one can see from Eq. (21), there are other components of the wave function which do not have a counterpart within the standard nucleon-nucleon description. These are the components when the pion-nucleon subsystem is in the $S_{11}$ or in the $P_{13}$ channel. We show these components of the deuteron wave function in Fig. 3 for the various projections of the helicity $m_2$ that are allowed by angular momentum selection rules. Comparing with Fig. 2 we see that these components are roughly one order of magnitude smaller than the standard ones. The total probability of the wave functions is 0.11% (0.10%) when the pion-nucleon subsystem is in the $S_{11}$ ($P_{13}$) channel.

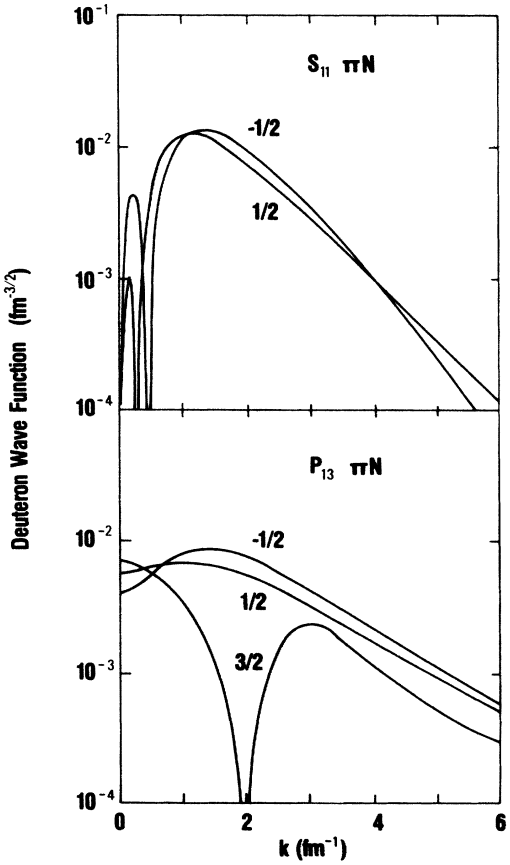


FIG. 3. Additional components of the "three-body" deuteron wave function which correspond to the $\pi$N subsystem being in the $S_{11}$ or $P_{13}$ channels. The curves are labeled according to the helicity $m_2$ of the $\pi$N subsystem.

Finally, we should mention the states $J^P=0^+,0^-,2^-$, which are not connected to the nucleon-nucleon system and which are very weak. The $J^P=0^+$ state is weakly attractive (the Fredholm determinant is slightly less than one) and the other two are weakly repulsive (the Fredholm determinant is slightly larger than one).

## V. RESULTS FOR THE ISOSPIN TWO CHANNEL

In Fig. 4 we show the results for states with isospin two, namely for $J^P=0^-$, $1^-$, and $2^-$. The states $0^+$, $1^+$, and $2^+$ are repulsive and therefore irrelevant with respect to the question of possible bound states or resonances. The curves of case (a) were calculated using as input only the pion-nucleon $P_{33}$ channel; those of case (b) including in addition the remaining pion-nucleon channels; and those of case (c) with both the pion-nucleon and nucleon-nucleon channels. The models $A$, $B$, $C$, and $D$ correspond to the form factor [Eq. (28)] of the $P_{33}$ channel with ranges $\beta=2000$, 4000, 7000, and 10 000 MeV/$c$ that were constructed in Ref. 9.

As we see, the effect of the range in this case is the op-

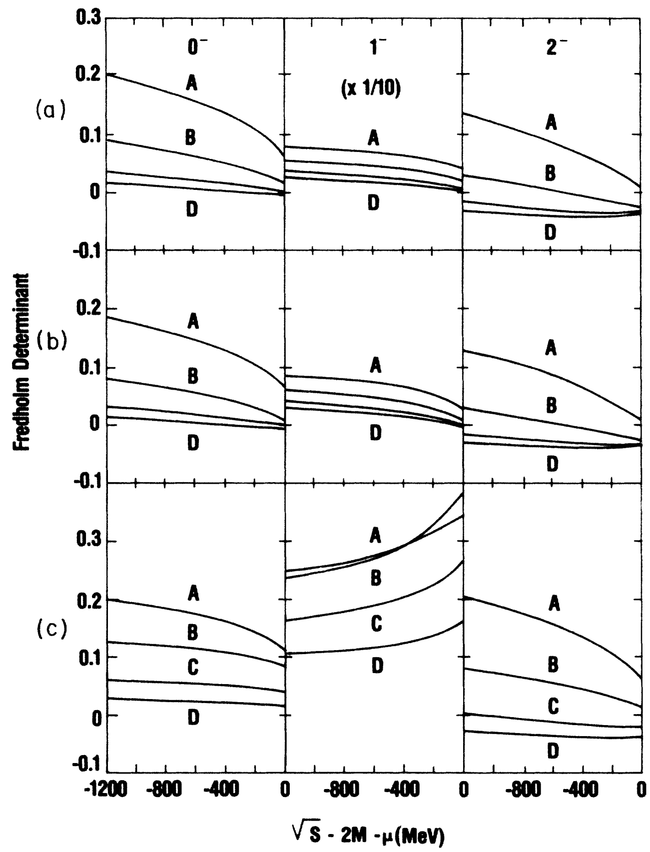


FIG. 4. The Fredholm determinant for the three states with isospin two $J^P=0^-$, $1^-$, and $2^-$, as a function of the energy. The curves labeled $A-D$ correspond to the models of the $P_{33}$ channel given by Eq. (28) with ranges $\beta=2000$, 4000, 7000, and 10 000 MeV/$c$, respectively. (a) Only the pion-nucleon $P_{33}$ channel, (b) all the pion-nucleon channels, and (c) all the pion-nucleon and nucleon-nucleon channels are taken into account in the calculation.

posite from that with isospin zero, since now a larger range $\beta$ leads always to increased attraction. We see that for the inclusion of only the pion-nucleon interaction the three channels can have a bound state for sufficiently large values of $\beta$, although the channel $2^-$ is the one with the strongest attraction. If we include in addition the nucleon-nucleon interaction, the bound states disappear in the $0^-$ and $1^-$ states and only that one in the $2^-$ state remains.

We can also see from Fig. 4 that in the case of the $0^-$ and $2^-$ states the pion-nucleon $P_{33}$ channel is the dominant two-body interaction, while the other pion-nucleon and nucleon-nucleon channels have only a very small effect. These two states have positive space parity [see Eq. (25)], and since the pion will be mainly in a $P$ wave with respect to the two nucleons, the nucleons themselves must also be in a relative $P$ wave (spin triplet); therefore the strong nucleon-nucleon singlet $^1S_0$ channel will not contribute. Thus, in the case of the $2^-$ state the $P$-wave pion will have its magnetic projection parallel to the spins of the two nucleons. That means that for each pion-nucleon pair the $\pi$N magnetic projection will be $\frac{3}{2}$ and the pion will form the delta 3,3 resonance with full strength with each of the nucleons, since in addition the isospins are also parallel. This configuration gives rise to a three-body resonance of spin 2 and isospin 2, which we can call the 4,4 resonance, in analogy to the 3,3 resonance of the two-body $\pi$N system that has spin $\frac{3}{2}$ and isospin $\frac{3}{2}$.

As we see in Fig. 4 models $A$ and $B$ give rise to a resonance while models $C$ and $D$ give rise to bound states. Since the existence of bound states has essentially been ruled out by two recent experiments,[5–7] this also rules out the models [Eq. (28)] of the pion-nucleon $P_{33}$ channel with range $\beta \gtrsim 5000$ MeV/$c$. However, if we accept the experimental evidence found by Lichtenstadt *et al.*,[7] namely the existence of a resonance at 38 MeV above the $\pi$NN threshold, we can find a model which gives rise to a 4,4 resonance of such a mass with quantum numbers of spin and isospin equal to two and positive space parity.

In order to search for the position of the resonance, we parametrized the Fredholm determinant of the $J^P=2^-$ channel as

$$\frac{1}{D(\sqrt{S})}=\frac{A}{\sqrt{S}-\sqrt{S_R}+\frac{i}{2}\Gamma(k)}+C \tag{32}$$

with

$$\sqrt{S}=(m_\pi^2+k^2)^{1/2}+(4m_{\rm N}^2+k^2)^{1/2} \tag{33}$$

and

$$\Gamma(k)=Bk^3 \tag{34}$$

corresponding to a $P$-wave resonance. The parameters $A$, $S_R$, $B$, and $C$ were fitted to the values of the Fredholm determinant below threshold, that is for $k=i\kappa$ with $\kappa$ real and positive. Since the resonance, however, must be located in the second Riemann sheet (defined by $k=-i\kappa$), we have to go from the physical to the unphysical sheet. This can be achieved within our simple parametrization [Eqs. (32)–(34)] by replacing $A\rightarrow A$, $\sqrt{S_R}\rightarrow\sqrt{S_R}$, $C\rightarrow C$, and $B\rightarrow -B$. Applying this procedure to our model of the $P_{33}$ channel [Eq. (28)], we reproduced the correct experimental mass with the parameters $\alpha=182.126$ MeV/$c$, $\beta=4830$ MeV/$c$, $\gamma=-0.1640368$ fm$^2$, and $A=0.625$. The result for the width, however, is between 2 and 3 MeV, which is much smaller than reported by Lichtenstadt *et al.*[7]

## VI. SUMMARY

We have described a relativistic three-body theory of the bound-state problem and applied it to the case of the $\pi$NN system with total isospin zero and two. Our results for isospin zero are in qualitative agreement with the behavior expected from the low-energy nucleon-nucleon phase shifts. In the case of the deuteron bound state, we find that the wave function of our "three-body" deuteron is very similar to those of other models.

In the case of isospin two, we find that the state with the most attraction has total angular momentum two and positive space parity. Since—according to experimental searches—the state is not bound, it will appear as a 4,4 resonance. Taking into account these experimental results, we can set a limit to the range of the pion-nucleon $P_{33}$ form factor within our model which must be less than 5000 MeV/$c$. More general, if the experimental finding of Ref. 7 will be confirmed by other groups, our calculations have shown the appointement of the resonance to a specific channel ($J^P=2^-$) and a strong connection of the position of the resonance to off-shell features of the underlying pion-nucleon interaction in the $P_{33}$ channel.

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*On leave from Escuela Superior de Física y Matemáticas, Instituto Politécnico Nacional, México 14 D.F., México.

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